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# Hurst-Kolmogorov dynamics in paleoclimate reconstructions

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## 1. Abstract

Our understanding of the climate system is linked to our knowledge of past climate, mainly due to the role played by the variability of climate on long scales in shaping our perception of the climate system behaviour. Therefore, paleoclimate data are an important source of information, whose study should be accompanied by that of the related uncertainties, determined by an appropriate statistical framework. The Hurst-Kolmogorov dynamics, also known as long-term persistence, has been detected in many long hydroclimatic time series and is stochastically equivalent to a simple scaling behaviour of climate variability over time scale. We demonstrate that this behaviour is dominant in paleoclimate reconstructions of Holocene and Pleistocene (0.01 – 3 million years) and has a serious impact on the estimation of uncertainty. The comparison between the classical statistical framework and the Hurst-Kolmogorov approach results in significant differences, particularly in the implied uncertainty.

## 2. Motivation

- Studies of paleoclimate records are often used to reduce uncertainties in our prediction of the future.
- During this process (paleo-) climatologists make extensive use of the well-known of classical statistical procedures. However, there is evidence that several hypotheses underlying the classical statistics (e.g. independence in time) are not valid in climatic processes. The latter seem to be more consistent with a Hurst-Kolmogorov behaviour.
- The comparison between the statistical estimators of classical statistics (CS) and Hurst-Kolmogorov statistics (HKS) has shown that the variance (and therefore system uncertainty) is underestimated by the CS (Koutsoyiannis & Montanari, 2006). This difference becomes quite serious as the Hurst coefficient, which is the index of long-term persistence intensity, approaches 1.
- Temperature reconstructions of length of the order of 1000 years exhibit this kind of behaviour as it was demonstrated by Koutsoyiannis (2003); here we investigate its presence in longer time scales and examine the sequences in the implied uncertainty.

## 3. Hurst-Kolmogorov dynamics

- Hurst-Kolmogorov dynamics, else known as scaling behaviour, has been discovered by Hurst (1951), while investigating the flow of the Nile River and other natural processes, although Kolmogorov (1940) had studied the stochastic process that describes this behaviour 10 years earlier.
- Since then, the Hurst-Kolmogorov dynamics, has been detected in many hydro-climatic time series, such as global mean temperatures (Bloomfield, 1992), flows of the River Nile (Eltahir, 1996), monthly and daily inflows of Lake Maggiore, Italy (Montanari et al., 1997) and indexes of North Atlantic Oscillation (Stephenson et al., 2000).
- It was demonstrated that the multiple-scale variability of a time series can explain the Hurst phenomenon (Koutsoyiannis, 2002).
- This behaviour can be described in terms of invariance properties of the time series aggregated (or averaged) on different time scales, and therefore quantified through the so-called Hurst exponent (or coefficient), $H$, which is described by the relationship:

$$\sigma^{(k)} = k^{H-1}\sigma$$

  where $\sigma^{(k)}$ is the standard deviation for time scale $k$ ($\sigma = \sigma^{(1)}$). In a white noise series (random) series $H$ is 0.5, whereas in real-world time series $H$ is usually greater.
- To establish a pragmatic representation of hydrometerological processes, it is very useful to examine observed long time series. Small sample sizes often hide the real behaviour of the studied system and therefore paleoclimate reconstructions are suitable for this type of analysis.

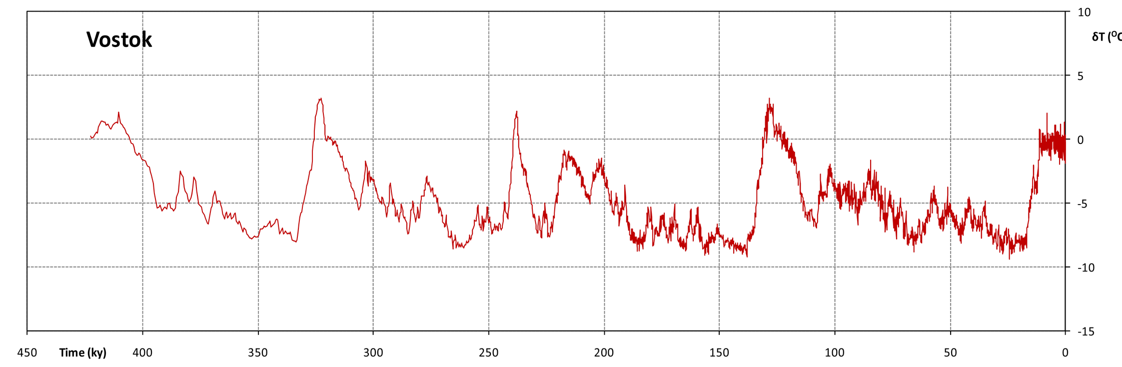
## 4. The sample data

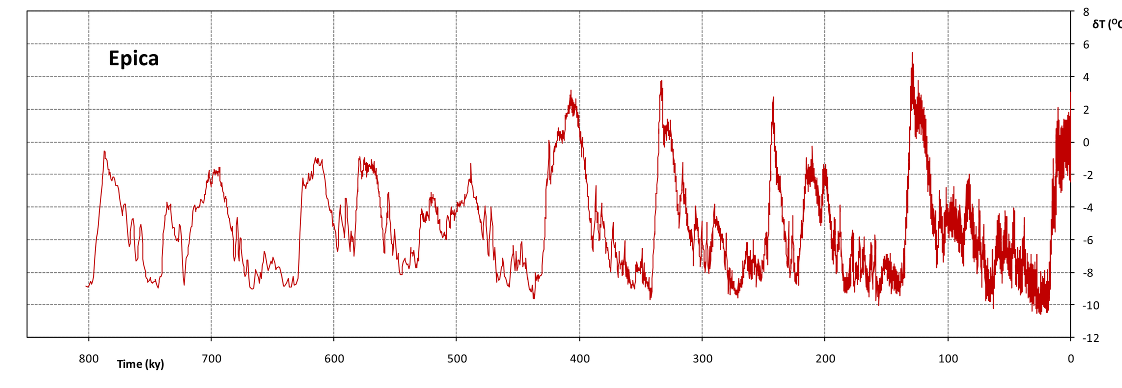
| Data series | Vostok | Epica | Hu06 | Li05 |
|---|---|---|---|---|
| Recon. Length (ky) | 423 | 802 | 2580 | 5320 |
| Proxy Type | Ice core | Ice core | Sediment | Sediment |
| Location | Antarctica | Antarctica | N. & Eq. Atlantic | Atlantic |
| Variable | δT (1 ky) | δT (1 ky) | δ18O | δ18O |
| Reference | Petit et al. (2001) | Jouzel et al. (2007) | Huybers (2006) | Lisiecki & Raymo (2005) |

Ice core from the EPICA deep drilling at Dome C, Antarctica. *(Laurent Augustin, CNRS/LGGE, Grenoble, France)*

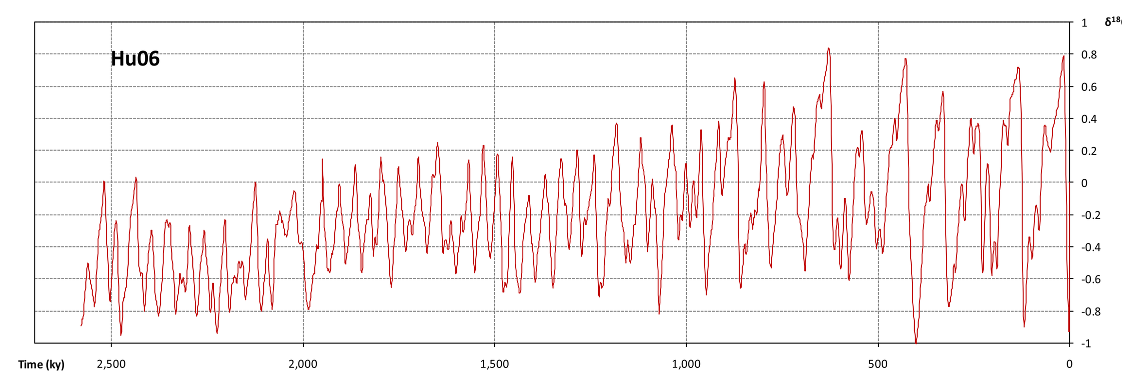
- Ice-core proxy temperature is calculated using a deuterium/ temperature gradient of 9‰/°C after accounting for the isotopic change of sea-water.
- Temperature difference (δT) is the difference between the mean recent time value (1 ky) and the paleoclimate value.
- Sediment proxies that were used in Hu06 and Li05 reconstructions where both of planktic and benthic type.
- All reconstructions were aggregated to longer time scales, because of the variable temporal resolution of the samples, except Hu06, which was available on a constant time step.

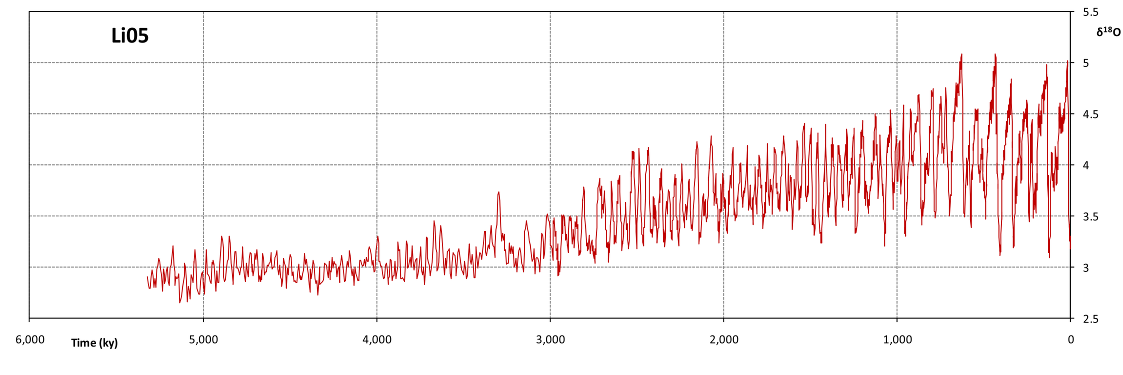
## 5. Ice-core reconstructions



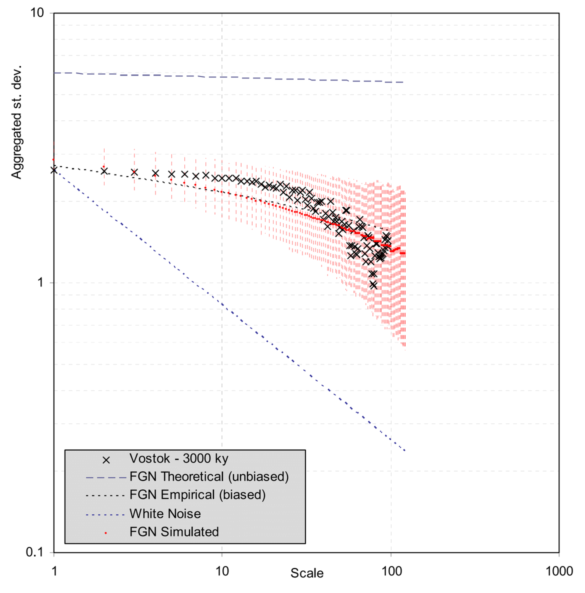



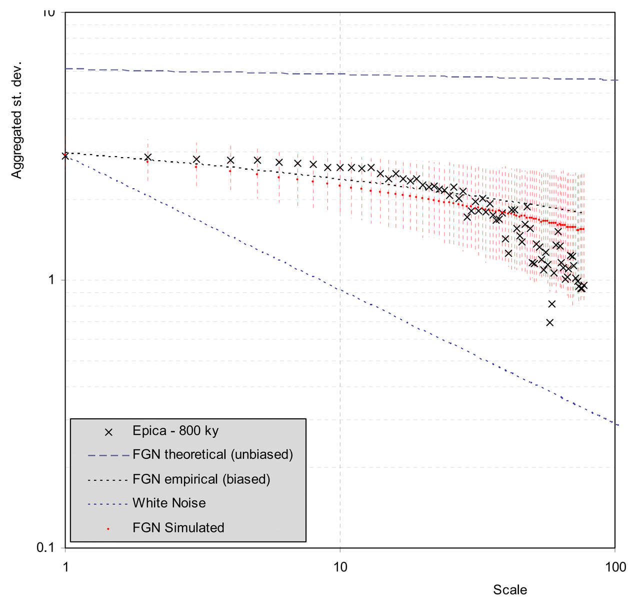

## 6. Sediment reconstructions



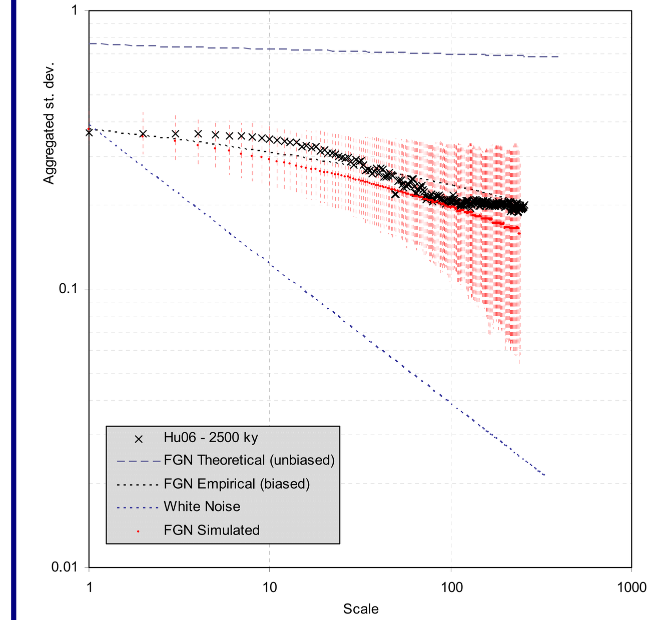

## 7. Long term persistence in ice-core reconstructions

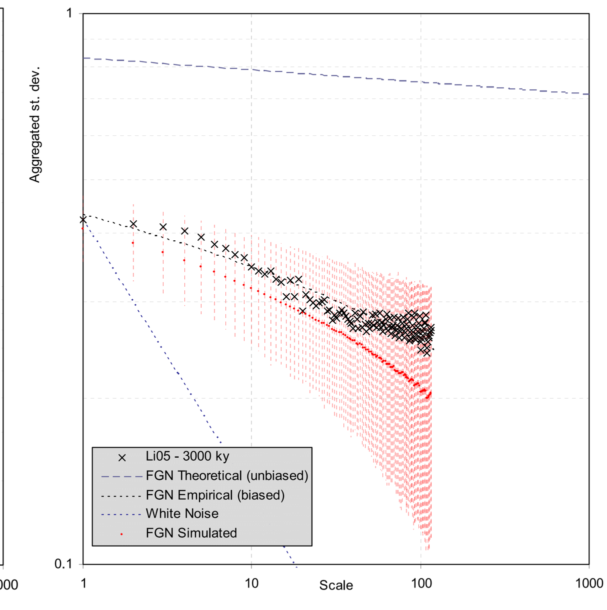



The classical statistical estimator of standard deviation is biased for HK processes (FGN Theoretical), and therefore the unbiased estimator, proposed by Koutsoyiannis (2003) was used (FGN Empirical).

## 8. Long term persistence in sediment reconstructions




The bias and 95% prediction limits were determined by Monte Carlo simulation (200 simulations with length equal to each paleoclimate series).

The synthetic time series, used in the simulation, were created by multiple time-scale fluctuation method, proposed by Koutsoyiannis (2002).

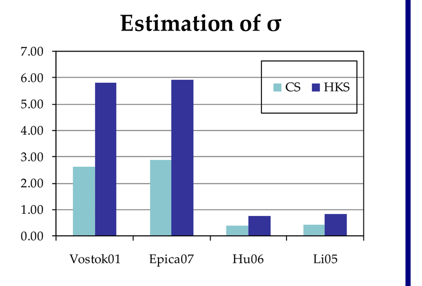
## 9. Comparison of Hurst-Kolmogorov statistics (HKS) and classical statistics (CS)

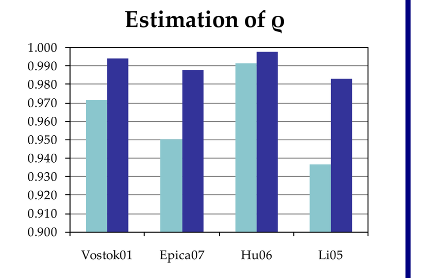
| True values → | Mean, $\mu$ | Standard deviation, $\sigma$ | Autocorrelation $\rho_l$ for lag $l$ |
|---|---|---|---|
| Standard estimator | $\underline{\bar{x}} := \frac{1}{n}\sum_{i=1}^{n} \underline{x}_i$ | $\underline{s} := \sqrt{\frac{1}{n-1}} \sqrt{\sum_{i=1}^{n} (\underline{x}_i - \underline{\bar{x}})^2}$ | $\underline{r}_l := \frac{1}{(n-1)\underline{s}^2}\sum_{i=1}^{n-l} (\underline{x}_i - \underline{\bar{x}})(\underline{x}_{i+l} - \underline{\bar{x}})$ |
| Relative bias of estimation, CS | 0 | $\approx 0$ | $\approx 0$ |
| Relative bias of estimation, HKS | 0 | $\approx \sqrt{1 - \frac{1}{n'}} - 1 \approx -\frac{1}{2n'}$ | $\approx -\frac{1/\rho_l - 1}{n' - 1}$ |
| Standard deviation of estimator, CS | $\frac{\sigma}{\sqrt{n}}$ | $\approx \frac{\sigma}{\sqrt{2(n-1)}}$ | |
| Standard deviation of estimator, HKS | $\frac{\sigma}{\sqrt{n'}}$ | $\approx \frac{\sigma\sqrt{(0.1\, n + 0.8)^{\lambda(H)}(1 - n^{2H-2})}}{\sqrt{2(n-1)}}$ where $\lambda(H) := 0.088\,(4H^2 - 1)^2$ | |

Note: $n' := n^{2-2H}$ is the "equivalent" or "effective" sample size: a sample with size $n'$ in CS results in the same uncertainty of the mean as a sample with size $n$ in HKS (Koutsoyiannis, 2003; Koutsoyiannis & Montanari, 2006).

## 10. Hurst-Kolmogorov statistics and paleoclimate reconstructions

| Data series | Vostok | Epica | Hu06 | Li05 |
|---|---|---|---|---|
| Recons. Length (ky) | 400 | 800 | 2500 | 3000 |
| Aggregated scale (ky) | 0.5 | 1.0 | 1.0 | 2.5 |
| Sample size, $n$ | 800 | 800 | 2500 | 1200 |
| Hurst coefficient, $H$ | 0.98 | 0.98 | 0.98 | 0.98 |
| St. deviation, $\sigma$ (CS) | 2.61 | 2.90 | 0.39 | 0.42 |
| St. deviation, $\sigma$ (HKS) | 5.80 | 5.93 | 0.75 | 0.82 |
| Autocorrelation, $\rho$ (CS) | 0.971 | 0.950 | 0.992 | 0.937 |
| Autocorrelation, $\rho$ (HKS) | 0.994 | 0.988 | 0.998 | 0.983 |
| Eq. sample size, $n'$ | 1.3 | 1.3 | 1.4 | 1.4 |




## 11. Conclusions

- The Hurst-Kolmogorov behaviour, also known as long-term persistence, has been detected in paleoclimate reconstructions of both ice-core and sediment origin, dating back up to 3 million years.
- All reconstructions indicate high values of the Hurst coefficient, $H$ (approx. 0.98); accordingly, the differences in the estimates of the statistical properties of the data, using the CS and the HKS, are very high. In particular, the standard deviation, estimated by HKS, is approximately double that of the CS estimation.
- Estimation bias and 95% prediction limits are determined theoretically and by Monte Carlo simulation (200 simulations with length equal to each paleoclimate series). The empirical data points lie within the Monte Carlo prediction limits of the HK model.
- The analysis allows the conclusion that classical statistics is inconsistent with climatic processes and describes only a portion of the natural climate system variability. In contrast, all paleoclimate reconstructions seem to be consistent with the simple HK model.

## 12. References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