1st International Summit on Hurricanes and Climate Change,
May 27-June 1 2007, Crete, Hellas

Theoretical Estimation of the Mean Rainfall Field in Tropical Cyclones

A. Langousis, D. Veneziano
Department of Civil and Environmental Engineering, Massachusetts Institute of Technology (M.I.T.), U.S.A.

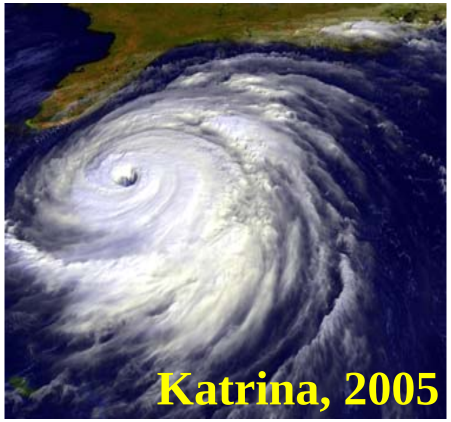
S.S. Chen
RSMAS/University of Miami, U.S.A

Objectives

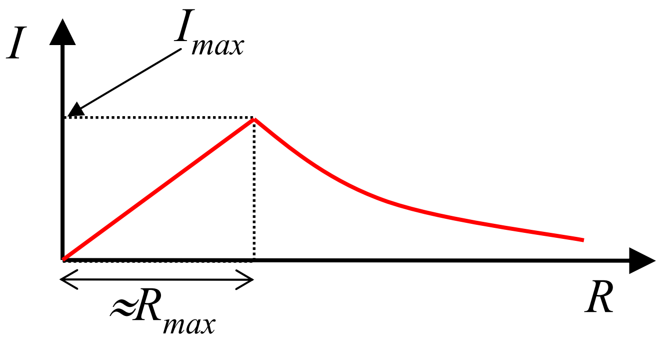
➢ ***Develop a simple model for the mean rainfall field in TCs***

Study how mean rainfall varies with TC parameters:

- cyclone intensity V_{max}
- radius of maximum winds R_{max}
- Holland's B parameter
- translation velocity V_c of the cyclone

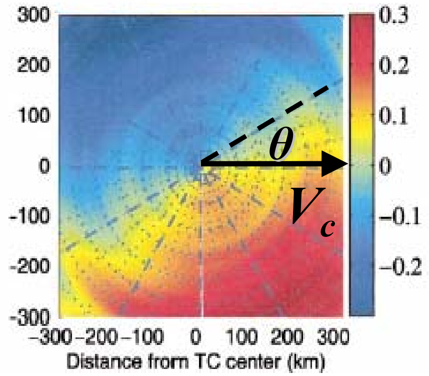


Axi-symmetric component

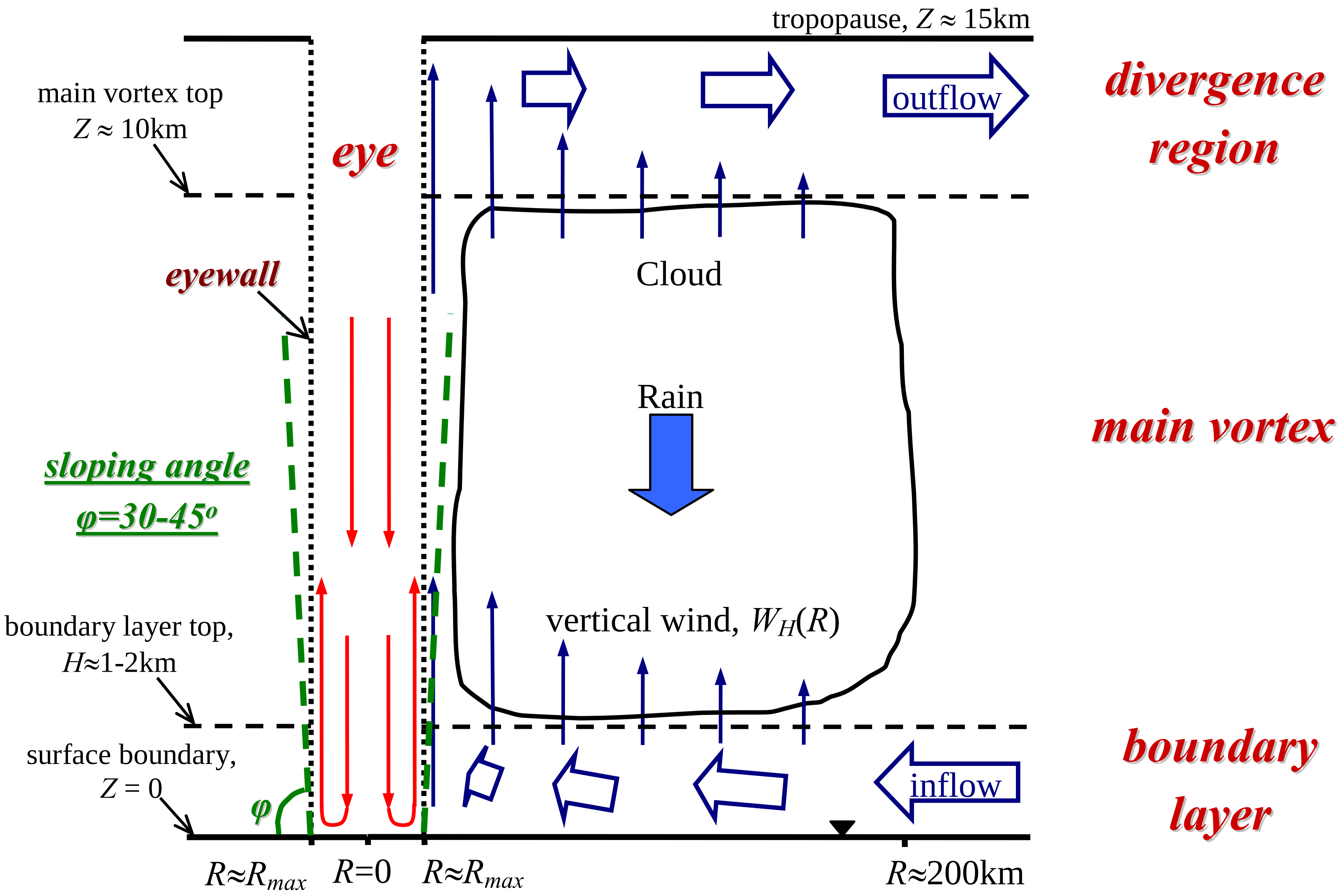


+

Asymmetry due to motion



Schematic structure of a TC



What goes up must come down…

Assumption:

rainrate= *upward* ***water vapor flux*** *at the top of the boundary layer*

$$I(R,\theta)=\left(\frac{\rho_{air}}{\rho_{w}}\,q_{w}(T)\right)W_{H}(R,\theta)$$

mean rainfall intensity

dry air density

liquid water density

water vapor mixing ratio

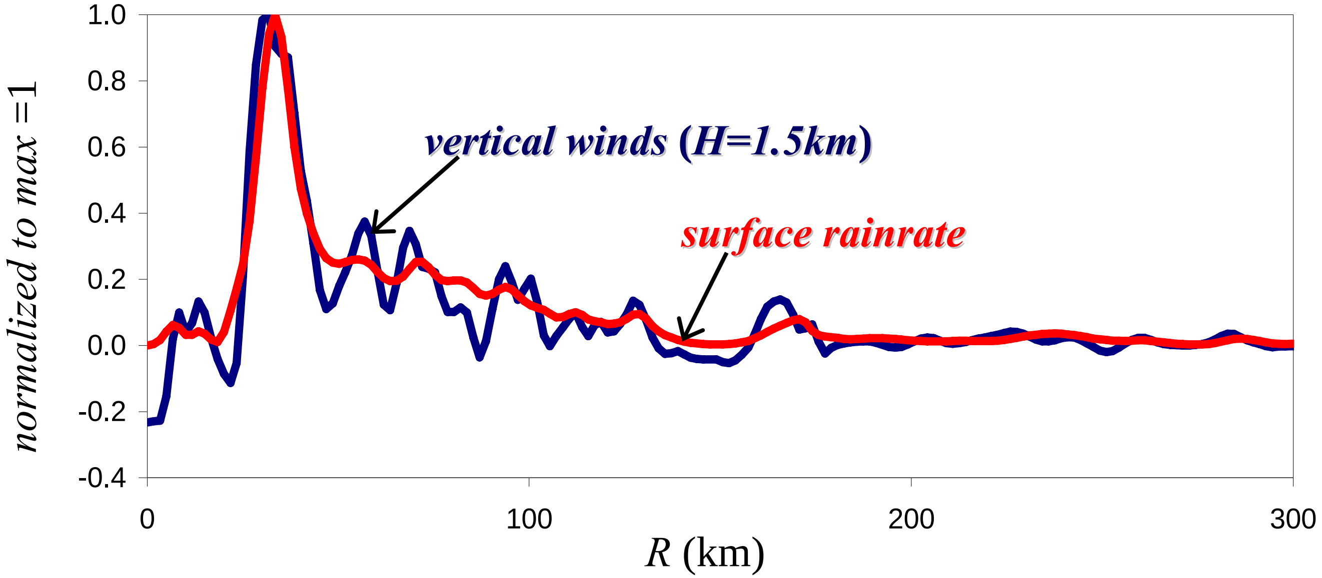
depth averaged temperature

vertical wind velocity

Almost constants:

- $T\approx 20^{o}$C
- Satur. ≈ 0.8
- $q_w \approx 10\text{-}13$ gr/kgr

➢ ***Validation using MM5!***



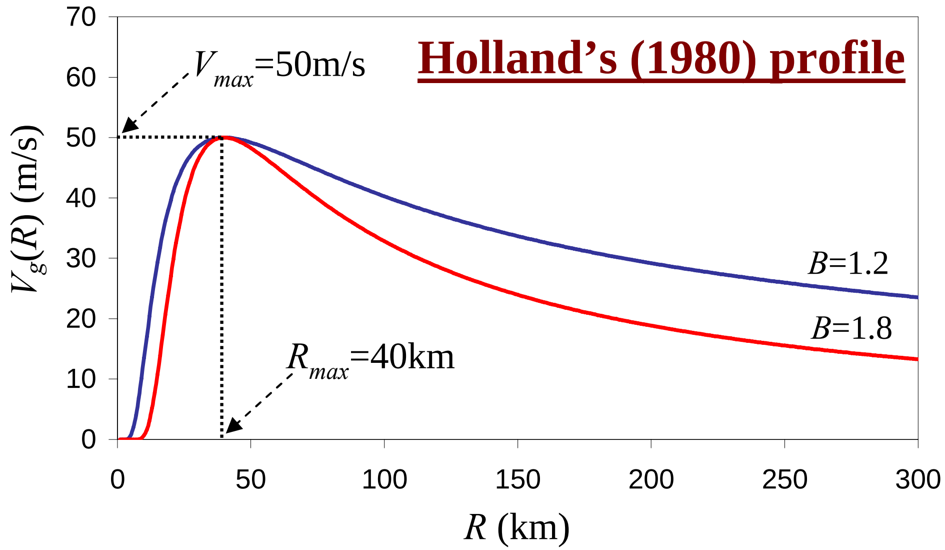
Use a BL model to calculate W_H…

Solving the Boundary Layer (BL)…

Conditions at BL top

$$U=0 \quad V=V_g$$

$$\frac{\partial W}{\partial Z}=\frac{\partial U}{\partial Z}=\frac{\partial V}{\partial Z}=0$$



W_H

Main vortex

BL top (Z=H)

BL equations

$$U\frac{\partial U}{\partial R}+\frac{V}{R}\frac{\partial U}{\partial \theta}+W\frac{\partial U}{\partial Z}+\frac{V_g^2-V^2}{R}+f(V_g-V)=K\frac{\partial^2 U}{\partial Z^2}$$

$$U\frac{\partial V}{\partial R}+\frac{V}{R}\frac{\partial V}{\partial \theta}+W\frac{\partial V}{\partial Z}+\frac{UV}{R}+fU=K\frac{\partial^2 V}{\partial Z^2}$$

$$\frac{1}{R}\frac{\partial (RU)}{\partial R}+\frac{1}{R}\frac{\partial V}{\partial \theta}+\frac{\partial W}{\partial Z}=0$$

vertical diffusion coef.

$K \approx$ 10-50m^2/s

geostrophic cond. *eyewall*

U: radial *V: tangential*

W: vertical

Surface boundary (Z=0)

Stress conditions with drag coefficient C_d ($C_d \rightarrow \infty$ for non-slip)

Boundary layer model 1: Kepert (2001)

Features:

✓ ***Analytical*** *and* ***depth resolving***

- BCs at Z=0 and Z=$H \rightarrow \infty$
- BL scale thickness: $\boldsymbol{\delta(R,\theta)}$

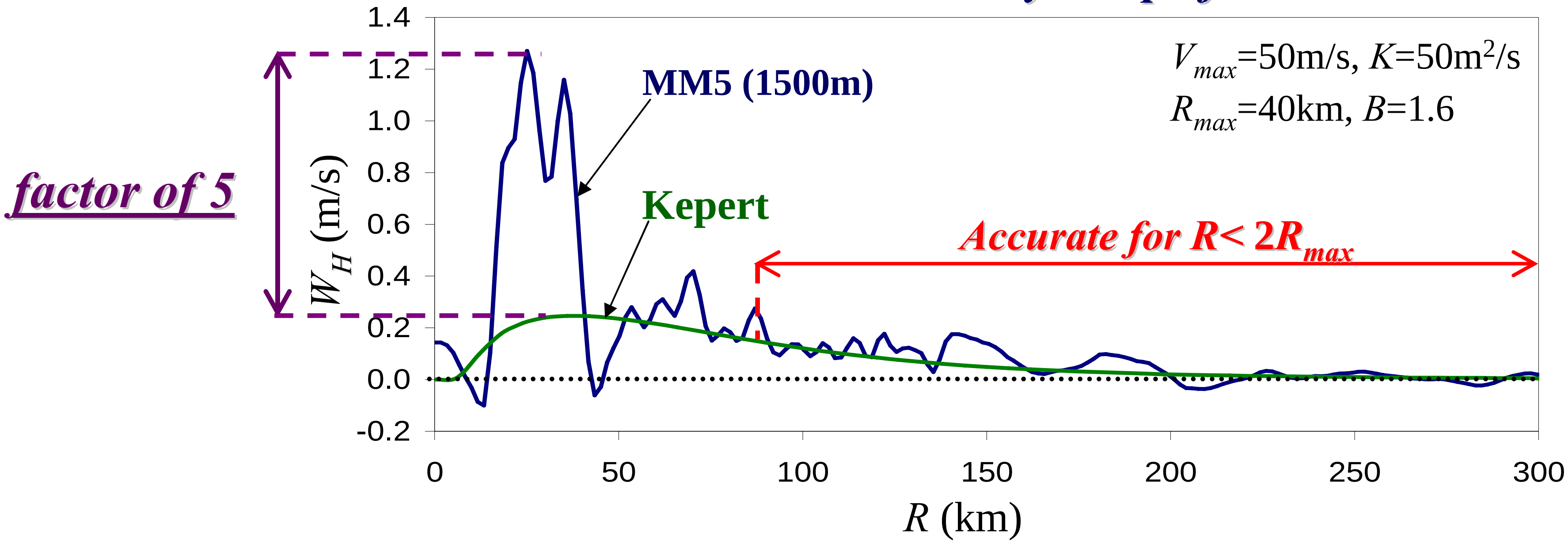
✓ Accounts for ***storm translation***

× ***Linearized version of BL equations***

Model breaks:

- *for large horizontal gradients* $\Rightarrow$ $\boldsymbol{R< 2R_{max}}$
- *for large vertical gradients* $\Rightarrow$ $\boldsymbol{C_d \rightarrow \infty}$
- *for high translation velocities* $\Rightarrow$ $\boldsymbol{V_c}$ **>5m/s**
- *under inertial neutrallity* $\Rightarrow$ $\boldsymbol{B}$ **>1.8**

Vertical wind velocity at top of BL



Boundary layer model 2: Shapiro (1983)

Features:

✓ Vertically averaged ✓ Accounts for ***storm translation***

Issues:

× *High horizontal velocities*

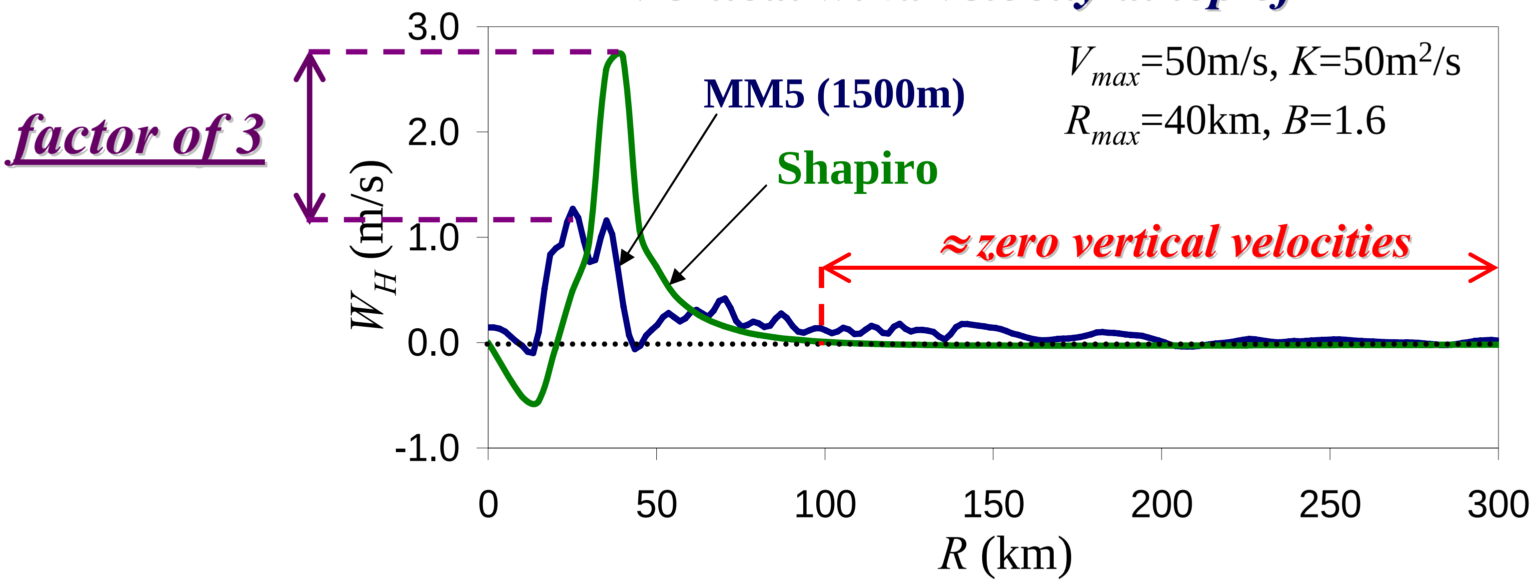
× ***Stability for $R>R_{max}$ requires***

constant boundary layer depth ***H*=1000m**

vertical diffusion coefficient ⇒ ***K*=50000m²/s**

discretization step ⇒ ***ΔR = 5km***

Vertical wind velocity at top of BL



Boundary layer model 3: Smith (1968)

Karman & Pohlhausen momentum integral method:

- ***Assume*** that dependence of V and U on Z is of the ***Ekman*** type:

$$V(R,Z)=V_g(R)\, f[Z/\delta(R)] \qquad U(R,Z)=E(R)\, V_g(R)\, g[Z/\delta(R)]$$

gradient winds (V_g) *BL scale thickness* (δ) *amplitude coef.* (E)

Smith (1968): Ekman solutions

$$f(\eta) = -e^{-\eta}\,(a_1 \sin\eta + a_2 \cos\eta)$$
$$g(\eta)=1-e^{-\eta}\,(a_1 \cos\eta + a_2 \sin\eta)$$

- ***Substitute*** U and V into the BL equations
- ***Integrate*** *in the* ***vertical*** *direction* accounting for boundary conditions
- ***Solve*** ordinary differential equations (ODEs) for $E(R)$ and $\delta(R)$

Limitations:
- ***Stationary hurricanes***
- $a_1, a_2 = const. \Rightarrow$ ***Applies only for non-slip BCs***

Modification of Smith (1968) for a moving storm

- **Wind speeds (relative to the moving vortex):**

$$\left.\begin{aligned} V(R,\theta,Z) &= V_g(R)\, f\left[\frac{Z}{\delta(R,\theta)}\right] \\ U(R,\theta,Z) &= E(R,\theta)\; V_g(R)\; g\left[\frac{Z}{\delta(R,\theta)}\right] \end{aligned}\right\} \Rightarrow W_H(R,\theta) = -\frac{1}{R}\int_0^{\infty}\frac{\partial(RU)}{\partial R} + \frac{\partial V}{\partial \theta}\, dZ$$

f & g functions:

$$\begin{cases} f(\eta) = -e^{-\eta}\,[a_1(R,\theta)\sin\eta + a_2(R,\theta)\cos\eta] \\ g(\eta) = 1 - e^{-\eta}\,[a_1(R,\theta)\cos\eta + a_2(R,\theta)\sin\eta] \end{cases}$$

Stress surface boundary
($C_d \to \infty$ for non-slip)

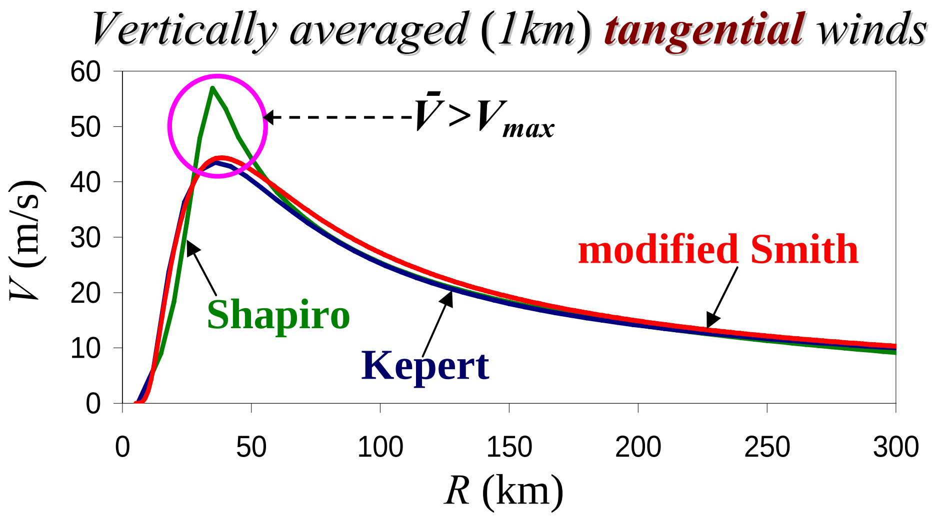
Analytical expressions for
a_1 and a_2

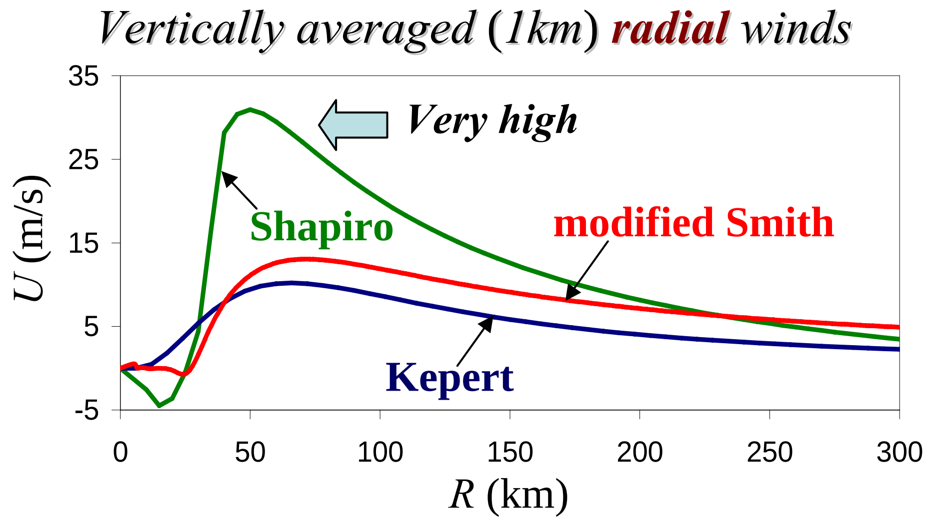
- ***Solve*** a non-linear system of partial DEs for $E(R,\theta)$ and $\delta(R,\theta)$

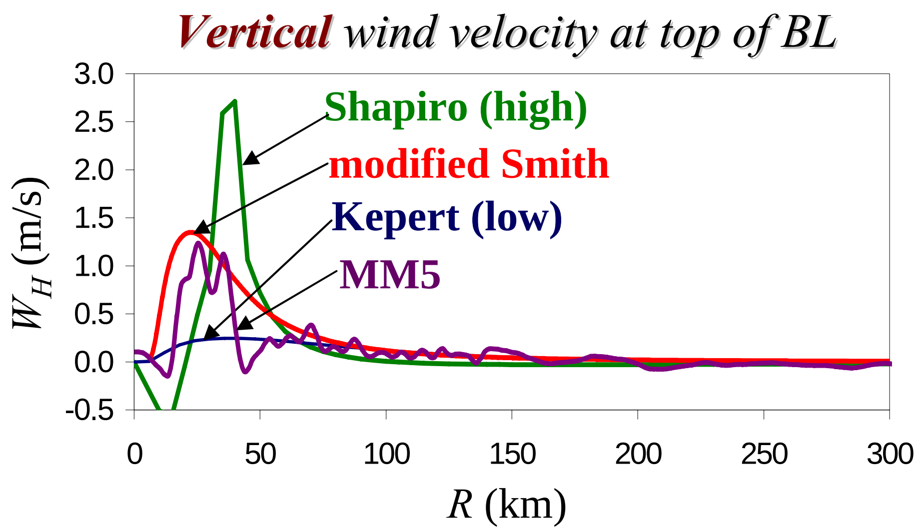
✓ *Numerically stable and fast formulation*

Model comparison: Stationary hurricane

(V_{max}=50m/s, R_{max}=40km, B=1.6, K=50m²/s)





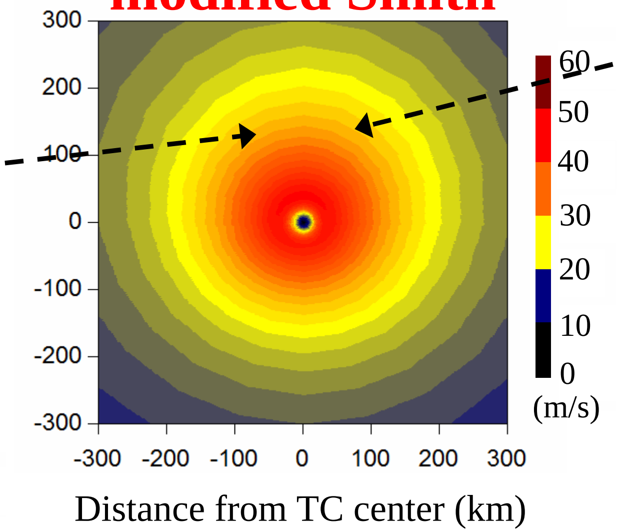


Model comparison: Moving hurricane (5m/s)

(V_{max}=50m/s, R_{max}=40km, B=1.2, K=50m^2/s)

Vertically averaged (1km) ***tangential*** *winds*:

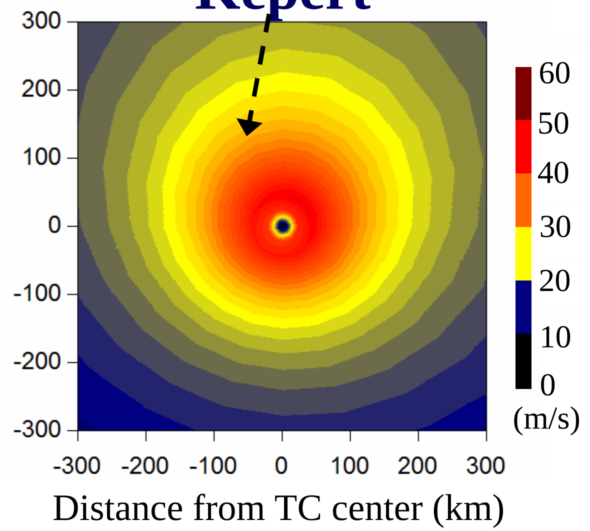
modified Smith



Intensification due to motion

Small differences…

Kepert

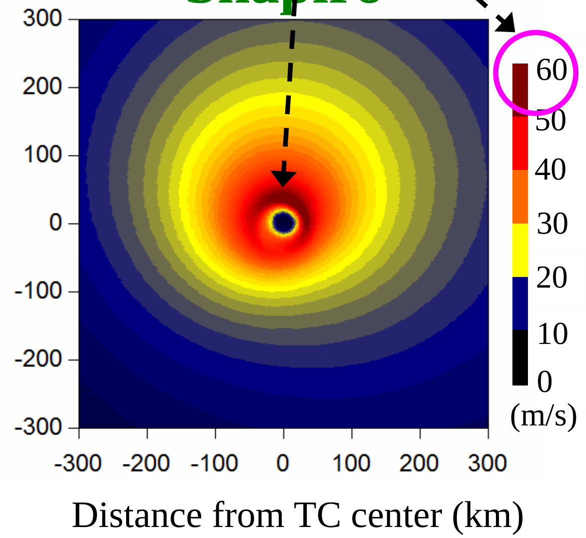


Direction of motion

$\bar{V}_c$=5m/s

$\bar{V} > V_{max}$

Shapiro

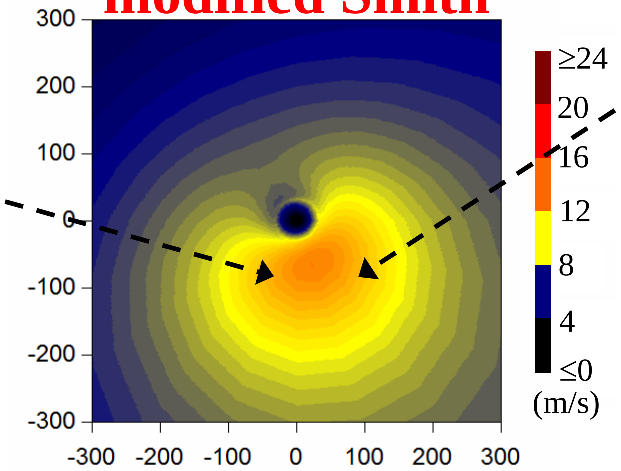


Model comparison: Moving hurricane (5m/s)

Vertically averaged (1km) ***radial*** *winds:*

Kepert solution is ***more sensitive*** *to* V_c (missing non-linear terms)

modified Smith

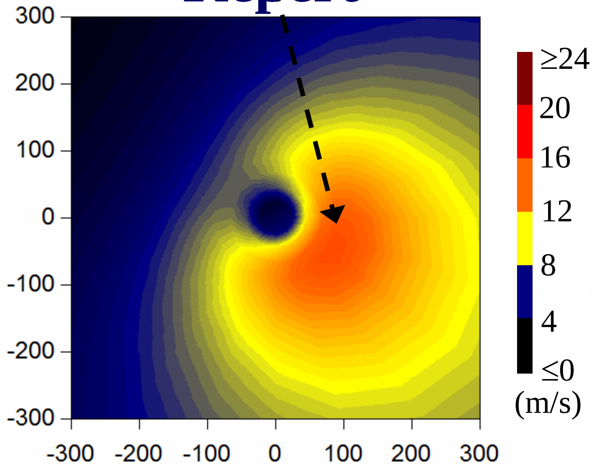


Distance from TC center (km)

Intensification due to motion

Very high radial wind velocities

Kepert

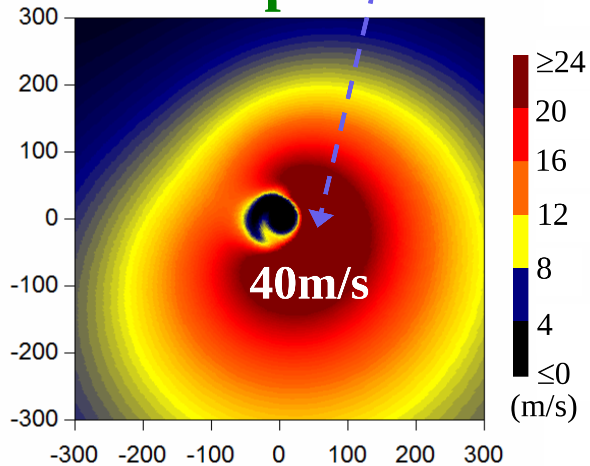


Distance from TC center (km)

Direction of motion

$\bar{V}_c$=5m/s

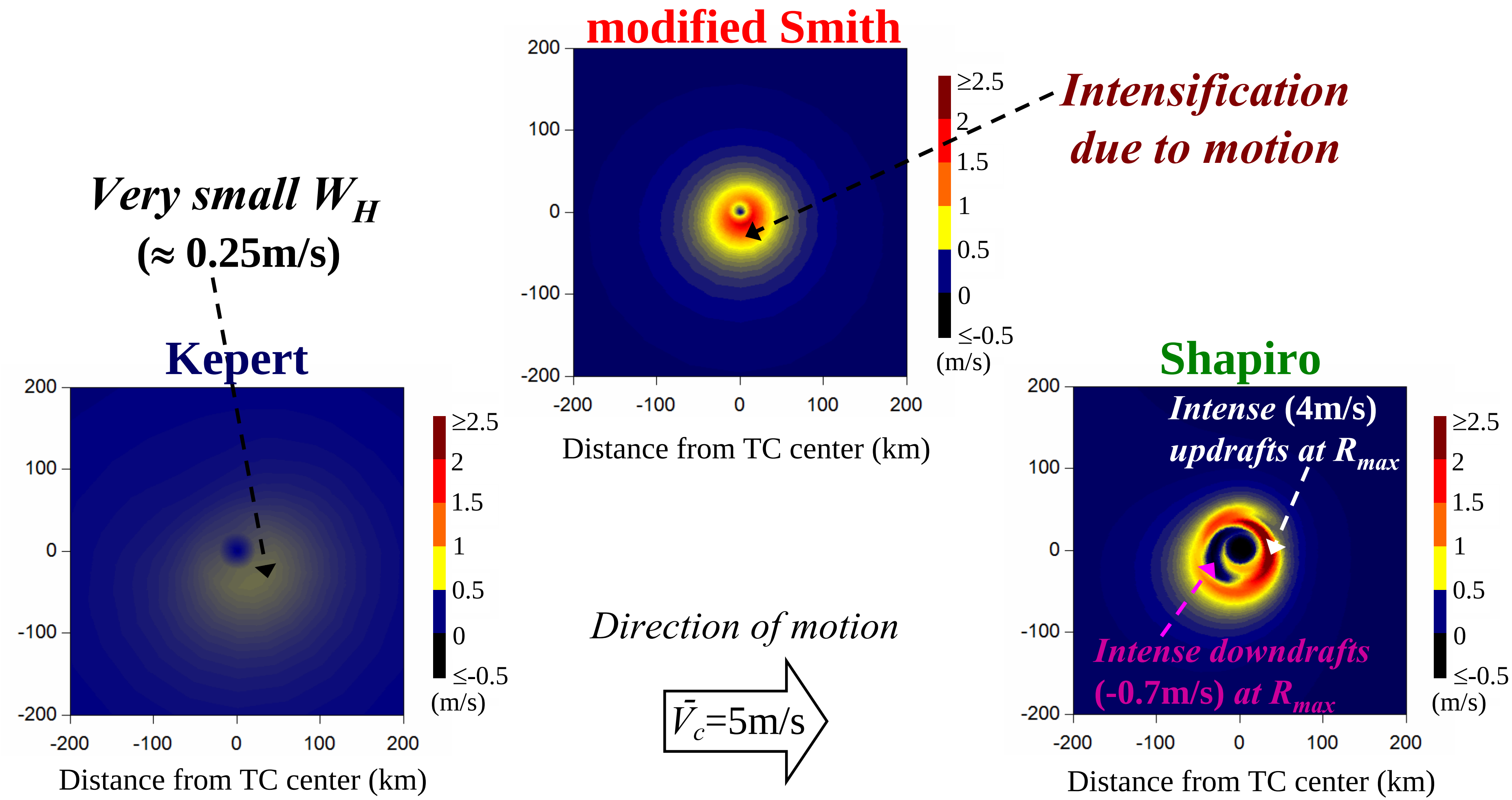
Shapiro



Distance from TC center (km)

Model comparison: Moving hurricane (5m/s)

Vertical wind velocity at top of BL:



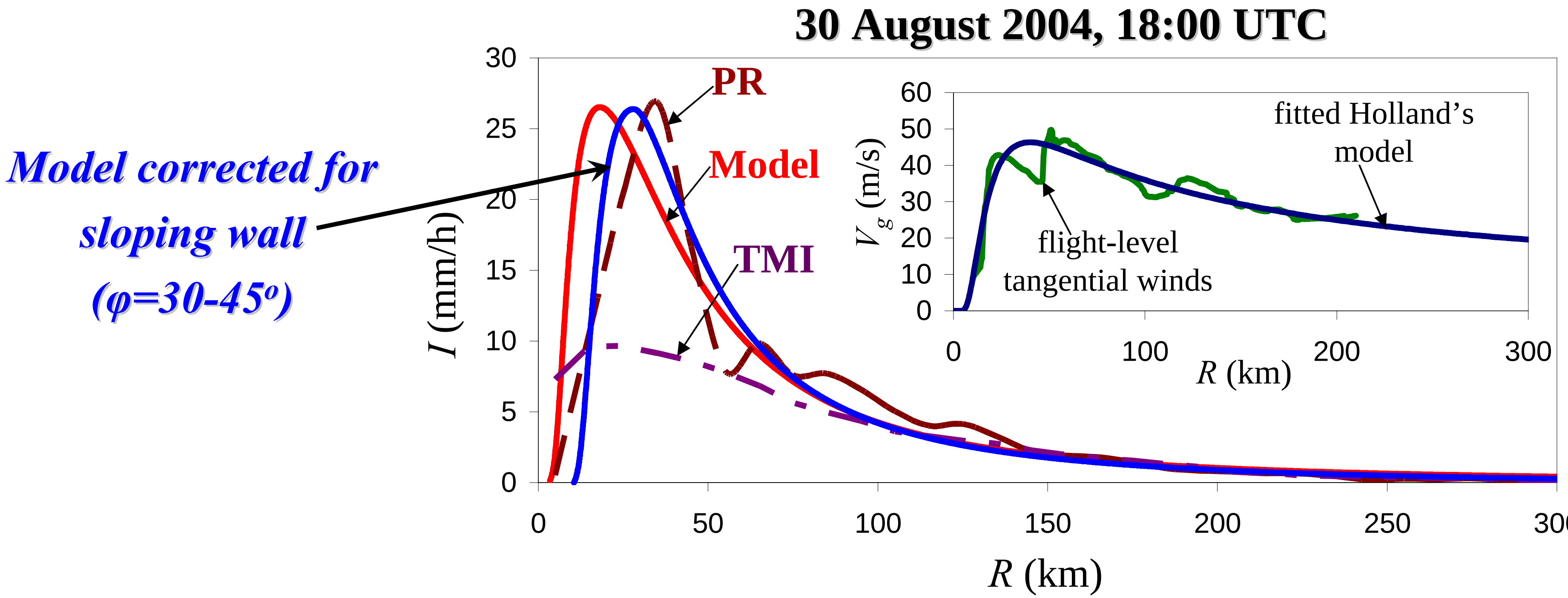
Model validation: Axi-symmetric rainfall

Application to Frances 2004:

- ✓ ***Fit* Holland's (1980) profile to *flight-level* tangential wind *data***
- ✓ ***Use* modified Smith model to calculate $W_H(R)$**
- ✓ ***Calculate* the azimuthally averaged rainfall intensity I:**

$$I(R) = \left(\frac{\rho_{air}}{\rho_w} q_w(T)\right) W_H(R)$$

(q_w=11gr/kgr; T=20°C, S=0.8)



Model validation: Rainfall asymmetry

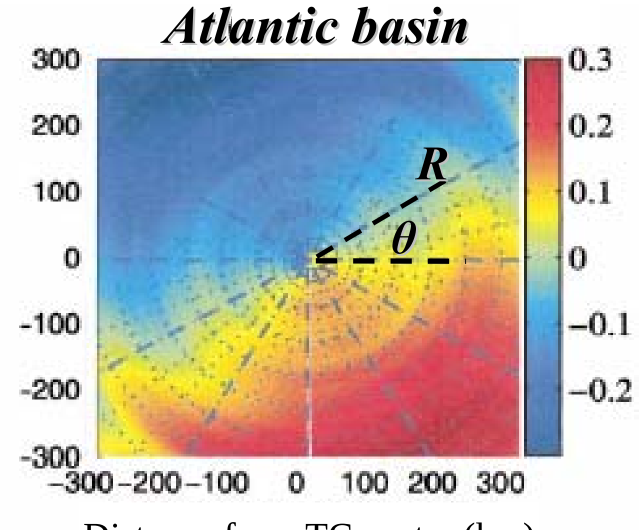
Rainfall asymmetry

$$\gamma_I(R,\theta) = \frac{I(R,\theta) - \bar{I}(R)}{\bar{I}(R)}$$

rainfall intensity at (R,θ)

azimuthal average

Atlantic basin



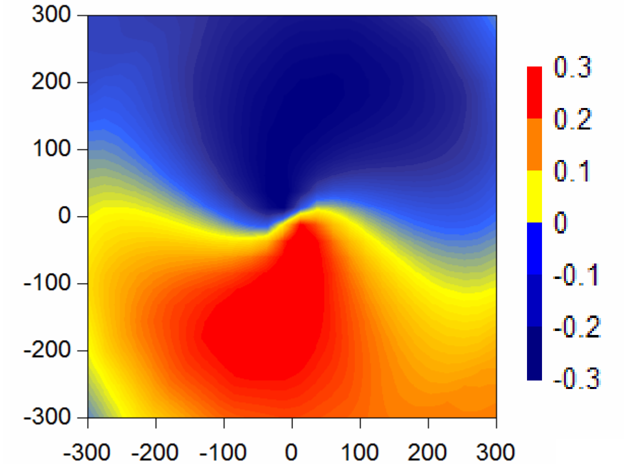
Distance from TC center (km)

Averaged over 476 storms (Lonfat et al. 2004)

Direction of mean translation

$\bar{V}_c$=5m/s

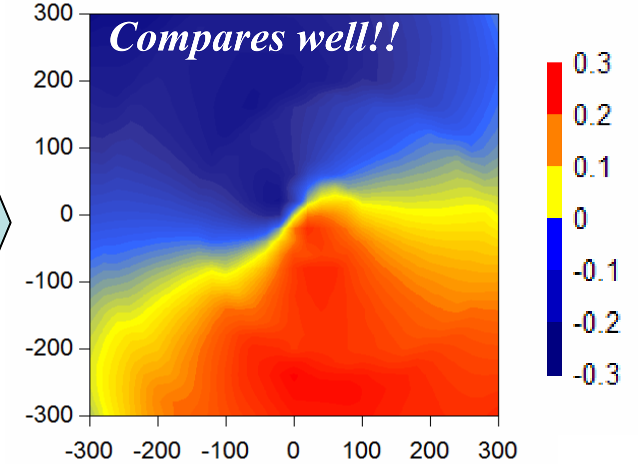
Asymmetry of W_H (model)



Distance from TC center (km)

Rainwater redistribution

Rainfall asymmetry (model)



Distance from TC center (km)

Conclusions

- *We developed a* ***simple*** *model for the* ***mean rainfall*** *field in hurricanes*
- *The model is* ***parameterized*** *through* [V_{max}, R_{max}, B, V_c, q_w]
- ***Products:*** $\left\{ \right.$ ***Axi-symmetric component*** ***and*** ***Asymmetry due to motion***
- *Validated through MM5+TRMM*
- *The model runs in approximately* ***5 min…***

⇨ ***Suitable for long-term risk analysis***

Future research:

- *Effect of* ***vertical*** *wind* ***shear*** *on rainfall asymmetry.*
- *Model* ***rainbands*** *and* ***small-scale*** *rainfall* ***fluctuations.***

- ***Application to rainfall risk!***

Thank you for your time!