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The Rainfall Areal Reduction Factor: a Multifractal Analysis

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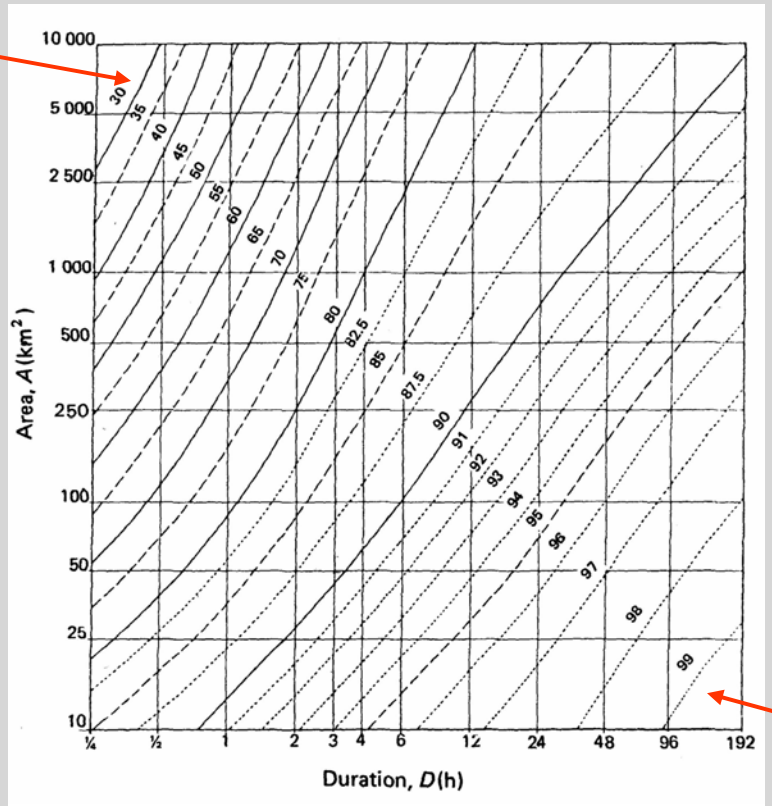
The Areal Reduction Factor: Definition

- $I(A, D, T)$ = T-year average rainfall intensity in $(A \times D)$ (IDAF curves)

$$\Rightarrow \quad ARF(A, D, T) = \frac{I(A, D, T)}{I(0, D, T)} = \frac{IDAF}{IDF}$$

N.E.R.C.
T = 2-3 years



N.E.R.C. (1975)
Flood Studies Report, Vol. II, p. 40

ARF Analysis

1. **IDAF $\Rightarrow$ ARF under exact multifractality**

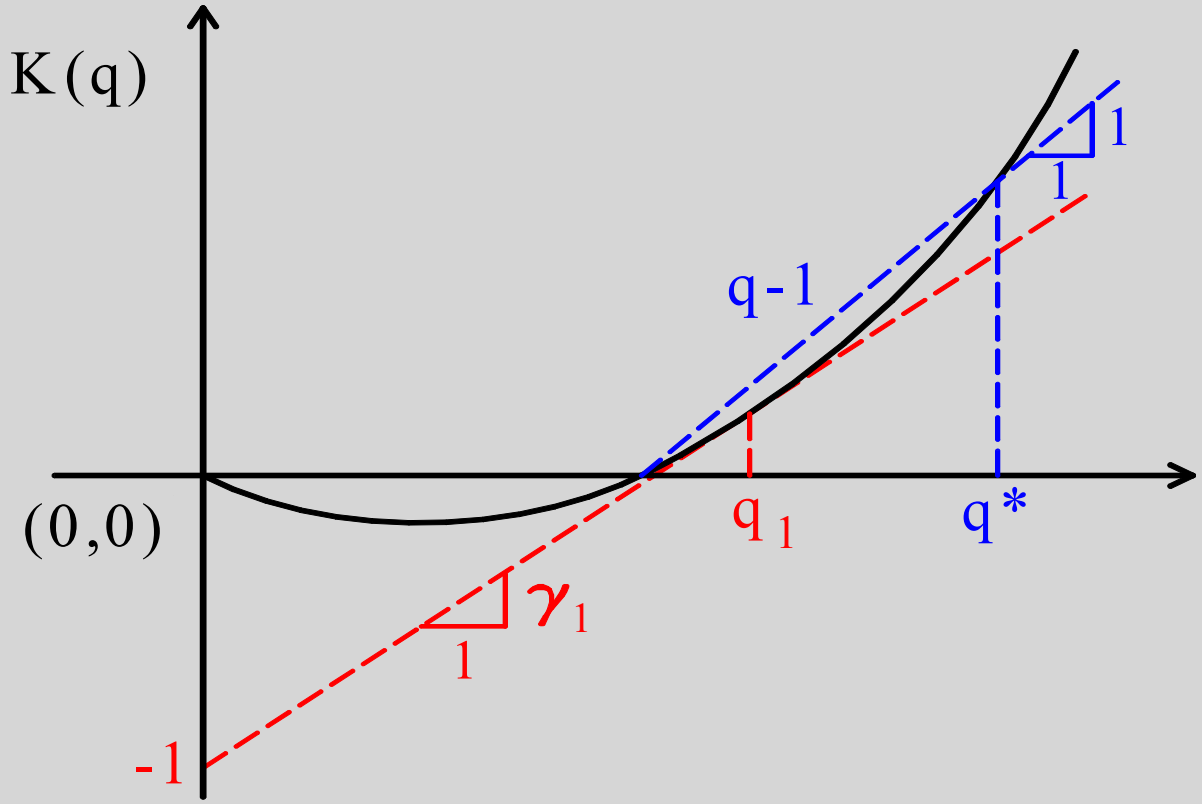
2. **Factors that affect the ARF**

   - Advection
   - Deviations from multifractality (e.g. bounded cascades)
   - Discrete sampling of rainfall fields

3. **Numerical example**

➢ IDF (reminder)

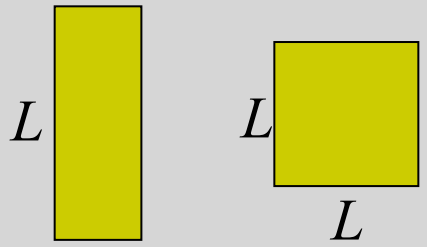
$$I(D,T) \sim \begin{cases} D^{-\gamma_1} T^{1/q_1} \;, \text{ for} \begin{cases} D \to 0 \\ T \text{ finite} \end{cases} \\ D^{-1} T^{1/q_*} \;, \text{ for} \begin{cases} D \text{ finite} \\ T \to \infty \end{cases} \end{cases}$$



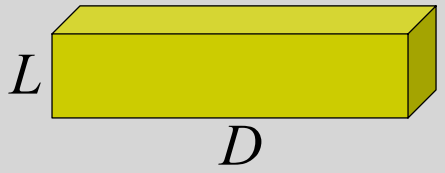
➢ IDAF

$I(A, D, T) \Rightarrow I(L, D, T)$

L = largest linear size of the region



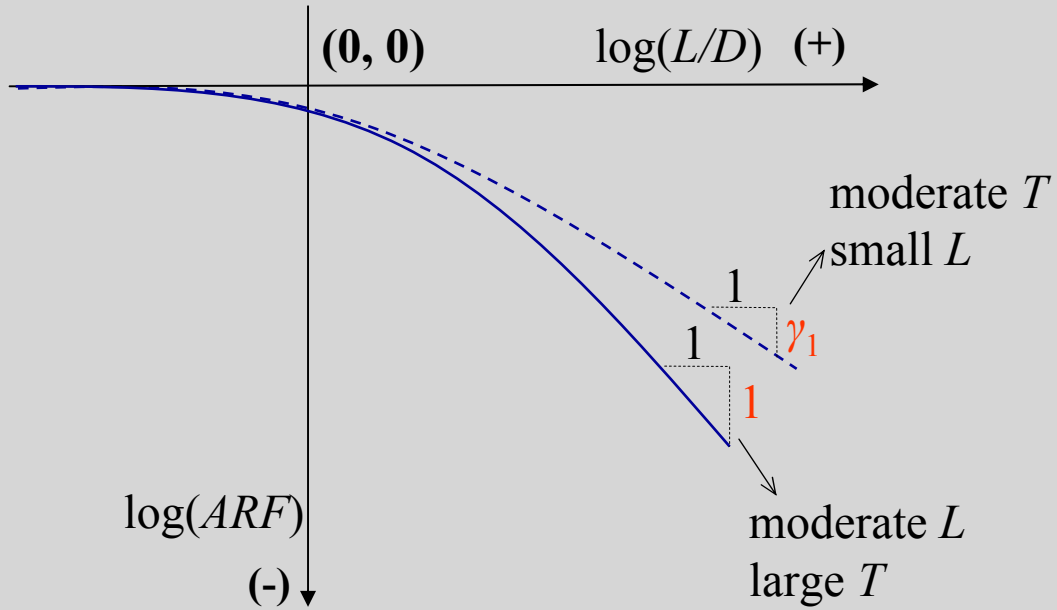
- **Scaling depends also on *L*/*D***



$L/D \to 0$	$L/D \to \infty$
$I(L, D, T) \sim \begin{cases} D^{-\gamma_1} T^{1/q_1}, & \text{for} \begin{cases} D \to 0 \\ T \text{ finite} \end{cases} \\ D^{-1} T^{1/q_*}, & \text{for} \begin{cases} D \text{ finite} \\ T \to \infty \end{cases} \end{cases}$	$I(L, D, T) \sim \begin{cases} L^{-\gamma_1} T^{1/q_1}, & \text{for} \begin{cases} L \to 0 \\ T \text{ finite} \end{cases} \\ L^{-1} T^{1/q_*}, & \text{for} \begin{cases} L \text{ finite} \\ T \to \infty \end{cases} \end{cases}$
(no dependence on L) **(same as IDF)**	(no dependence on D)

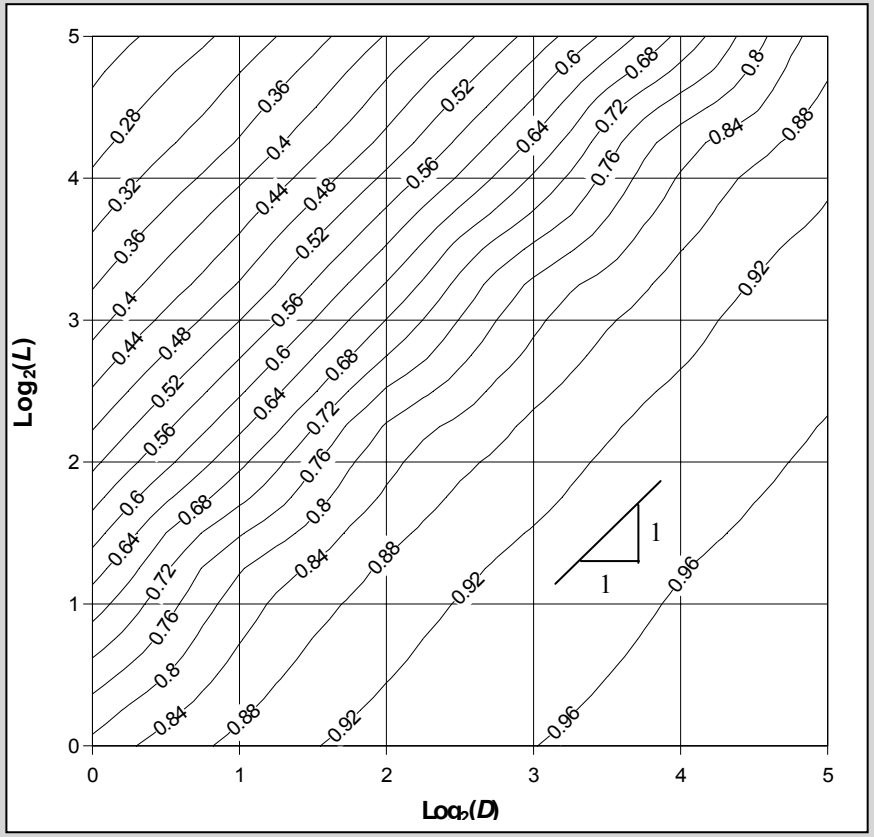
$$\text{➢ } ARF = \frac{IDAF}{IDF}$$

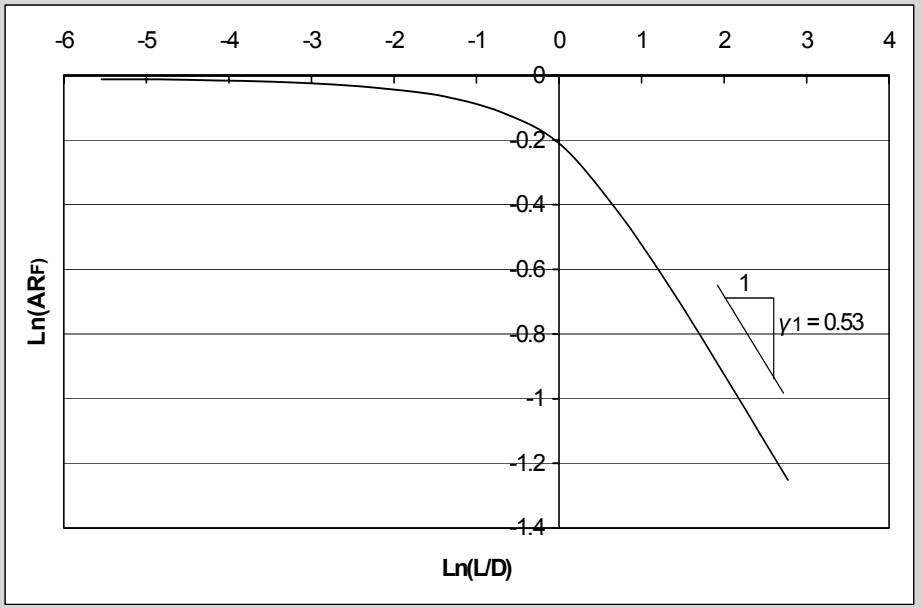
$L/D \to 0$	$L/D \to \infty$
$ARF(L, D, T) \approx 1$	$ARF(L, D, T) \sim \begin{cases} \left(\frac{L}{D}\right)^{-\gamma 1}, & \text{for} \begin{cases} L \to 0 \\ T \text{ finite} \end{cases} \\ \left(\frac{L}{D}\right)^{-1}, & \text{for} \begin{cases} L \text{ finite} \\ T \to \infty \end{cases} \end{cases}$



➢ Numerical validation 3D *LN* cascade

Cascade Parameters $\begin{cases} C_1 = 0.1 \\ \gamma_1 = 0.53 \\ T = 1 \text{ year} \end{cases}$

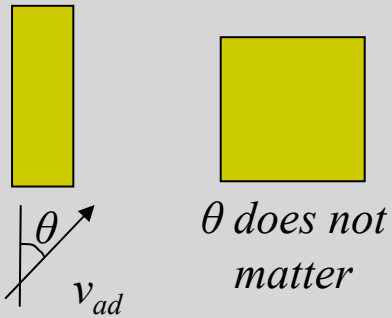




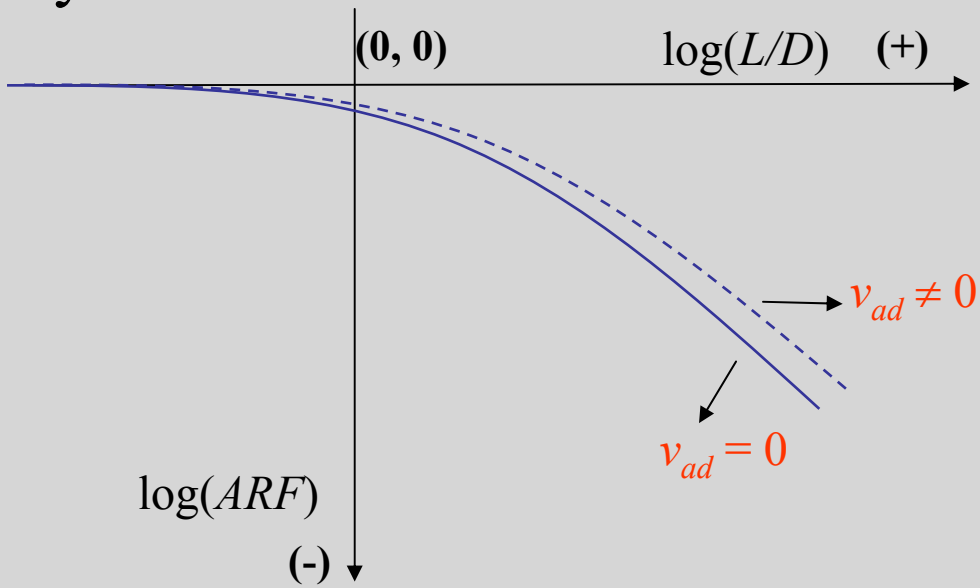
Various Effects on the ARF

➢ Advection

- Qualitatively: $v_{ad} \Rightarrow \begin{cases} D_{eff} > D \\ T_{eff} > T \\ L_{eff} \le L \end{cases}$
- Effect depends on the shape of the region:



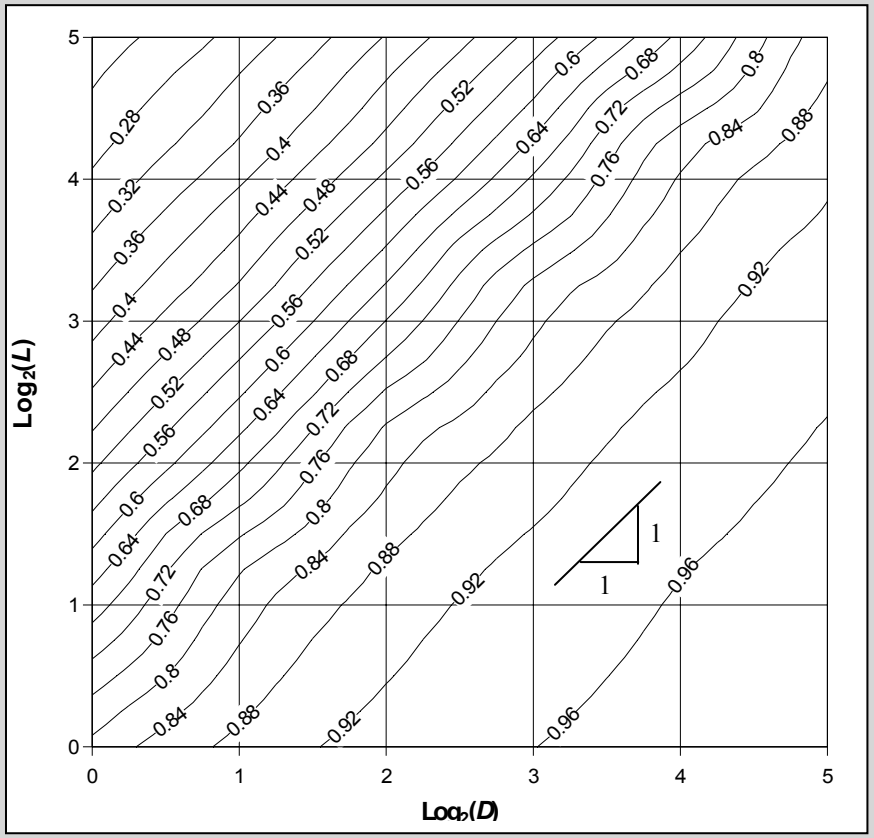
- Typically:



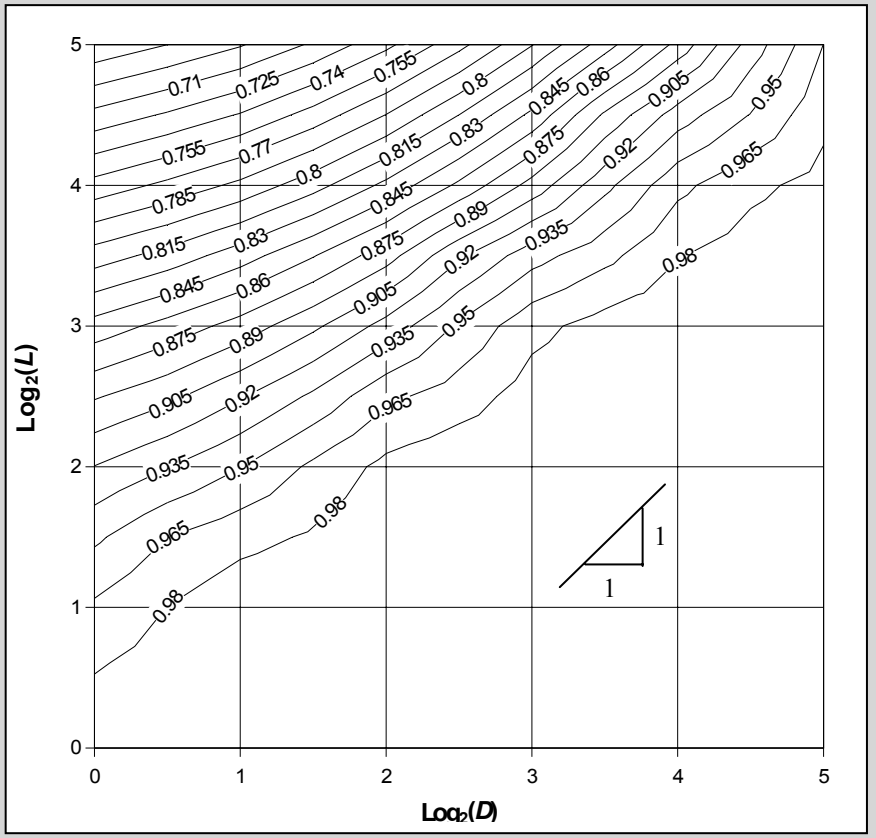
Deviations From Multifractality

- **Bounded Cascades**

Exact Multifractal



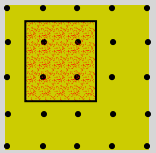
Bounded Cascade

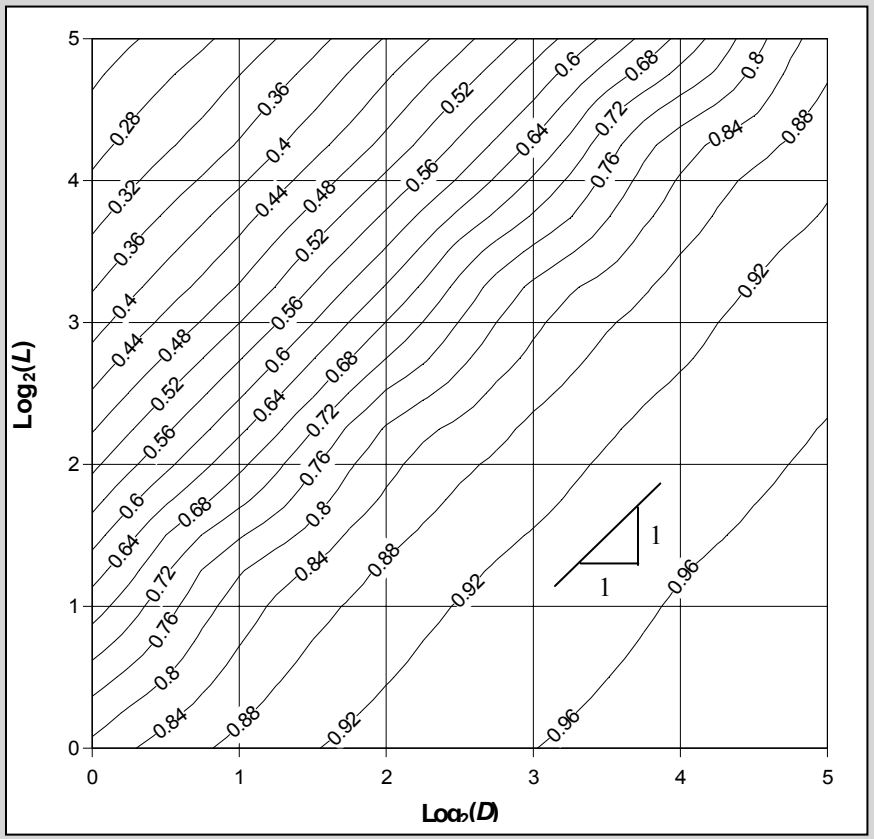


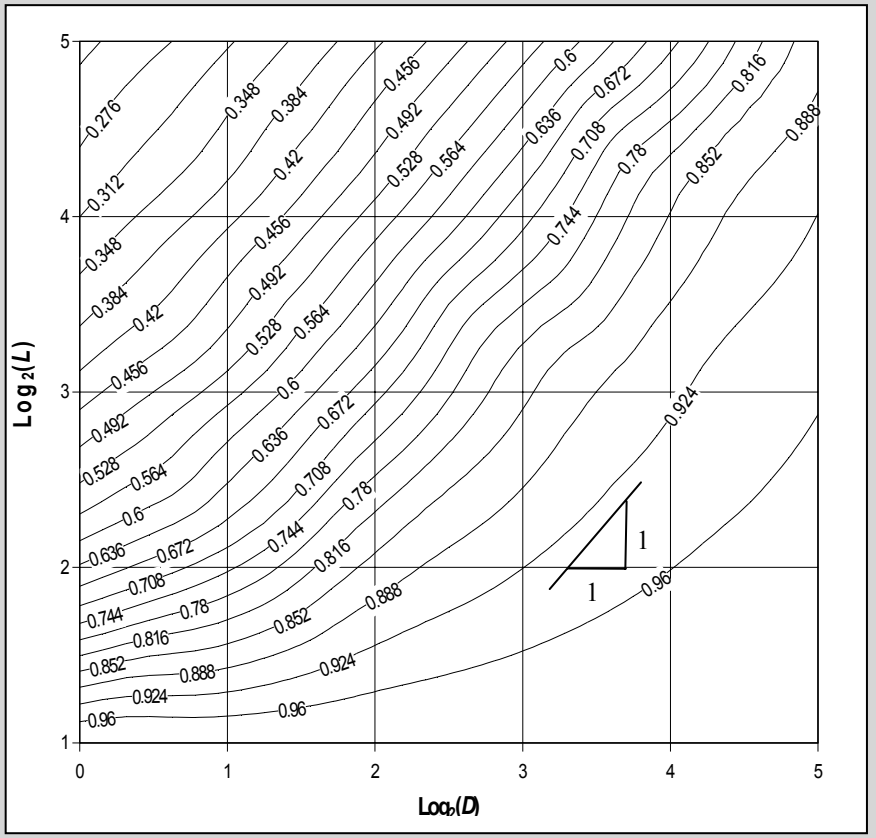
➢ Sampling

Continuous Recording (Exact)

Sparse Sampling

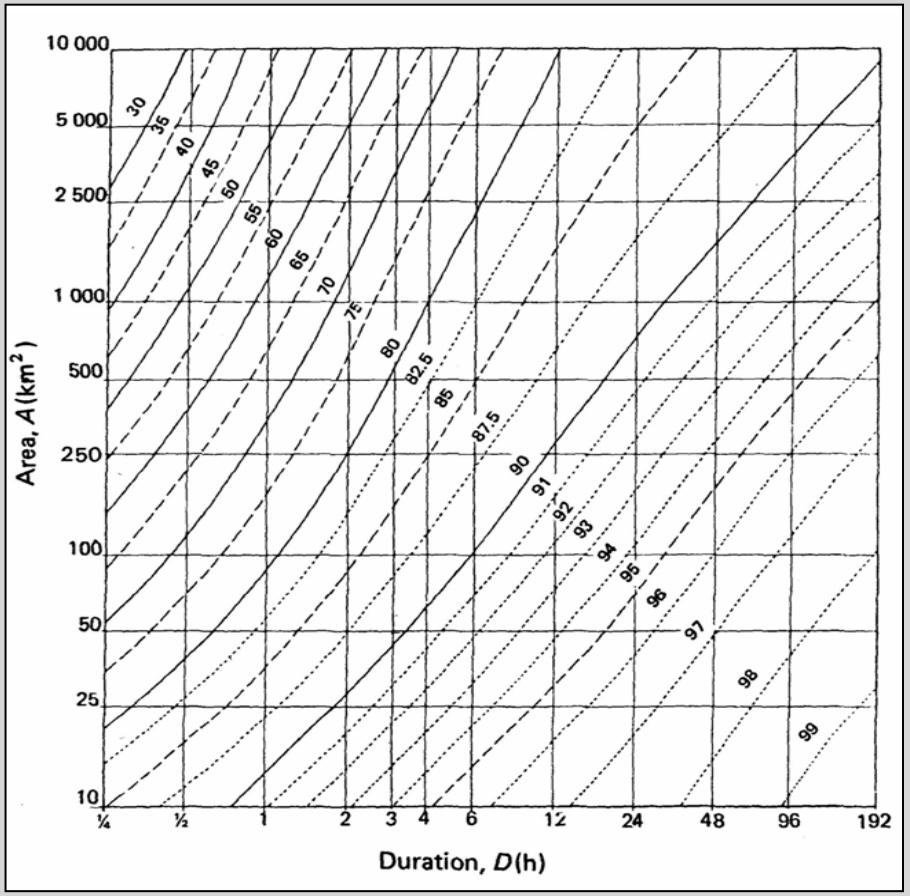




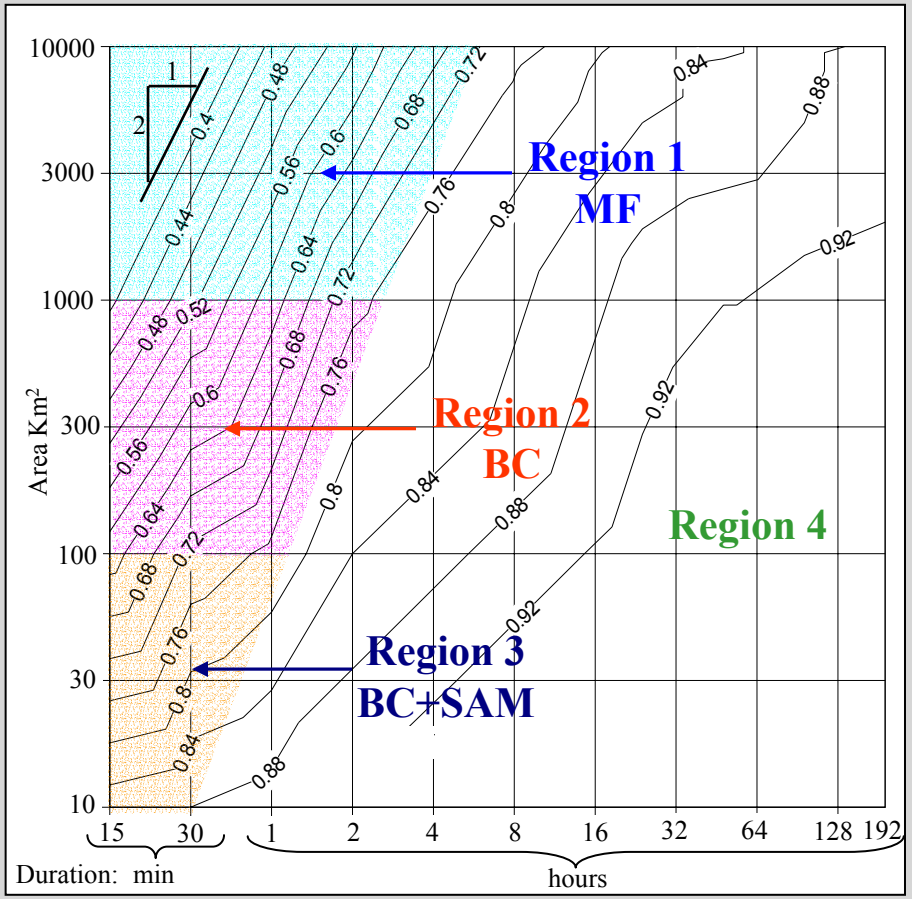


Application to ARF Data

N.E.R.C. Interpolation



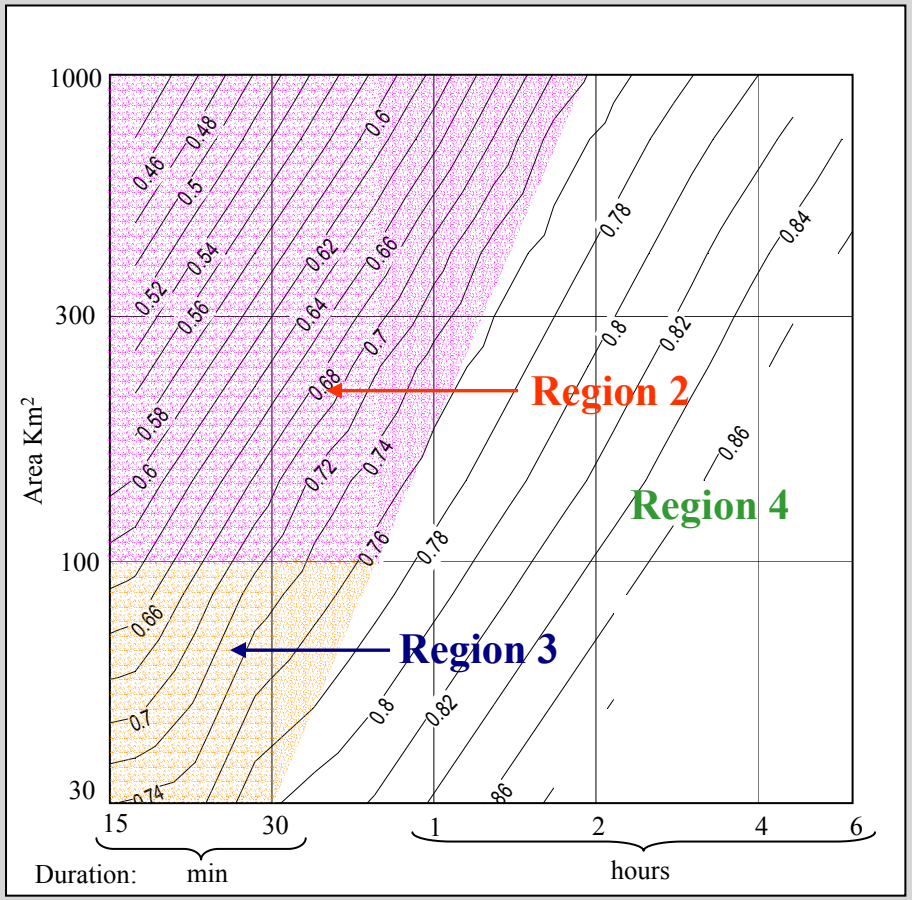
Our Interpolation



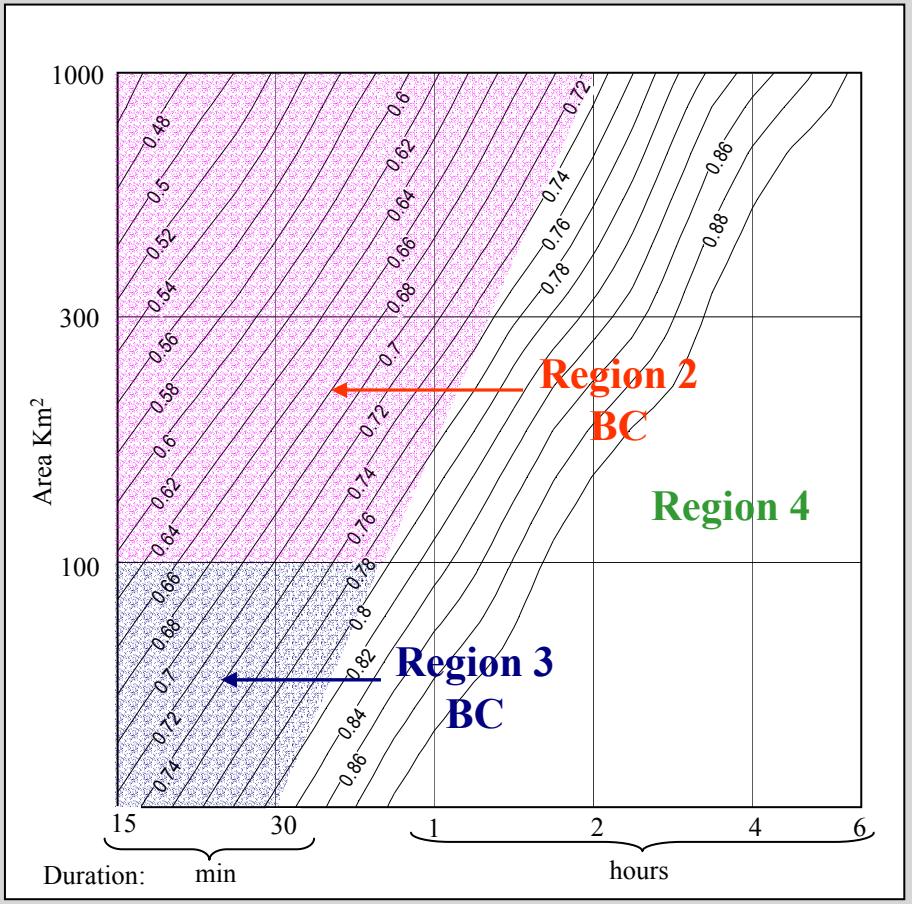
➢ Regions 2 and 3 - Bounded Cascade

Parameters from: { Menabde and Sivapalan (2000)
Perica and Foufoula-Georgiou (1996)

Our Interpolation (Magnification)

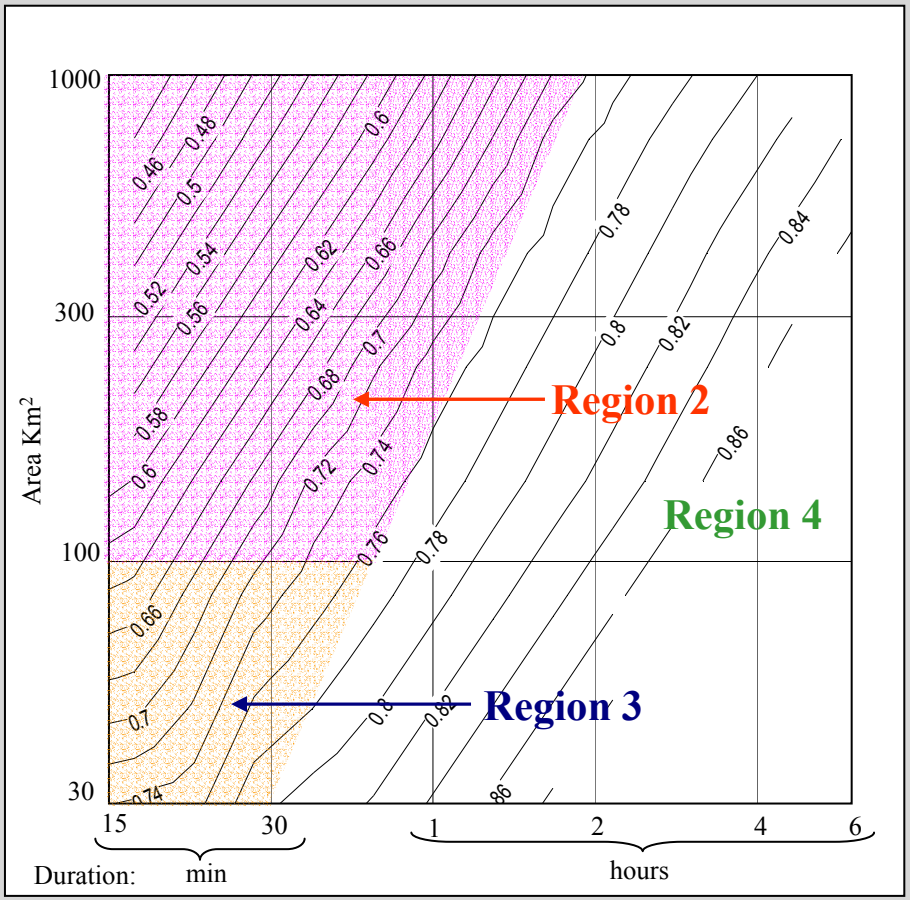


Simulation

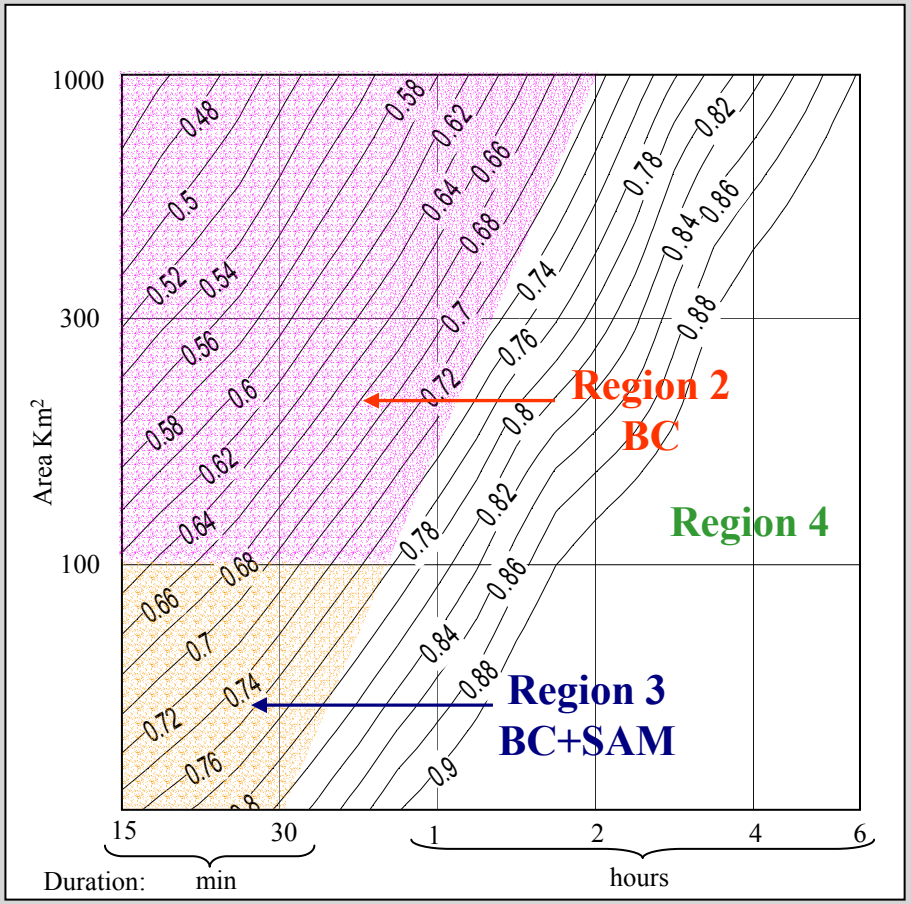


➢ Regions 2 and 3 - Bounded Cascade + Sampling

Our Interpolation (Magnification)



Simulation



SUMMARY & CONCLUSIONS

1. **Found limiting scaling of IDAF curves $\Rightarrow$ ARF factors**

2. **Studied distortions of ARF due to:**

   - Storm advection
   - Deviations from multifractality
   - Sparse sampling

3. **Reproduced various features of observed ARF factors**

4. **Behavior of observed ARF for large *D*/*L* (Region 4) to be explained**