Theoretical Model of Rainfall in Tropical Cyclones for the Assessment of Long-Term Risk

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Abstract

We propose a theoretical model to evaluate the rainfall intensity field due to large-scale horizontal wind convergence in tropical cyclones (TCs). The model is intended as one component of a methodology to assess the risk of extreme rainfall intensities from TCs. The other components are a recurrence relation for the model parameters and track and a statistical representation of the deviations of rainfall intensity from model predictions. The latter are primarily caused by rainbands and local convective activity and is the focus of an upcoming communication. The vertical flux of moisture and the associated surface rain rate are calculated using basic thermodynamics and a simple numerical model for the vertical winds inside the TC boundary layer. The tropical cyclone is characterized by the radial profile of the tangential wind speed at gradient level, the storm translation velocity, the surface drag coefficient, and the average temperature and saturation ratio inside the TC boundary layer. A parametric analysis shows the sensitivity of the symmetric and asymmetric components of the rainfall field to various storm characteristics.

Index Terms: Precipitation (3354), Theoretical Modeling (3367), Tropical Meteorology (3374), Boundary layer processes (3307), Floods (1821).
1. Introduction

Tropical cyclones (TCs) are atmospheric disturbances capable of producing extreme rainfall with devastating social and economic impact (Landsea, 2000; Rappaport, 2000). Consequently, there is much interest in the assessment of the rainfall hazards posed by TCs, either in real time (with leads of hours or days) or in the long run; see e.g. Marks et al. (1998). For the latter purpose, when interest is in the rate at which different rainfall intensity levels are exceeded, one needs to parameterize the storms and for each set of parameters evaluate rainfall at the site or over the region of interest as a random process in time or a random field in space-time. In principle, the stochastic rainfall model could be directly fitted to data from historical events, but the large number of parameters and the relative lack of historical data make an empirical model identification and fitting approach unfeasible. Moreover, it would be difficult in such an approach to incorporate knowledge of the physics of the phenomenon. A better approach, which we follow here, is to formulate a physically-based rainfall model. The model should be simple enough that it can be run under a very large set of scenario conditions; hence detailed numerical TC models would not be suited for this purpose.

Neither simple nor sophisticated TC models can produce accurate statistical estimates of space-time rainfall for a given set of global TC parameters. Therefore, any deterministic rainfall model must be complemented by a statistical representation of the rainfall “residuals”, defined as the difference between observed rainfall and model prediction. For example, the model developed here ignores the rainfall fluctuations due to rainbands and local convection. The statistical characterization of these fluctuations (residuals) is the focus of a separate communication.
The third and final component of a long-term TC rainfall risk analysis method is the recurrence model, which specifies the frequency with which different TC parameter combinations occur in the region of interest. This component has been the subject of numerous studies, as the recurrence relation is common to the assessment of any TC-related risk, such as wind, waves and surges; see for example Vickery and Twisdale (1995), Vickery *et al.* (2000), Willoughby and Rahn (2004) and Powell *et al.* (2005).

In the late 1950s, R.H. Kraft (as referenced by Pfost, 2000, and Kidder *et al.*, 2005) used raingauge rainfall depths to estimate the maximum 24-hr rainfall accumulation due to the passage of a TC. According to Kraft, this maximum is 100 inches (254cm) divided by the storm translation speed in knots (1knot = 0.514m/s). Limitations of Kraft’s analysis are that it does not provide information on the spatial distribution of rainfall and does not account for TC characteristics such as size and intensity.

Riehl and Malkus (1961), Goodyear (1968) and more recently Simpson and Riehl (1981) have addressed some of these limitations. From the examination of 46 TCs making landfall along the Gulf Coast of the United States, Goodyear (1968) concluded that the 48-hr maximum rainfall depth is about 150mm and occurs 40-80km inland and 40-80km to the right of the storm. Using a similar approach, Riehl and Malkus (1961) and Simpson and Riehl (1981) found that for hurricane-strength cyclones rainfall intensity averages about 33mm/h within 37km from the cyclone center and for larger distances decays almost exponentially. While these studies extend and improve upon Kraft’s rule, they too fail to resolve the dependence of rainfall on storm characteristics.

NASA’s Tropical Rainfall Measuring Mission (TRMM) (Simpson *et al.*, 1988) produced vast amounts of TC rainfall data, making it possible to conduct more systematic statistical analyses.
Lonfat et al. (2004) extracted 2121 tropical cyclone microwave images from the TMI TRMM data set to find how the azimuthally averaged rainfall intensity varies with distance $R$ from the TC center in three storm intensity ranges: tropical storms (TSs) with maximum tangential wind speed $V_{\text{max}}$ in the range 18-33m/s; CAT12 cyclones with $V_{\text{max}} = 34-48$m/s and CAT35 cyclones with $V_{\text{max}} > 49$m/s. The study concluded that TC rainfall intensifies with increasing $V_{\text{max}}$ and the symmetric component of the rainfall intensity reaches its maximum at a distance from the hurricane center close to the radius of maximum winds $R_{\text{max}}$. For larger distances, rainfall intensity decays approximately as a power law; see their Figure 11. Due mainly to storm translation and vertical wind shear, rainfall intensity lacks circular symmetry and varies also with the azimuth relative to the directions of shear and motion.

Chen et al. (2006) used the same TRMM storms to further assess the dependence of rainfall on vertical wind shear $S$, defined as the difference between the 200 and 850-hPa horizontal wind velocities in the annular region between 200 and 800km from the TC center. The study calculated the average rainfall asymmetry, defined as the ratio of the wavenumber-1 Fourier amplitude to the azimuthal average of the rainfall intensity, for the nine combinations of the 3 intensity categories in Lonfat et al. (2004) and three shear magnitude ranges ($S < 5$m/s, $5 \leq S \leq 7.5$m/s, and $S > 7.5$m/s). Chen et al. (2006) found that, in storms in the Northern (Southern) hemisphere with high wind shear ($S > 5$m/s), rainfall intensifies downshear and downshear-left (-right) of the storm.

Parametric rainfall models have also been developed. Using the radial rainfall profiles of Lonfat et al. (2004), Tuleya et al. (2007) suggested one such model for 24-hr rainfall totals (R-CLIPER) based on climatological and persistence information. The model assumes that storms are symmetric and therefore ignores vertical wind shear and storm motion. Lonfat et al. (2007)
built on the R-CLIPER algorithm to construct a parametric rainfall model (PHRaM) that includes shear-related asymmetries according to the results of Chen et al. (2006).

Due to data limitations, R-CLIPER and PHRaM use a coarse and incomplete storm parameterization: the effects of storm intensity and vertical wind shear are modeled by interpolating from 3 classes of each variable, the size of the vortex $R_{max}$ is only implicitly taken into account by allowing the location of the maximum rainrate depend on the intensity of the storm according to the results of Lonfat et al. (2004), while other factors (e.g. the radial wind velocity profile in the main vortex, the surface roughness, and the storm translation velocity) are ignored. Another limitation is that the Lonfat et al. (2004) profiles on which R-CLIPER and PHRaM are based use ensemble averages of storms with significantly different $R_{max}$ values. Since rainfall intensity has a sharp peak near $R_{max}$, this averaging operation depresses the maximum rainfall estimate. For example, for CAT35 storms Lonfat et al. (2004) find maximum rainfall intensities around 12mm/h, which is 2.5-3 times lower than the values most often reported in the literature; see for example Riehl and Malkus (1961), Jiang et al. (2006), Trenberth et al. (2007) and the rainfall intensities implied by the radar reflectivities in Marks (1985) and Kepert (2006a,b). Finally, the Lonfat et al. (2004) profiles are based on TMI rainfall products, which are known to be biased towards low values for high rainfall intensities and towards high values for low rainfall intensities (Viltard et al., 2006).

Here we develop a simple theoretical model of TC rainfall based on the vertical outflow of water vapor from the TC boundary layer (BL). This water vapor flux originates from the low-level convergence of the horizontal flow. The analysis combines a user-specified tangential wind profile at gradient level, an Ekman-type solution for the horizontal and vertical winds inside the boundary layer (BL), and basic thermodynamics. Evaluation of the BL winds is based on Smith’s
(1968) axi-symmetric formulation, modified by Langousis et al. (2008) to account for storm motion. The resulting models of wind and rainfall are referred to as the modified-Smith (MS) BL model and the modified-Smith-for-rainfall (MSR) model, respectively.

The MSR model produces asymmetric rainfall fields that explicitly depend on: the maximum tangential wind velocity at gradient level $V_{max}$, the radius of maximum winds $R_{max}$, Holland’s $B$ parameter (Holland, 1980), the surface drag coefficient $C_D$, the storm translation velocity $V_t$, the vertical diffusion coefficient of the horizontal momentum $K$, and the average temperature $\bar{T}$ and saturation ratio $\bar{Q}$ inside the TC boundary layer.

An important departure from previous studies is that we parameterize asymmetries in terms of storm motion not vertical wind shear. The degree to which TC motion and shear contribute to wind, lightning, and rainfall asymmetries has been intensely discussed in the literature; see for example Black et al. (2002), Corbosiero and Molinari (2002, 2003), Rogers et al. (2003), Lonfat et al. (2004) and Chen et al. (2006). Separation of the two effects through data analysis is made difficult by the high correlation between the directions and magnitudes of motion and shear in any given geographical region (Corbosiero and Molinari, 2003; Lonfat et al., 2004; Chen et al., 2006). As a consequence, the calculated rainfall asymmetry is almost the same when storms are aligned in the direction of motion or shear, except for a region-specific rotation; see e.g. Corbosiero and Molinari (2003) and Section 5 below. Another consequence is that, in risk analysis, one may equivalently use shear or motion as conditioning parameter. Since it is easier to include motion than shear when modeling rainfall and the historical records readily provide storm motion information (e.g. Vickery and Twisdale, 1995, and Vickery et al., 2000), we have chosen to develop a motion-based rather than shear-based parameterization of rainfall asymmetry.
Section 2 summarizes the boundary layer model developed by Langousis et al. (2008) and Section 3 uses the vertical fluxes from that model to estimate surface rainrates in the case of stationary (i.e. symmetric) cyclones. Model predictions are compared to MM5 simulations and R-CLIPER rainrate estimates. The choice of MM5 is based on the fact that this code has been successfully used to simulate a number of TCs, including Hurricanes Bonnie (1998) (Rogers et al. 2003, 2007), Floyd (1998) (Tenerelli and Chen, 2001, Rogers et al. 2007) and Frances (2004) (Chen et al., 2007). Section 4 validates the symmetric MSR predictions using precipitation radar (PR) rainfall products from 38 TRMM frames. The PR rainfall products are less biased than the microwave imager (TMI) data used in previous studies, especially in the core region where rainfall intensities are high (Viltard et al., 2006). Section 5 extends the analysis to translating TCs, which generate asymmetric rainfall fields, assesses the effect of motion on the spatial variation of TC rainfall, and suggests a motion-based parameterization of rainfall asymmetry. Section 6 assesses the sensitivity of the symmetric and asymmetric rainfall components to various TC parameters and Section 7 summarizes the main conclusions.

2. Modified Smith boundary layer model for moving tropical cyclones

A number of studies (Myers and Malkin 1961; Shapiro 1983; Kepert 2001; Langousis et al. 2008) have developed theoretical boundary layer (BL) models for moving tropical cyclones. These models derive the radial, tangential and vertical winds inside the boundary layer from an assumed radial profile of the tangential wind velocity under gradient balance, $V_{gr}(R)$, and suitable surface boundary conditions. For example, a widely used gradient wind profile is (Holland, 1980)

$$V_{gr}(R) = V_{max} \sqrt{(R_{max}/R)^b \exp[1-(R_{max}/R)^b]}$$

(1)
where $V_{\text{max}}$, $R_{\text{max}}$, and $B$ are TC-specific parameters. According to equation (1), the tangential velocity $V_{\text{gr}}$ increases radially to a maximum $V_{\text{max}}$ at $R = R_{\text{max}}$ and for $R \gg R_{\text{max}}$ decays approximately as a power-law of distance with exponent $-B/2$. The shape parameter $B$ varies in the range [1, 2], with typical values around 1.4 (Willoughby and Rahn, 2004). Next we briefly describe the boundary layer model of Langousis et al. (2008) and in Sections 3-5 use this model to calculate water vapor fluxes that are responsible for rainfall.

The model of Langousis et al. (2008) corrects Smith’s (1968) BL formulation for the case of stress surface boundary conditions and accounts for storm motion. Like in Smith (1968), vertical diffusion of the horizontal momentum is parameterized through a vertical diffusion coefficient $K$. The horizontal momentum equations are written in cylindrical coordinates that move with the storm and solved using the Karman and Pohlhausen momentum integral method. In this method, one specifies vertical profiles for the radial $U$ and tangential $V$ wind velocity components, which satisfy the boundary conditions at the surface (elevation $Z = 0$) and for $Z \to \infty$ tend to the gradient winds, for example the profile in equation (1). The boundary conditions are modeled using a surface stress formulation with drag coefficient $C_D$.

For $U$ and $V$, Langousis et al. (2008) use functions of the Ekman type with parameters $E$ (amplitude coefficient) and $\delta$ (dimensionless BL scale thickness) that vary both radially and azimuthally. The horizontal momentum equations are vertically integrated through the BL to produce a system of two partial differential equations, which are solved numerically to obtain $E$ and $\delta$ as functions of radius $R$ and azimuth $\theta$ relative to the direction of storm motion. Once the horizontal wind components $U$ and $V$ are obtained, the vertical wind velocity $W$ is calculated using mass conservation, as
\[ W(R, \theta, Z) = \frac{1}{R} \left[ \int_{0}^{Z} \frac{\partial(U)}{\partial R} dZ + \int_{0}^{Z} \frac{\partial V}{\partial \theta} dZ \right] \]  (2)

For stationary cyclones \((V_t = 0)\), there is no azimuthal variation of \(V\) and \(U\) and equation (2) reduces to

\[ W(R, Z) = \frac{1}{R} \frac{d}{dR} \left( R \int_{0}^{Z} U dZ \right) \]  (3)

\(W(R, Z)\) in equation (3) is also the symmetric component of the vertical wind speed for a storm that translates with velocity \(V_t \neq 0\).

The above modified Smith (MS) scheme is computationally very efficient and stable over a wide range of parameter values. Model predictions are close to MM5 simulations and to observed wind speeds; see Langousis et al. (2008) for details.

3. Estimation of the symmetric component of rainfall

Estimates of rainfall intensity are obtained assuming that, with corrections to be made later, the surface rain rate \(i\) is proportional to the water vapor up-flux at a reference height \(H\). Similar approaches have been used in the past to evaluate the rainfall potential of extra-tropical storms (Palmen, 1958), orographic precipitation (Alpert, 1986) and latent heat (Magaki and Barros, 2004), as well as to predict rainfall extremes (Abbs, 1999; Wilson and Toumi, 2005).

To verify how strongly rainfall intensity is related to the vertical velocity \(W_{th}(R, \theta) = W(R, \theta, Z=H)\) from equation (2) at different elevations \(H\), we used MM5 simulations. Figure 1 shows the correlation between the two quantities using 12 frames of Hurricane Frances, simulated at 6 hr intervals for the period Aug. 29-Sep. 01, 2004. The correlation is maximum around 0.85 at an elevation of 2-3km, which can be taken as the reference height \(H\). The inset of
Figure 1 compares the MM5 radial profiles of the simulated rainfall intensity and vertical wind velocity at 3km elevation for the 06:00UTC Aug. 29, 2004 frame. Both profiles are normalized to have unit maximum value. This detailed comparison shows that the correlation coefficient is below 1 due mainly to fluctuations of the rainfall intensity caused by rainbands and other local convective phenomena. If these fluctuations in the MM5 profiles are smoothed out, which is what the present MSR model effectively does, the surface rainfall intensity and vertical wind speed are in even better agreement.

To complete the symmetric rainfall model one needs the proportionality constant between rainfall intensity and vertical wind speed. From simple calculations using a lapse-rate of about 6-7°C/km (Rogers and Yau, 1996), one obtains that at elevations in excess of 6-8km the water vapor mixing ratio is close to zero. Consequently, one may accurately assume that the upward water vapor flux from the TC boundary layer equals the downward flux of rainwater. To keep the rainfall model simple, we assume that below the reference height $H$ the temperature $T$ and saturation ratio $Q$ are constant and equal to the depth-averaged values $\bar{T}$ and $\bar{Q}$. For cyclones over tropical and sub-tropical waters, $\bar{T}$ ranges between 20-24°C and $\bar{Q}$ is between 75-85%; see Gray et al. (1975), Frank (1977) and Smith (2003). Under these conditions, the symmetric rainfall intensity $i_{sym}$ is given by

$$
    i_{sym}(R) = \begin{cases} 
    \alpha(\bar{T}) \bar{Q} W_{th}(R), & W_{th}(R) > 0 \\
    0, & W_{th}(R) \leq 0 
\end{cases}
$$

(4)

where $\alpha(\bar{T})$ is the volume of liquid water per unit volume of saturated air after complete condensation (see below), and $W_{th}(R) = W(R,Z=H)$ is the vertical wind velocity in equation (3) for $Z = H$. The function $\alpha(\bar{T})$ is obtained by combining the ideal gas law with the Clausius-Clapeyron equation. Using a liquid water density $\rho_w = 1000$kg/rm$^3$, this gives
\[ \alpha(T) = \frac{1.324 \times 10^{-3}}{T + 273} \exp \left( \frac{17.67}{T + 243.5} \right) \]  

(5)

where \( T \) is in °C. Notice that in downdraft regions where \( W_H \) is negative, equation (4) sets the rainfall intensity to zero. This means that rainfall generation is limited to regions where moist air updrafts. However, due to the slant of the wall updrafts and the cyclonic advection, rainfall may be nonzero also in downdraft regions. This effect is modeled below through a rainfall redistribution scheme.

3.1 Correction for the sloping angle of the wall

Flight observations (e.g. Jorgensen, 1984b; Marks and Houze, 1984) show that the wall updraft of a tropical cyclone slopes outward to altitudes \( H_0 \approx 5-7 \) km, with an angle \( \psi_0 \) from the vertical in the 45°-60° range. The MS model of Langousis et al. (2008) assumes fixed vertical profiles of the radial and tangential wind velocities and therefore does not account for such sloping angle. Consequently, equation (4) tends to underpredict the radius of maximum rainfall.

To include radial advection of the rainwater by the wall updraft while avoiding discontinuities in the radial distribution of rainfall, we assume that the angle of the updrafts decreases exponentially with distance \( R \) from the storm center, as

\[ \psi(R) = \psi_0 \exp \left( -\frac{R - R_m}{R_m} \right) \]  

(6)

where \( R_m \) is the location where \( i_{sym} \) and \( W_H \) in equation (4) are maximum. The outward radial displacement \( \Delta R \) of the rainwater due to the sloping updrafts is then

\[ \Delta R = H_0 \tan \psi \]  

(7)
Notice that estimating rainfall intensities at distance $R$ from the cyclone center as $i_{sym}(R-\Delta R)$ is technically incorrect because the model does not satisfy mass conservation. However, we have verified that the error is very small and negligible in practice.

### 3.2 Comparison with MM5 and R-CLIPER

Figure 2 compares the azimuthally averaged rainfall intensities $i_{sym}$ for Hurricane Frances (2004) estimated by MM5, R-CLIPER (see Introduction), and the present modified-Smith-for-rainfall (MSR) model. The MM5 and MSR curves are the ensemble averages of 12 rainfields simulated at 6 hr intervals during the period Aug. 29-Sep. 01, 2004, using the two models. The MM5 simulations were conducted at 1.67km resolution using the nested grid capability at the University of Miami (Houze et al., 2006; 2007), whereas the MSR estimates were obtained as follows:

1) For each frame, the parameters $V_{max}$ and $R_{max}$ in equation (1) were extracted from the azimuthally averaged tangential winds simulated by MM5 at 5km elevation;

2) Holland’s (1980) gradient wind profile with $B=1$ was used in the model of Langousis et al. (2008) to calculate the vertical wind profile $W_H(R)$ at elevation $H=3$km;

3) Equations (4) and (5) were used to estimate how the azimuthally averaged rainfall intensity $i_{sym}$ varies with distance $R$ from the TC center;

4) Finally, the results were corrected for sloping-updrafts using equations (6) and (7) and averaged over the 12 frames.

Setting Holland’s $B$ to 1 reproduces well the MM5 rainfall fields, as well as the PR rainfall estimates from TRMM; see Section 4.

The model of Langousis et al. (2008) requires also specification of the Coriolis parameter $f$, the vertical diffusion coefficient $K$, and the surface drag coefficient $C_D$. In our simulations we
have set $f = 4.7 \times 10^{-5} \text{ sec}^{-1}$, which corresponds to latitude 19° North (the approximate latitude of TC Frances during the period considered), $K = 50 \text{ m}^2/\text{s}$, and $C_D = 0.002$. Values of $K$ near 50 m$^2$/s are often quoted in the literature (e.g. Smith, 1968; Shapiro, 1983; Kepert, 2001; Kepert 2006b) and are consistent with back-calculations from MM5 simulations (Melicie Desflots, 2007, personal communication). The value 0.002 is representative of drag coefficients extracted from oversea MM5 simulations and to values in the literature for winds in the hurricane range (e.g. Kepert, 2001; Powell et al., 2003; Donelan et al., 2004). The vertically averaged temperature $\bar{T}$ (over a depth of 3km) and saturation ratio $\bar{Q}$ in equation (4) have been set to 22°C and 80%, respectively. These values correspond to a depth-averaged mixing ratio of approximately 13 gr/kg, which is slightly lower than the ensemble average value of 15 gr/Kg extracted from MM5 simulations for Hurricane Frances (Melicie Desflots, 2007, personal communication). For the wall updraft correction in equations (6) and (7), we have assumed an outwards slope of $\psi_0 = 50^\circ$ from the vertical to an altitude $H_0 = 6$ km.

The solid lines in Figure 2 are the profiles of $i_{sym}$ before the correction for sloping updrafts (thin lines) and after that correction (thick lines). The rainfall estimates from the MSR model are close in shape and magnitude to the MM5 profiles. This is especially true after the correction for out-sloping updrafts. Differences are mostly due to local rainfall intensifications in MM5 caused by rainbands. By contrast, the rain rates of Lonfat et al. (2004), which form the basis of the R-CLIPER algorithm, agree with MM5 in the far field but severely underestimate rainfall in the near-core region. As discussed in the Introduction, reasons for the much-reduced rain rate maximum in R-CLIPER are the smoothing effect of ensemble averaging and the bias of the TMI rainfall retrievals used by Lonfat et al. (2004).
4. Validation of symmetric MSR predictions

Figure 3 compares PR and MM5 rainfall estimates with rainfall intensities generated by the present MSR model using the procedure described in Section 3. Figure 3.a shows a scatterplot of the ratio between the PR and MSR rainfall estimates as a function of the normalized distance \( R/R_{\text{max}} \) from the storm center, using a 5km × 5km grid of spatial locations and the 38 TRMM frames in Table 1 (a total number of 48483 points). The number of points in different ranges of \( R/R_{\text{max}} \) is shown in Table 2. The MSR estimates where generated using the \( V_{\text{max}} \), \( R_{\text{max}} \) and latitude information in the extended best track record (Demuth et al., 2006; M. DeMaria, 2008; personal communication). Figure 3.b shows a similar scatterplot of the ratio between the MM5 and MSR rainfall estimates. In this case the comparison is based on the 12 simulated rainfields of Hurricane Frances, for a total of 43919 points. All MSR simulations were performed using \( B = 1 \), \( K = 50 \text{m}^2/\text{s} \) and \( C_D = 0.002 \). Both Figures 3.a and 3.b show a large dispersion, which reflects the significant small-scale variability of rainfall intensity due to rainbands and local convection. Those fluctuations are not resolved by the MSR model.

Figures 3.c and 3.d show the moving average and standard deviation of the ratios in Figures 3.a and 3.b, using a window of 2000 points. Except for a small region close to the core (\( R < 1.5R_{\text{max}} \)), the local average in Figure 3.c fluctuates around 1. This means that on average the MSR model generates unbiased rainfall profiles for radial distances up to 15\( R_{\text{max}} \) from the TC center. For distances \( R < 1.5 \) \( R_{\text{max}} \) the MSR model tends to overpredict the PR rainrates.

As noted above, the large local standard deviations in Figure 3.c reflect the significant small-scale variability of TC rainfall. It is interesting that the standard deviation tends to increase as the distance from the TC center increases. This is in accordance with the findings of other studies.
(Jorgensen, 1984a; Powell, 1990, and Molinari et al., 1994) that the outer TC environment exhibits more cellular structure and higher small-scale variability relative to the inner region.

Figure 3.d shows that for radial distances up to $8R_{\text{max}}$ the MSR model tends to underpredict the MM5 rainfall intensities by about 50%, whereas for larger distances the opposite is true. Since the MSR model displays good skills in reproducing the PR rain rates, it is possible that these differences reflect MM5 biases. This is consistent with what other studies have found when comparing MM5 rainfall estimates to empirical and radar observations; see e.g. Fall et al. (2007), Juneng et al. (2007), Chen et al. (2007) and Rogers et al. (2007). The higher standard deviations in Figure 3.d compared to Figure 3.c further suggest that MM5 may enhance local convective activity. One should however caution that these observations are based on just one simulated hurricane and should be validated through a more extensive comparison.

5. Asymmetry of the rainfall field

In the case of a moving TC, equation (4) becomes

$$i(R,\theta) = \begin{cases} \alpha(T) \bar{Q} W_H(R,\theta), & W_H(R,\theta) > 0 \\ 0, & W_H(R,\theta) \leq 0 \end{cases}$$

where the vertical wind speed $W_H$ depends on both $R$ and $\theta$ and is given by equation (2) for $Z = H$. In this asymmetric case the rainfall intensities from equation (8) must be corrected both radially using equations (6) and (7) and azimuthally to account for the redistribution of rainwater due to cyclonic circulation; on the latter, see Corbosiero and Molinari (2002), Black et al. (2002) and Rogers et al. (2003).

To keep the correction simple, we perform the azimuthal redistribution uniformly within an angular interval $[\theta, \theta + \Delta \theta]$ where $\Delta \theta$ is given by
The angle $\Delta \theta$ is in radians (positive clockwise in the Northern hemisphere), $V_{gr}$ is the tangential wind velocity at gradient level (equation (1)), $t_f \approx 30\text{min}$ is the time needed for rain generating features like convective cells to develop (Weisman and Klemp, 1986; Rogers and Yau, 1996) and $t_r$ is the time needed for a raindrop at height $H$ to reach the ground. A rough estimate of $t_r$ comes from assuming an average raindrop velocity of 2-3m/s and a boundary layer depth $H \approx 2.5-3\text{km}$. This gives $t_r \approx 25\text{min}$.

Next we use equations (8) and (9) for $t_f + t_r = 60\text{min}$ to assess the effect of motion on the spatial variation of TC rainfall and propose a motion-based, rather than shear-based, parameterization of rainfall asymmetry.

### 5.1. Motion-based versus shear-based parameterization of rainfall asymmetry

MSR is a boundary layer model that generates spatial rainfall without explicitly considering vertical shear $S$. Rather, rainfall asymmetries are linked to storm motion. Since most of the rainfall originates at low altitudes relative to those that define wind shear, one may expect this to be a suitable approach.

To verify this assertion, Figure 4 compares the shear-aligned rainfall asymmetry from TRMM with the motion-aligned rainfall asymmetry from MSR. In both cases, asymmetry is defined as

$$A(R,\theta) = \frac{i(R,\theta) - i_{\text{sym}}(R)}{i_{\text{sym}}(R)}$$  \hspace{1cm} (10)  

where $i(R,\theta)$ is rainfall intensity at $(R,\theta)$ and $i_{\text{sym}}(R)$ is the azimuthal average. More specifically, Figure 4.a shows the average of the rainfall asymmetries in Figure 7 of Chen et al. (2006) over all TC-intensities and shear magnitudes after aligning the shear vector to point North. For shear
we have used the distribution in Figure 6 of the same study, whereas for TC intensity we have
used the discrete distribution in Table 1 of Lonfat et al. (2004).

Similarly, Figure 4.b was generated by averaging rainfall asymmetries from the MSR model
over a range of TC intensities and translation velocities. Storms are assumed to move in the
Northern hemisphere at an angle of 75° west of the shear-direction in Figure 4.a. This is the
average angle between shear and motion from Figures 3 and 12 of Chen et al. (2006) and is in
the range reported by Corbosiero and Molinari (2003). For storm intensity we have used the
same discrete distribution as in Figure 4.a, setting $V_{\text{max}} = 30\text{m/s}$ for tropical storms, $V_{\text{max}} = 42\text{m/s}$
for CAT12 and $V_{\text{max}} = 60\text{m/s}$ for CAT35 systems. The distribution of the translation velocity was
taken from Figure 11 of Chen et al. (2006). All other storm parameters have been kept constant,
with values $f = 4.7 \times 10^{-5} \text{sec}^{-1}, R_{\text{max}} = 40\text{km}, B = 1, \bar{T} = 22^\circ\text{C}, \bar{Q} = 0.8, K = 50\text{m}^2/\text{s},$ and $C_D = 0.002.$

One sees that the two asymmetries are very similar in both pattern and magnitude, validating
the contention that for rainfall risk one can use the MSR model with motion as the driver of
asymmetry. Differences between Figures 4.a and 4.b occur mainly far away from the core
($R > 250\text{km}$), but these differences are statistically not significant and inconsequential for risk
analysis.

6. Sensitivity analysis

Figures 5 and 6 show the sensitivity of the MSR model results to various tropical cyclone
characteristics: the tangential wind speed under gradient balance (parameterized by $V_{\text{max}}, R_{\text{max}}$
and $B$; see equation (1)), the vertical diffusion coefficient $K$, the surface drag coefficient $C_D$, the
depth-averaged temperature $\bar{T}$ inside the BL and the translation velocity $V_t$ of the storm. Since
rainfall intensity is proportional to the depth-averaged saturation ratio $\bar{Q}$ (see equations (4) and
(8)), dependence on $\bar{Q}$ is not illustrated.
Figure 5 shows the sensitivity of the azimuthally averaged rainfall intensity $i_{sym}$ to $V_{max}$, $R_{max}$, $B$, $K$, $C_D$ and $\bar{T}$. Parameters are varied one at a time around the base-case values $V_{max} = 50\text{ m/s}$, $R_{max} = 40\text{ km}$, $B = 1$, $K = 50\text{ m}^2/\text{s}$, $C_D = 0.002$, $\bar{T} = 22^\circ\text{C}$ and $\bar{Q} = 0.8$ (solid lines). The figure shows that the maximum tangential velocity $V_{max}$ and the roughness of the surface boundary (expressed through $C_D$) have significant effects on rainfall intensity and that lower values of $R_{max}$ produce rain rates that are more peaked and more concentrated near the TC center.

Dependence of the azimuthally averaged rainrate $i_{sym}$ on $V_{max}$ of the type produced by the model has been observed in TC rainfall data (Lonfat et al., 2004, Tuleya et al., 2007; see Introduction). For example, the expressions used by the R-CLIPER parameterization (Tuleya et al., 2007) indicate that when $V_{max}$ increases from 50 to 70 m/s, the maximum rainrate increases by a factor of about 1.5. This is also what the MSR model predicts. However, to our knowledge the effect of $C_D$ and $R_{max}$ on $i_{sym}$ have not been isolated from data. The effect of surface roughness can be qualitatively assessed using the finding in Trenberth et al. (2007) that low-level horizontal wind convergence is by far the dominant factor for TC rainfall. Hence, if one considers that low-level convergence increases with increasing surface drag (Shapiro, 1983; Kepert, 2001; Langousis et al., 2008), one concludes that higher surface drag coefficients should cause TC rainfall to intensify.

The $B$ parameter has a small effect on the peak rainfall intensity, but influences significantly the rate at which rainfall decays with radial distance (higher values of $B$ resulting in faster decay). The azimuthally averaged rainfall intensity $i_{sym}$ has small sensitivity to temperature $\bar{T}$ and the vertical diffusion coefficient $K$. Consequently, setting those parameters to constant values (e.g. to $\bar{T} = 22^\circ\text{C}$ and $K = 50\text{ m}^2/\text{s}$, as was done in Sections 3-5) does not induce large errors.
Figure 6 shows the effect of the drag coefficient \( C_D \) and translation velocity \( V_t \) on rainfall asymmetry for a TC that translates northward in the Northern hemisphere. All other parameters are the same as for the base case in Figure 5. As expected and in accordance with findings in Lonfat et al. (2004), the asymmetry increases as \( V_t \) increases. The effect of \( C_D \) is more complex: at the front of the storm, rainfall asymmetry is insensitive to \( C_D \), whereas at the rear-right the rainfall asymmetry increases with decreasing \( C_D \).

7. Conclusions

We have developed a simple theoretical model for the large-scale rainfall intensity field generated by translating tropical cyclones (TCs). The model assumes that, with corrections for sloping updrafts and azimuthal redistribution, the upward water vapor flux originated from the boundary layer is a good predictor of rainfall intensity. Vertical moisture fluxes are calculated using elementary thermodynamic principles in combination with a boundary layer model that extends Smith’s (1968) analysis to moving storms.

The proposed modified-Smith-for-rainfall (MSR) model estimates the rainfall field from a given radial profile of the tangential wind speed at gradient level, the translation velocity \( V_t \) of the storm, the surface drag coefficient \( C_D \), and the average temperature \( \bar{T} \) and saturation ratio \( \bar{Q} \) inside the TC boundary layer. Model predictions are compared to MM5 simulations and R-CLIPER estimates and validated through precipitation radar (PR) rainfall products from TRMM. The MSR model displays good skills in reproducing the shape and magnitude of PR rainfall fields. We have also verified that the asymmetries produced by storm motion are close to those observed and often parameterized in terms of vertical wind shear. In a parametric analysis, we have studied how the model predictions depend on various storm characteristics.
The combination of a rich parameterization and computational efficiency makes the present model an attractive instrument for risk applications, where one must assess tropical cyclone rainfall under many storm and environmental scenarios. For the latter purpose one needs tools with computational times on the order of minutes. This constraint effectively rules out the use of full-physics high-resolution numerical weather prediction models. An important limitation of the MSR model relative to high-resolution schemes is that it does not account for local rainfall intensifications due to rainbands and local convection. As was explained in the Introduction, these phenomena contribute to the “residuals” of the present model, which for risk analysis must be modeled statistically. This is the focus of an upcoming manuscript. Another limitation of the MSR model is that it does not account for after-landfall conditions and therefore is applicable only to open-water or near-water sites. Extension of the model to inland conditions should be pursued in the future.

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program and the next-generation fully coupled atmosphere-wave-ocean models for

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E.S. Saltzman (2004) On the limiting aerodynamic roughness of the ocean in very strong


Table 1: Storm characteristics for the PR-TRMM rainfields used in Figure 3. The estimates of $V_{\text{max}}$ and $R_{\text{max}}$ are obtained from the extended best track record (M. DeMaria, 2008; personal communication).

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<th>Lon. (deg)</th>
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Table 2: Number of data shown in Figure 3.a that fall into different ranges of $R/R_{max}$.

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Figure captions

Figure 1: (a) Ensemble correlation function of the vertical wind velocity at different elevations and the surface rainfall intensity from MM5 simulations of Hurricane Frances. Ensemble averaging is over 12 frames (at 6 hr intervals) during the period Aug. 29-Sep. 01, 2004. (b) Normalized radial profiles of surface rainfall intensity and vertical wind velocity on Aug. 29, 2004 at 06:00UTC at 3km elevation.

Figure 2: Comparison of the ensemble average rainrates for Hurricane Frances 2004 during the period Aug. 29-Sep. 01, produced by the MSR, MM5 and R-CLIPER rainfall models.

Figure 3: Comparison of PR, MM5 and MSR point rainfall intensity estimates. (a) Scatterplot of the ratio between PR and MSR rainfall estimates as a function of the normalized distance $R/R_{\text{max}}$ from the storm center, for 38 TRMM frames; see Table 1. The number of data points in different ranges of $R/R_{\text{max}}$ is shown in Table 2. (b) Scatterplot of the ratio between MM5 and MSR rainfall estimates as a function of $R/R_{\text{max}}$, for hurricane Frances 2004 during the period Aug. 29-Sep. 01. (c) Local averages and standard deviation of the ratios in (a) using a moving window of 2000 points. (d) Same as (c) but for the ratios in (b).

Figure 4: Comparison of rainfall asymmetry from TRMM and the MSR model. (a) Ensemble average of rainfall asymmetries in Figure 7 of Chen et al. (2006) over all TC intensities and shear magnitudes. (b) Ensemble average of rainfall asymmetries from MSR over all TC intensities and translation velocities. In (b), the TC moves in the Northern hemisphere at an angle 75° to the west of the shear vector in (a).
Figure 5: Sensitivity of the azimuthally averaged MSR rainfall profiles. Solid lines correspond to
\[ V_{\text{max}} = 50\text{m/s}, \ R_{\text{max}} = 40\text{km}, \ B = 1, \ C_D = 0.002, \ K = 50\text{m}^2/\text{s}, \ \bar{T} = 22^\circ\text{C} \text{ and } \bar{Q} = 0.8. \] Each panel shows results under perturbation of one parameter.

Figure 6: Sensitivity of MSR rainfall asymmetry to the drag coefficient \( C_D \) and the storm translation velocity \( V_t \) for a tropical cyclone that moves northward. All other parameters are the same as for the base case in Figure 5.
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