Simple IDF Estimation Under Multifractality

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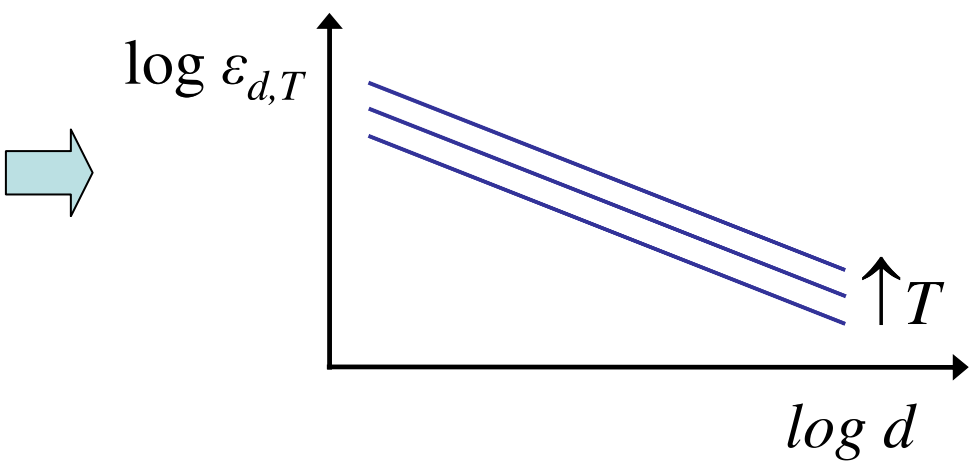
IDF Curve Definition

ε_d: average rainfall intensity over duration d

$\varepsilon_{d,max}$: annual maximum of ε_d

$\varepsilon_{d,T}$: value exceeded by $\varepsilon_{d,max}$ with probability $1/T$ (years)

IDF Curves



Methods of IDF Estimation

1) *From annual maxima*

- ***Separability assumption…***

$$\varepsilon_{d,T} = b(d)\ \alpha(T)$$

- $\varepsilon_{d,T}$ → *IDF value*
- $b(d)$ → *empirical function* **e.g. $(d+\eta)^{-\delta}$**
- $\alpha(T)$ → *empirical function* **e.g. T^k, log$T$** ***or*** From distribution of $\varepsilon_{d,max}$ **Koutsoyiannis *et al.* (1998)**

- Estimate ***a*(*T*)** and ***b*(*d*)** from the annual maxima time series

2) *From stochastic models of rainfall*

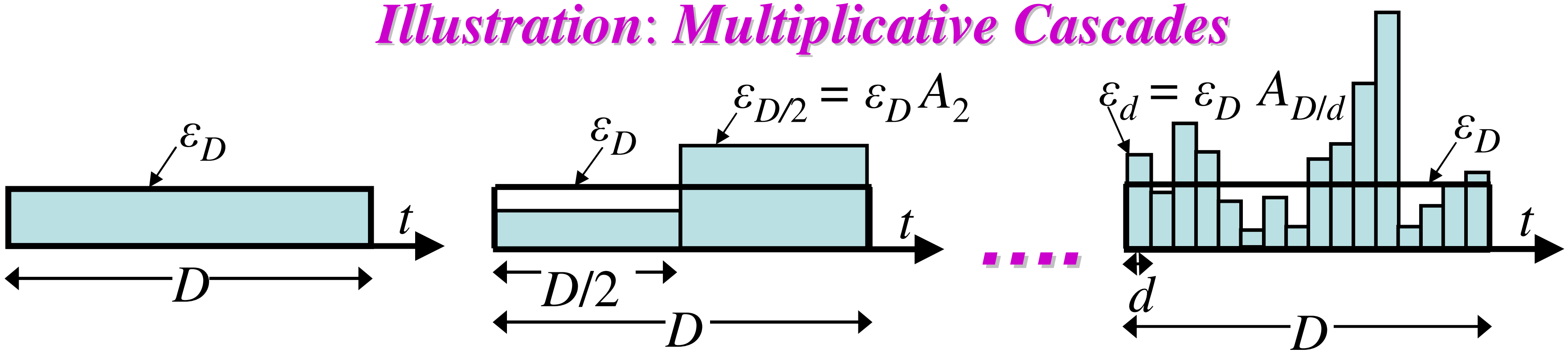
- Fit a model to the continuous rainfall record
- ***Calculate IDF curves from model*** ⇨ typically through MC simulation

Multifractal Rainfall Models

Definition:

Temporal *rainfall* is said to be *multifractal* (MF) if the *statistics* remain *unchanged* when the *observation axis is contracted by a factor* $r > 1$ and the *rainfall* intensity is *multiplied* by some random variable A_r.

Illustration: Multiplicative Cascades



Advantages

- ***Few parameters***
- ***No need for Monte Carlo simulation***
- ***No a-priori assumption on $\varepsilon_{d,T}$***
- ***Known asymptotic** IDF **scaling properties** for $d \to 0$ and $T \to \infty$*
- ***Use of the full historical record*** *(i.e. utilize information on the distribution of ε_d for $\neq d$)*

Asymptotic IDF Behavior Under Multifractality

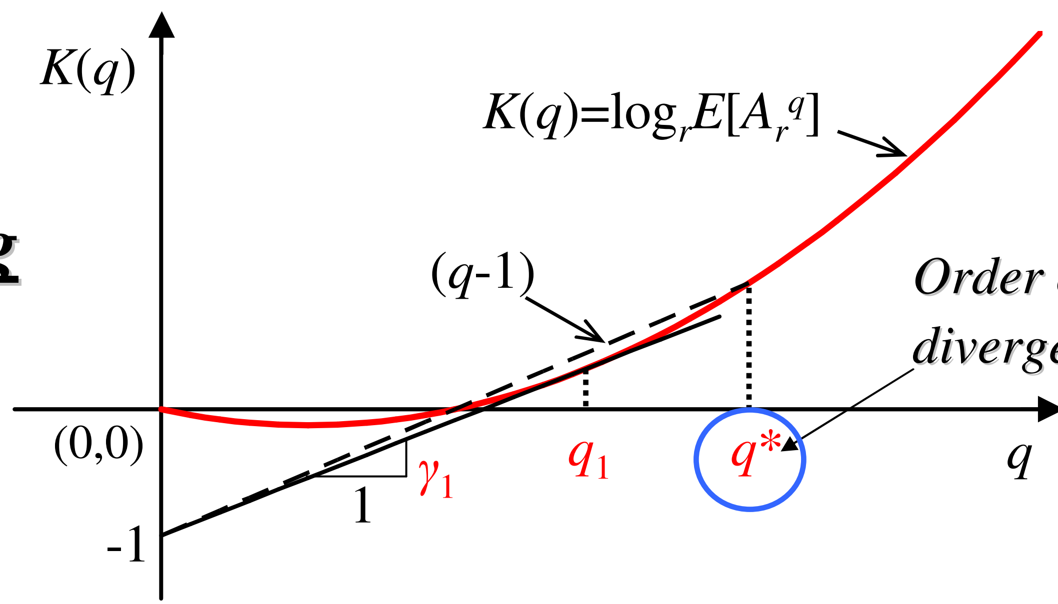
❖ *Multifractal models* have known *asymptotic IDF scaling properties* (Hubert *et al.*, 1999; Veneziano and Furcolo, 2002)

$$\varepsilon_{d,T} \propto \begin{cases} d^{-\gamma_1}\, T^{1/q_1}, & \text{for } d \to 0 \text{ and } T \text{ finite} \\ d^{-1}\, T^{1/q^*}, & \text{for } d \text{ finite and } T \to \infty \end{cases}$$

➢ Separability holds for very small d or very large T

$$\varepsilon_{d,T} \sim T^{k}\, d^{-\delta}$$

Moment scaling function



What's next…

- *Multifractal rainfall models…*

IDF calculation

Accurate and simple IDF calculation procedures **(+ *example application*)**

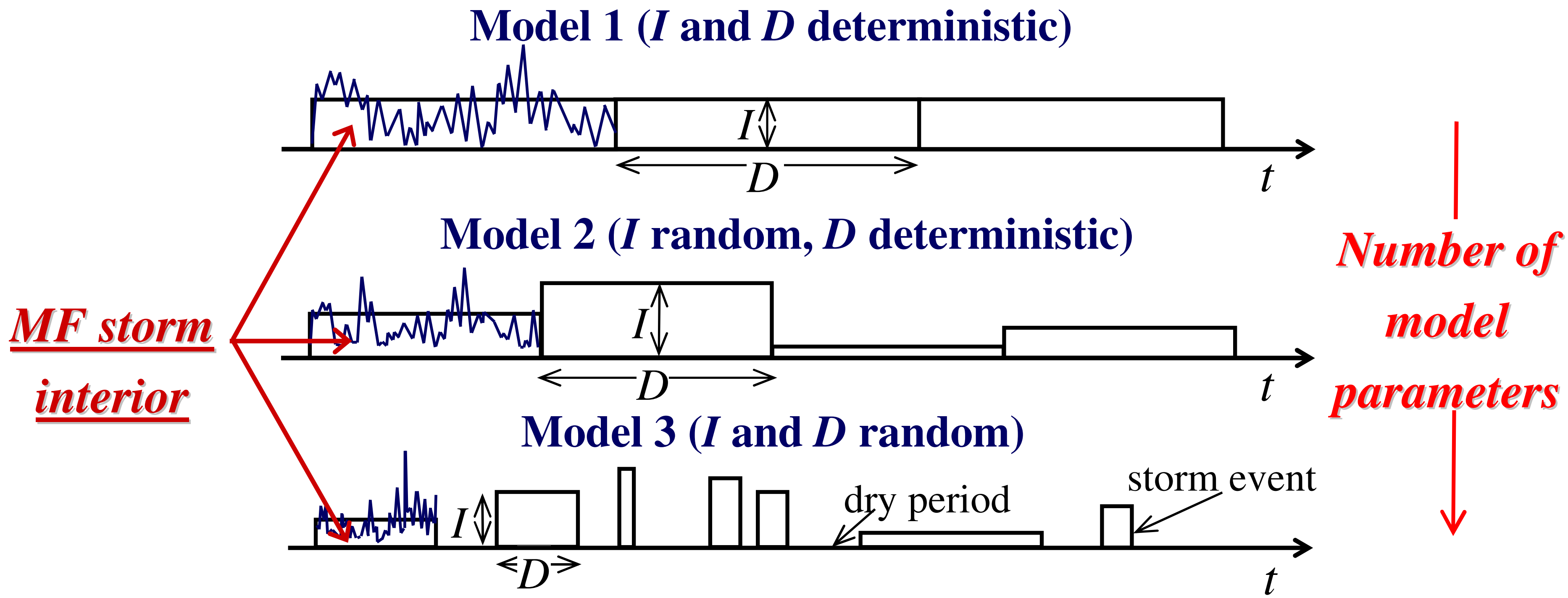
Identification of separability region

Bias of q estimates from tail fitting* **(*commonly done*)**

- *Comparison of annual maxima and MF methods…see poster*

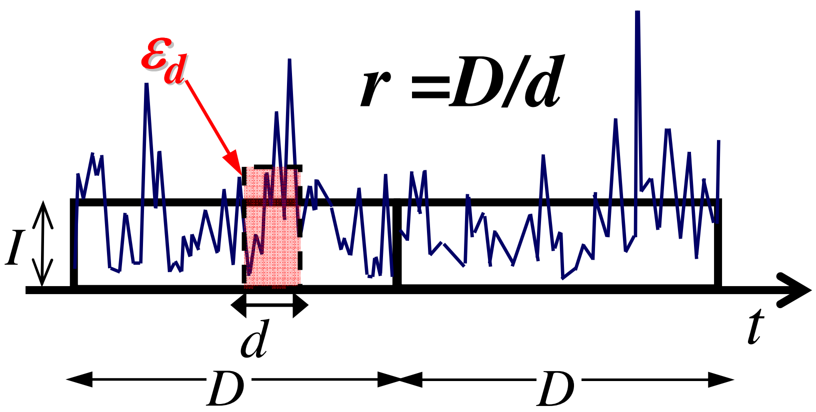
Rainfall Models With MF Interiors

Langousis and Veneziano (2007):



IDF Calculation Under Model 1

➢ ***Rainfall as a sequence of iid multiplicative cascades***



D: *Upper limit of Multifractality (average "storm" interarrival time)*

I: *Mean rainfall intensity*; **Set it to 1**

Mean 1 MF interior: A_r ~ (β-LN)

❖ $P[A_r = 0] = 1 - r^{-C_\beta}$

❖ $(\ln A_r | A_r > 0) \sim N[(C_\beta - C_{LN})\ln r,\ 2C_{LN}\ln r]$

➢ C_β: Intra-storm dry periods

➢ C_{LN}: Multiplicative fluctuations when it rains

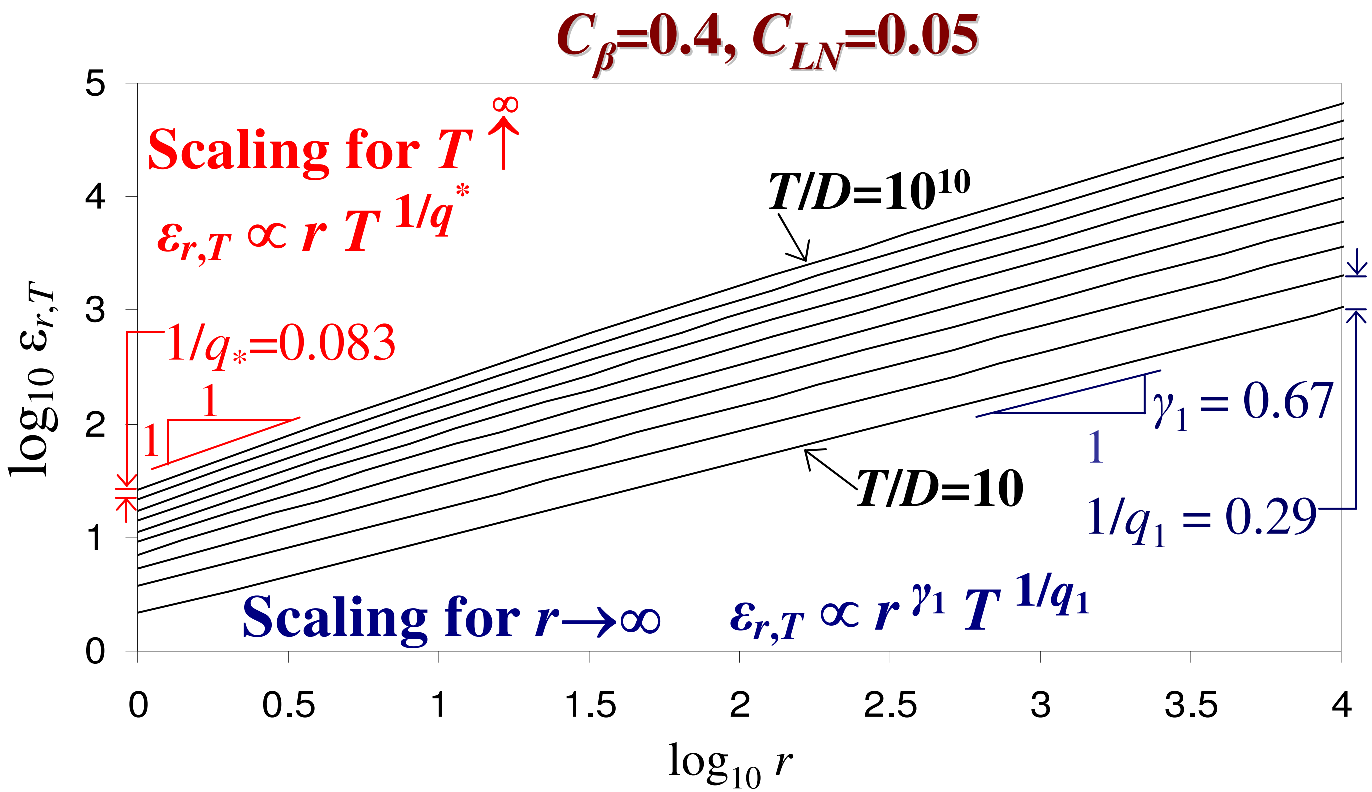
Asymptotic power law tail

$$\varepsilon_r = \varepsilon_{d=D/r} \overset{d}{=} A_r Z$$

$$\lim_{z\to\infty} P[Z > z] \propto z^{-q^*}$$

$$\lim_{\varepsilon\to\infty} P[\varepsilon_r > \varepsilon] \propto \varepsilon^{-q^*}$$

Exact IDFs Under Model 1



...IDF calculation is numerically heavy

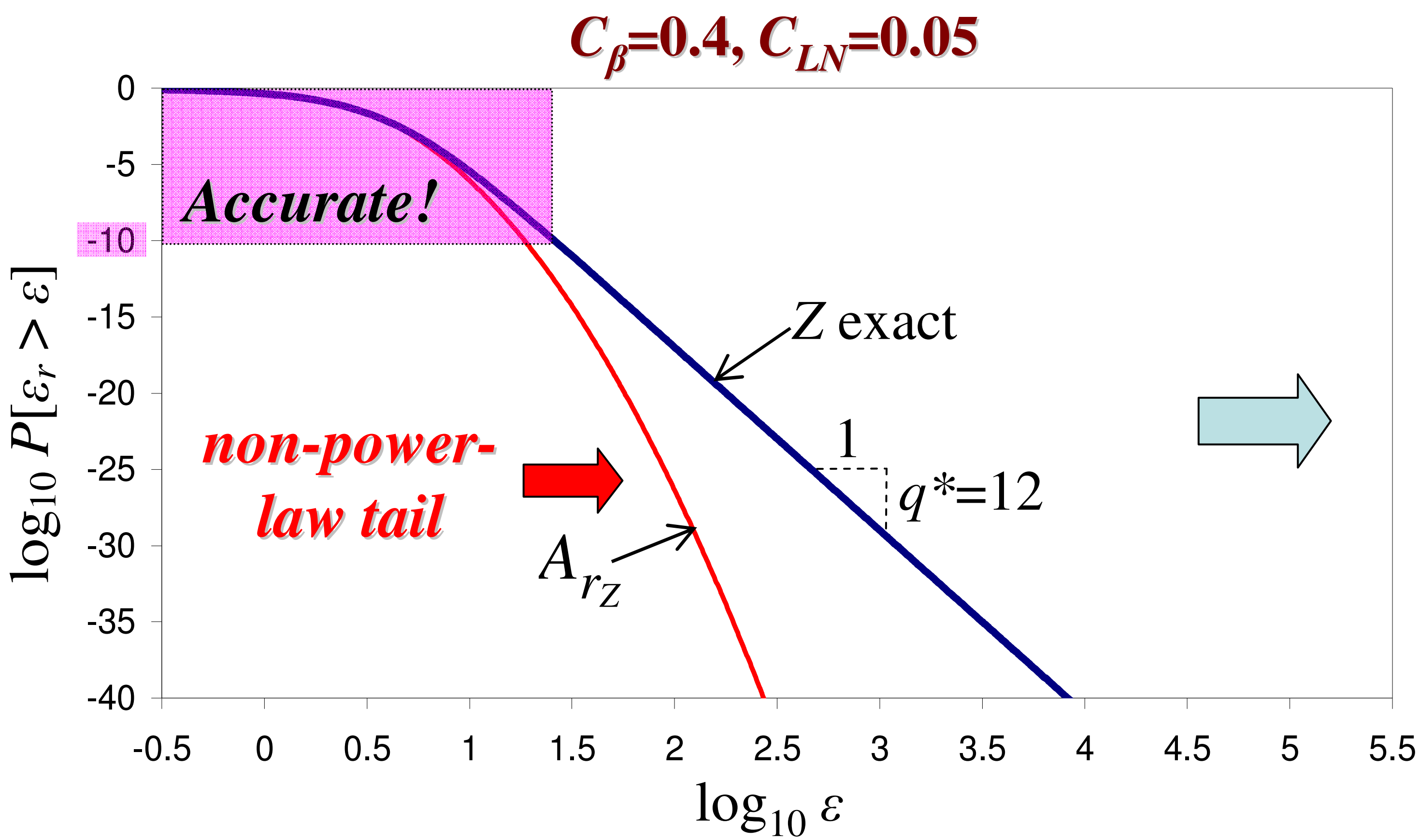
- ***Develop accurate but simple IDF calculation methods***

Approximation 1

I) *Replace Z with A_{r_Z}*

⇨ *Calculate r_Z so that A_{r_Z} matches some moment order q'* **of** *Z*

⇨ $q' \approx q^*/2 = (1-C_\beta)/2C_{LN}$ *works well….*



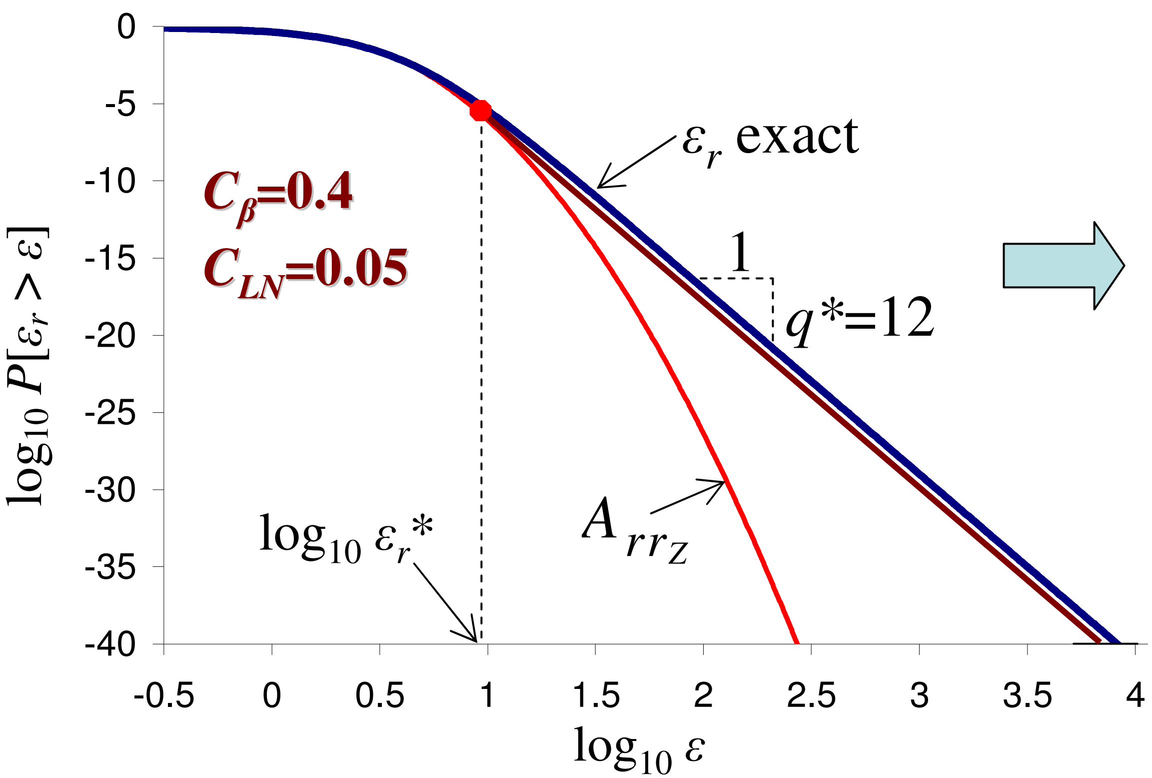
$$\varepsilon_r \overset{d}{=} A_{rr_Z}$$

Approximation 2

II) *Improvement over A_{rr_Z} for the upper tail region*

- *Find the point ε_r* at which the log-log slop of the distribution of A_{rr_Z} equals $-q^*$*

- *"Graft" a q^* power law tail to the distribution of A_{rr_Z} above ε_r**



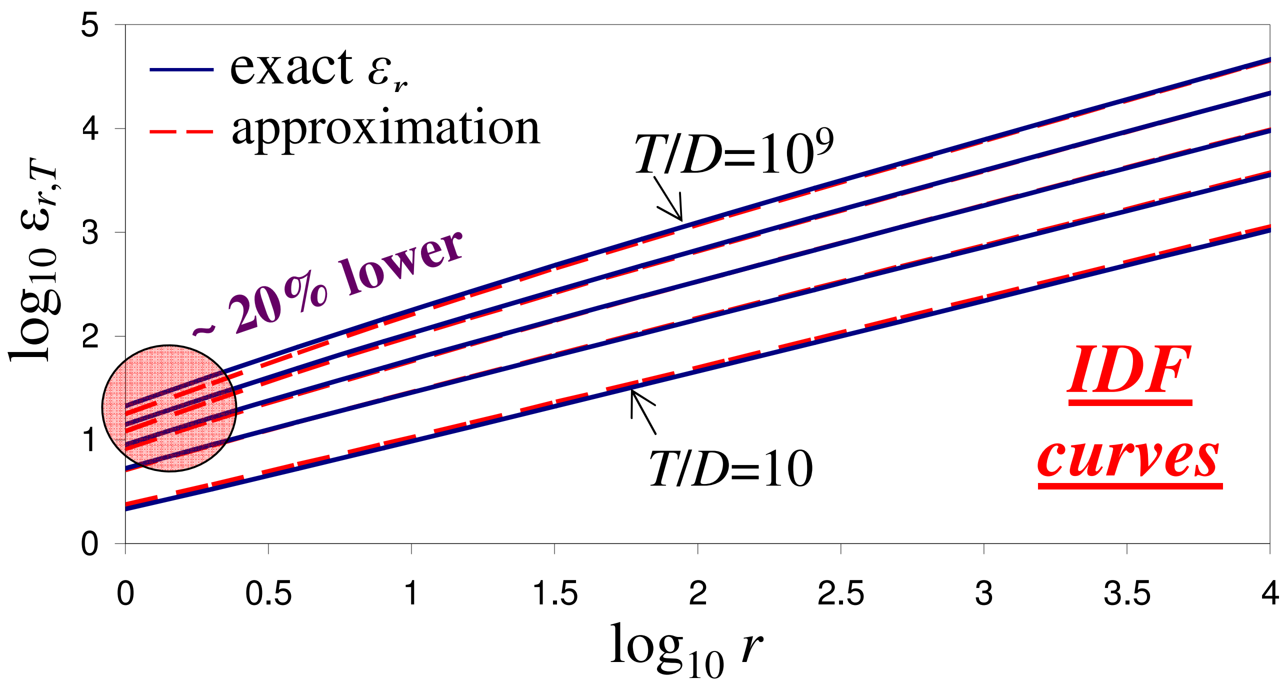
$\varepsilon_r \overset{d}{=} A_{rr_Z}$, **for** $\varepsilon \le \varepsilon_r^*$

$P[\varepsilon_r > \varepsilon] \propto \varepsilon^{-q^*}$, **for** $\varepsilon > \varepsilon_r^*$

Analytical Results

➢ ***Let $\varepsilon=(rr_Z)^\gamma$***

$$T_{r,\gamma} \approx \begin{cases} \dfrac{D}{r}\left[(2\pi)2C_{LN}\left(\dfrac{\gamma-C_\beta}{2C_{LN}}+\dfrac{1}{2}\right)^2 \ln(rr_Z)\right]^{1/2} (rr_Z)^{C_{LN}\left(\frac{\gamma-C_\beta}{2C_{LN}}+\frac{1}{2}\right)^2+C_\beta} & ,\ \gamma \le 2-C_\beta-C_{LN} \\ \dfrac{D}{r}\left[(2\pi)2\ln(rr_Z)\,\dfrac{(1-C_\beta)^2}{C_{LN}}\right]^{1/2} (rr_Z)^{[1+(\gamma-1)\frac{1-C_\beta}{C_{LN}}]} & ,\ \gamma > 2-C_\beta-C_{LN} \end{cases}$$



Practical IDF Estimation….

1) *Estimate the mean rainfall intensity* I *of the historical record.*

2) *Choose* C_β *and* C_{LN} *to match* $K(0)$ *and* $K(3)$

- $C_\beta = -K(0)$
- $C_{LN} = [K(3)+2K(0)]/6$

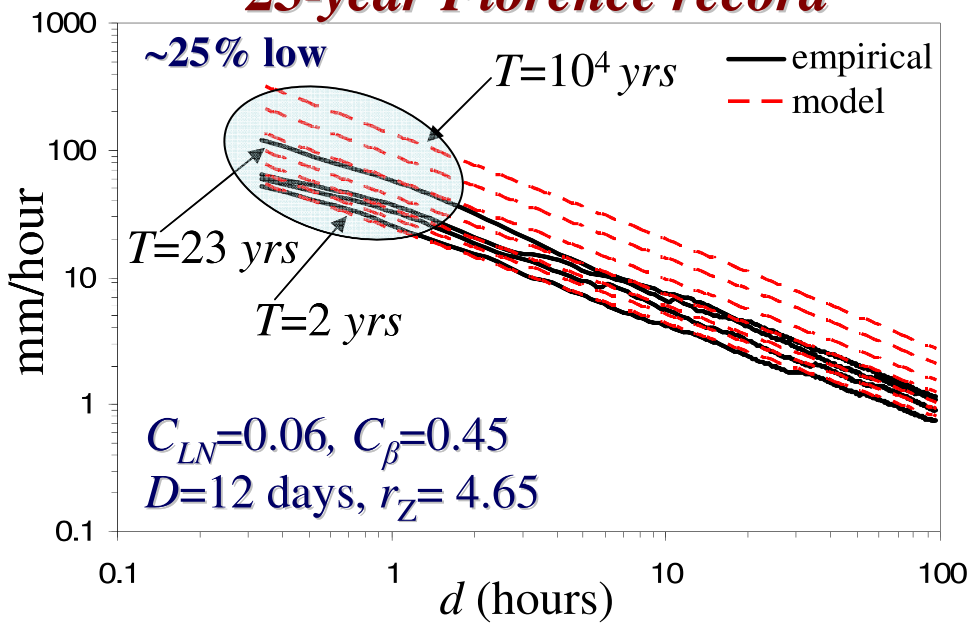
3) *Calculate* $r_Z(C_\beta, C_{LN})$ *from diagram*

4) *Set* D *such that the empirical moment* $E[\varepsilon_D^3]$ *equals the theoretical value* $I^3 r_Z^{K(3)}$.

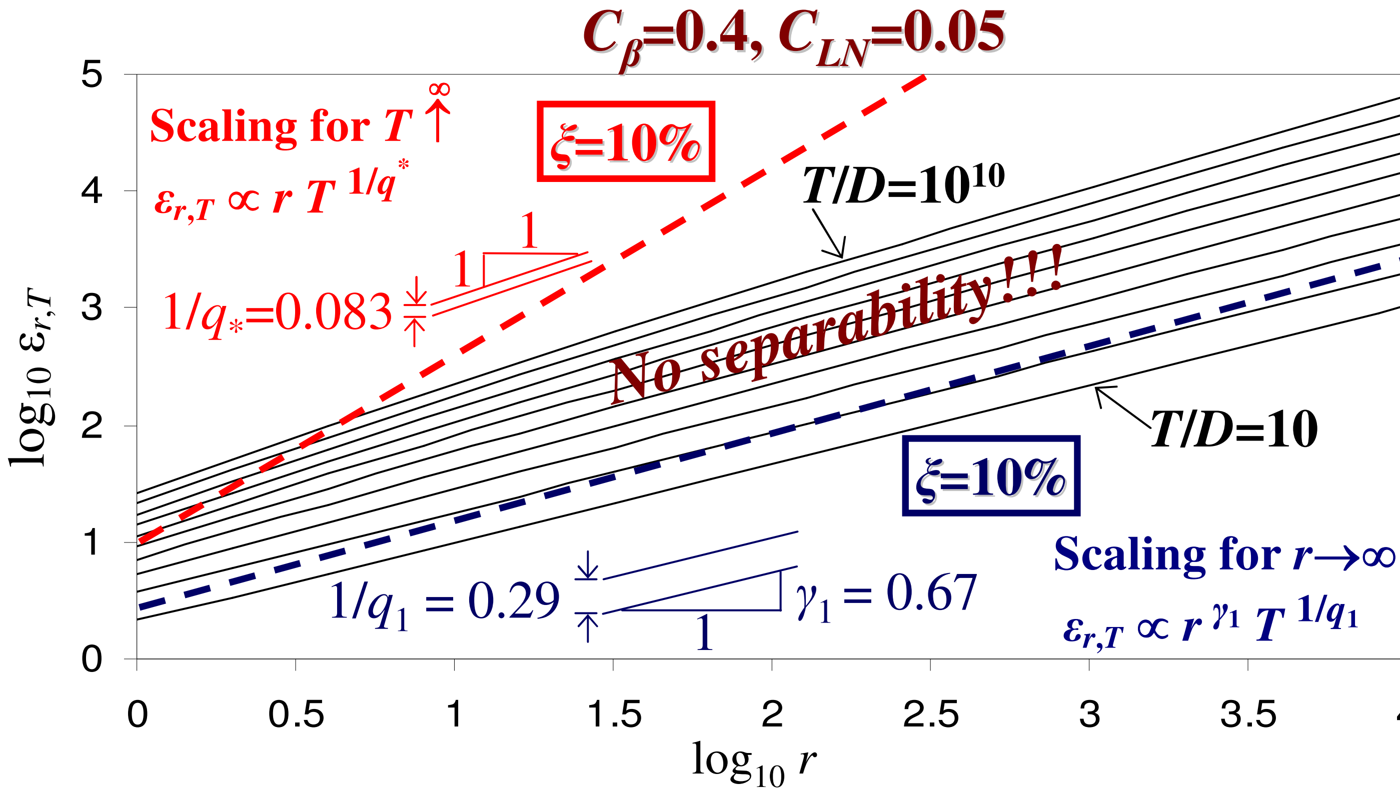
5) *Estimate the IDF values* $\varepsilon_{d,T}$ *using the previous equation.*

6) *Multiply the calculated IDF values by the average rainfall intensity* I.

23-year Florence record



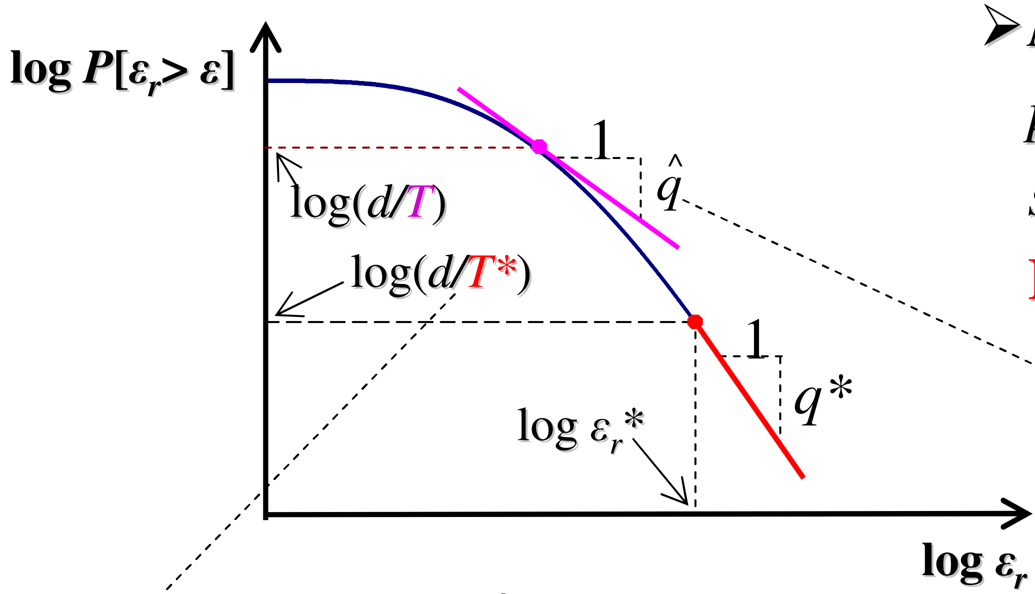
Ranges of Approximate IDF Scaling



➢Find regions where the actual slope and spacing are within *ξ %* of the asymptotic values

… analytical expressions for boundaries in Langousis *et al.* (2007)

Bias in the Empirical Estimation of q^*



- *Many studies infer multifractal parameters (C_β, C_{LN}) from the tail slope of the empirical distribution.*

Is there any bias?

$$\hat{q} = \lambda\, q^*$$

$$\lambda = \frac{\sqrt{C_{LN}\,[\log_{rr_Z}(rT/5D) - C_\beta]}}{1 - C_\beta}$$

$$T^* \approx \frac{5D}{r}\,(rr_Z)^{\left[\frac{(1-C_\beta)^2}{C_{LN}} + C_\beta\right]}$$

Minimum record length to observe the q* tail…

Example:

C_{LN}=0.06, C_β=0.45
D=12 days, r_Z= 4.65

d	r	T^* (yrs)
20 min	864	**$1.2\ 10^{16}$**
1 hour	88	**$8.5\ 10^{13}$**
1 day	12	**$5.4\ 10^{7}$**

Example: (T=50yrs of data)

d	λ	$\hat{q}$	q^*
20 min	**0.45**	**4.1**	**9.2**
1 hour	**0.47**	**4.3**	**9.2**
1 day	**0.56**	**5.15**	**9.2**

… range found in literature

Summary and Conclusions

1) *We developed IDF estimation methods assuming multifractality of rainfall.*

2) *Simple approximations led to analytical expressions suitable for engineering practice.*

3) *We identified the regions on the* (*T*,*d*) *plane where the IDF curves are separable and scaling* (*common assumption in MF literature*).
⇨ *Wide range of non-separability*.

4) *Bias when estimating the power-law slope of the distribution of* ε_d (*common procedure in MF literature*).

➢ ***Continuation…*** (Poster: XY0569, tomorrow, Halls X/Y)

- ***Comparison of annual maxima* and *MF methods***
  - *Accuracy*
  - *Robustness*
  - *Sensitivity to outliers*
- ***Are actual IDF's separable in d and T ?***
- ***Modifications for non scaling rainfall***

Thank you for your time!