Scaling versus Hurst

(By V. Klemeš)

A historic record of streamflows represented by a time series $[X]_n \equiv X_1, X_2, \ldots, X_t, \ldots, X_n$ is taken as the basis for the comparison of the two approaches to its analysis. Such series typically exhibits a more or less pronounced clustering of higher and lower values over irregular intervals of different lengths. This is referred to as the fluctuation of X at different (time) scales. The nature of this fluctuation can be analyzed by examining consecutive segments of k values of the series. Such analysis is here referred to as analysis at (time) scale k. This scale - the length of the segment - can vary from $k = 2$ to $k = n$, in which case the whole series constitutes one segment. However, since the essence of the analysis is a comparison of the series behaviour in several complete segments of a given length, the maximum value of k has to be smaller than $n/2$.

This scheme is regarded as the general principle of what is here understood as 'scaling' analysis.

As an example, around which this discussion will be evolving, a time series of n=30 terms, $[X]_{30}$, will be used and the analysis performed for scale k=5, so that there will be m=6 complete segments. In the top of Figure 1 this series is plotted in black; the "population" mean $\mathbf{X}_{n=30}$ is also shown (in the text, I am using bold face to denote averages since I can't place a bar over a letter).

In the **first step** the original series $[X]_{30}$ is 'scaled' by replacing it with the series of the segment averages, $[\mathbf{X}^{(5)}]_6 \equiv \mathbf{X}_i^{(5)}$, $i = 1,2,\ldots,6$, i. e. by "mean flows in successive periods of length k=5". The top of Fig. 1 shows its plot in blue.

Up to this point, the Hurst analysis proceeds in exactly the same way as the scaling approach: it splits the original series into the same segments and replaces it with the series of the segment means. The only difference is "operational" rather than conceptual: the scaling analysis represents both series in their original form, while Hurst represents them in the form of their "integral transforms", namely as their (residual) mass curves with ordinates Y and $Y^{(5)}$, respectively. In the bottom part of Fig. 1 their plots are shown again in black and blue and are labelled SY and $SY^{(5)}$ to emphasize the fact that it is the slopes of the curve segments, rather than their ordinates, which define the flow values in this plot. In other words, the upper and lower black and blue plots in Fig. 1 contain the same information about the original and the scaled series.

A conceptual difference comes in the **second step** where a specific feature (measure) of the scaled series is adopted as a characteristic of its behaviour, i.e. as a variable to be analyzed:

In the scaling approach, it is the computed segment mean itself. In Hurst's approach, it is the 'theoretical' minimum storage capacity of a reservoir needed to "physically produce" this mean as a constant reservoir release throughout the duration (length) of the whole *i*-th segment, given that the reservoir inflows were the *k* original flows *X*. This storage capacity is defined, for each segment taken separately (i.e., as if it represented the whole series used for sizing the reservoir), by the "adjusted range", $R^{(k)}$, of the corresponding segment of the "residual mass curve" of the original flow (i.e., reservoir inflow) series *X*; the label "adjusted" means that the range is computed with respect to the average outflow $\mathbf{X}_i^{(k)}$ represented by the slope $SY_i^{(k)}$. This is depicted in the lower plot in Fig. 1, with the definitions given at the bottom (the double slash, //, stands for "adjusted with respect to").

Note the essence of the difference: In the scaling approach, when computing the segment mean, one also must first accumulate (add up) the *k* individual values of *X* and "store" them, the computer memory playing the role of Hurst's reservoir storage. But, in "computing" the segment mean, this generating mechanism is not reflected in the result it produces and does not enter the picture, in contrast to Hurst's approach which takes it explicitly into account.

From a conceptual point of view, one could say that Hurst goes one step "deeper" into the problem: he chooses a measure which is based on the segment mean but one which also reflects the mechanism by which this mean can be "conceptually" produced (a practical implementation of this concept is still a different matter - see the highlighted remark at the bottom of p.5 of my 1994 paper "Statistics and probability...").

From a statistical point of view, the scaling approach employs a measure of **central tendency** which takes no account of the variability within the segment (called 'intra-segment' herefrom). In contrast, Hurst uses a measure which also reflects central tendency but, being a function of the deviations from it, it emphasizes the intra-segment **variability.**

This is the place in the analysis where it most clearly comes to light why it is important to appreciate the fact that Hurst started to work with a **streamflow** record and that his original objective was to find out how the *length of the record* used for the **sizing of a reservoir** influences its storage capacity needed to "equalize" the flows over the record length (see below his own explanation). Had he not been an engineer concerned with reservoir design, but instead, say, a meteorologist interested in temperature fluctuations over time, he most likely would not have analyzed "storages needed to equalize" them, but could well have adopted the 'scaling approach' which has nothing to do with "storing" any physical entity and thus does not invite the employment of "mass curves".

The above problem Hurst tackled was a crucial one because, in his time, it was taken for granted that the storage capacity for a given "safe draft" has a definite "correct" value which is approached the more closely the longer is the flow series used for its computation - in a similar way as a sample mean approaches the "population" mean with the sample size. This was textbook wisdom even in my time 50 years ago, and was the main reason why it was engineers who were stressing the need for long streamflow records.

Hurst was the first who dispelled this myth and showed that there is no "correct" value which the storage approaches with an increasing record length - that this "random variable" has no population mean but grows without limit. And I think (and have written it somewhere) that this was Hurst's most important, but still not fully appreciated, contribution to the practice of reservoir design. This I think was the crucial finding that led to the abandonment of the traditional "Rippl's method" and established "respectability" and later general adoption of probabilistic methods.

Digression into a distant past (to be skipped on first reading)

The problem with the probabilistic methods (Savarenskiy, Kritskiy & Menkel, Moran, Lloyd) was that they could be more-or-less easily formulated only for annual flow series. The effect of sub-annual flow fluctuations - which is substantial for smaller reservoirs - had to be added by some ad-hoc procedure (e.g., based on 'typical' annual hydrograph) not related to the design reliability (probability of non-failure year), thus compromising the value of the 'design reliability' of the project..

I chose to 'solve' this problem as part of my PhD work in the early 1960s. The solution was in fact based on a "scaling à la Hurst" on two levels, annual and monthly (or daily). The object was to figure out a probability distribution of the 'sub-annual' component of storage that could be appropriately added to that of 'annual storage' established by the standard probabilistic methods. I am enclosing the English (i.e., my 'tortured' high-school English) summary of my 1963 paper with two its Figures (14 and 15) illustrating the substance: Fig. 14 shows how the 'seasonal component' was separated out on the mass curve from the total storage obtained from the monthly (daily) record. As shown in red, it consists of two parts, one contributing at the beginning, the other at the end of a 'critical period'. My reasoning was that years with any seasonal patterns could occur at these two boundaries, so I constructed the possible hypothetical start-of-period and end-of-period contributions for every year and, having 30 years of record, I could produce reasonably defined empirical distribution functions of both. They are shown in Fig. 15, together with the graphically constructed (no computers then!) convolutions adding them together, as well as convolutions of the result with the distribution of annual (long-term) storage obtained via Savarenskiy/Moran-type methods. The final theoretical result of the exercise (highlighted in red on the second sheet of the enclosure) was the **seasonal storage component, β_{sez}, as a function of draft α, for a given reliability (probability) level P** (below it are shown examples of empirically obtained results for three different flow series, and opposite an example of such empirically derived relationship for several different reliability levels).
Cumbersome as it was, my method was favourably regarded by both Kritskiy and Moran, and at the time has become a standard 'textbook' method in Czechoslovakia (a part of its exposition in the 1966 Votruba & Broža textbook is also shown in the enclosure).

Back to Hurst:

In his 1951 paper Hurst explains: "The investigation began empirically, before any attempts to find a mathematical theory were made, by finding *R* for any available long series of river discharges. Such series were scarce and so the work was extended to rainfall data, which were more plentiful. When it was found that rainfall data gave results similar to the river data the work was extended to other long series of natural events" (p.783). In other words, he was using all the 'other' series just as proxy for streamflow and he still had in mind reservoir storage, rather than the series fluctuations as such.

Paradoxically, this 'narrow' engineering focus on reservoir sizing for a given streamflow record kept him 'out of trouble' scientifically, because residual mass curves of streamflow represent a record of storage fluctuations in an ('infinite') reservoir, i. e. record of a physically meaningful cumulative process. This cannot be said of residual mass-curve representation of many other records such as those of temperatures, tree rings, barometric pressure, etc., which all have often been represented by their 'cumulative departures from the mean' (= residual mass curves) on the basis of whose patterns 'scientific' inferences about the nature of time fluctuations of the original series (!) have been routinely made. Most notoriously, such inferences have been made about long-term fluctuations of streamflow and precipitation because their representation by residual mass curves has been standard practice in hydrology (as well as meteorology and climatology). However, the 'long-term cycles' in the 'computed' mass curves are just chimeras - misleading mathematical artefacts as I have explained in the enclosed 1987 'Drought' paper and, in more detail, in Klemeš & Klemeš (1988); some examples from the latter were reproduced in the "Geophysical time series..." paper. Only if the mass curves have been 'computed by nature' (produced by some physical process) do their fluctuation patterns have a real 'meaning'. Thus it was the very fact that Hurst did not aspire to use the patterns of his mass curves to 'analyze' fluctuations in streamflow series (as did, say, his younger American contemporary Williams) what saved him from 'barking up the wrong tree'.

The next element of the conceptual difference between "scaling" and "Hurst" appears in the **third step** where the type of 'processing' of the previously chosen characteristics ($\boldsymbol{X}^{(k)}$ and $R^{(k)}$, respectively) of the scaled series is decided: the former uses its **standard deviation**, $\sigma[\boldsymbol{X}^{(k)}]$, while the latter uses the **mean**, $\mu[R^{*(k)}]$, where $R^{*(k)}$ is called the "rescaled adjusted range" and is equal to $R^{(k)}$ divided by the intra-segment standard deviation of the corresponding sample $[X]^{(k)}$).

Note that here the difference is diametrically opposite to that in step two: Here it is "scaling" which uses a variability characteristic, while "Hurst" uses one for central tendency. Also note that it is only in this step where Hurst made an "attempt to find a mathematical theory": this is evident from his 'normalization' of $R^{(k)}$ by $\sigma[X]^{(k)}$. He noted that both these variables are functions of deviations from the mean (p.784) and, by using the latter as a normalizing factor, he sort of 'purified' *R* from the effect of the intra-segment σ. The enormous difference between R and R^* is apparent from their plots, for k=5, in the top part of Fig. 1.

While, in my opinion, this 'rescaling' of R has little direct engineering relevance (reservoir size depends on the actual R, not on how much of it is due to σ and how much due to something else), it has been praised by Mandelbrot as a stroke of (mathematical) genius of which Hurst himself was most likely oblivious. Mandelbrot and Taqqu (1979) see the mathematical significance of R^* in its "robustness against scale changes" and "against extreme deviations from normality, including the infinite variance syndrome".

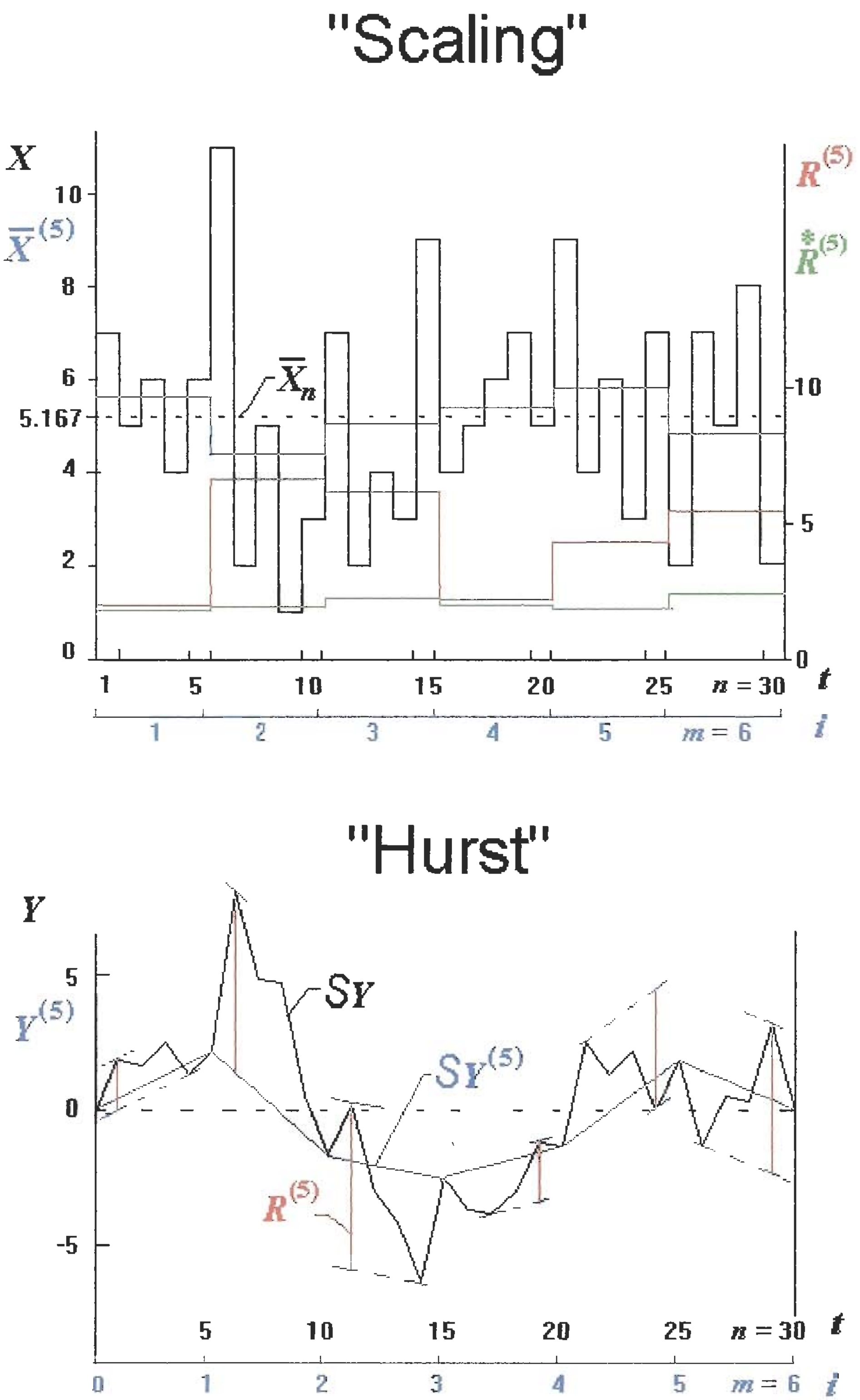
Be it as it may, the fact remains that it was the shape of the dependence of the mean of R^* on the segment length k, specifically the slope H of its log-log plot, that gave birth to the "Hurst phenomenon" and made it famous and notorious object of countless analyses.

However, the point of interest here is the following:

Why is the slope H of the log-log plot of mean $R^{*(k)}$ vs. k so similar (perhaps even identical?) to that of $\sigma[\boldsymbol{X}^{(k)}]$ vs. k, given the two above differences in the underlying procedures? Namely, "scaling" reflects the **variability of segment means**, while "Hurst" reflects the **mean of intra-segment variabilities** and, moreover, it also depends on the sequential order of the segment terms which the standard deviation ignores. The point is that, depending on this sequential order, R (as well as R^*) for a given segment can vary from some minimum to some maximum, the standard deviation remaining unchanged. Even if this variability is 'averaged out' in the mean of many segments, R^* still has different information content than σ. The more general difference is that "scaling" employs statistics of the series itself, while "Hurst" uses a statistics of its 'integral transform' - and, as follows from the preceding discussion, for the fluctuations of the series itself its own statistics should be more relevant than statistics of its mass curve, which in turn should be more relevant to storage fluctuations.

For somebody whose 'mathematical synapses and neurotransmitters' are still well oiled, it might be interesting to trace mathematically the root of the similarity between the estimates of H (I find it hard to believe that they are identical) obtained by the two approaches.

Reference:
Mandelbrot & Taqqu, Robust R/S analysis of long run serial correlation, RC 7936 (#34217) 10/4/79, Research Report, IBM Thomas J. Watson Research Center, Yorktown Heights, New York.



t	X_t
1	7
2	5
3	6
	4
	6
	11
	2
	5
	1
10	3
	7
	2
	4
	3
	9
	4
	5
	6
	7
20	5
	9
	4
	6
	3
	7
	2
	7
	5
	8
30	2

i	$\bar{X}_i^{(5)}$	$R_i^{(5)}$	$\overset{*}{R}_i^{(5)}$
1	5.6	1.8	1.76
2	4.4	6.6	1.86
3	5.0	6.0	2.30
4	5.4	2.2	2.16
5	5.8	4.4	2.06
6	4.8	5.6	2.57

$$Y_t = \sum_1^t (X_t - \bar{X}_n)$$

$$R_i^{(5)} = (\max Y - \min Y)_i \,/\!/\, SY_i^{(5)}$$

$$Y_i^{(5)} = \sum_1^i (\bar{X}_i^{(5)} - \bar{X}_n)$$

$$\overset{*}{R}_i^{(5)} = R_i^{(5)} / \sigma[X]_i^{(5)}$$

Fig. 1

β_{d1} — многолетняя составляющая емкости,
$\beta_{(2)}$ — композиция составляющих $\beta_{sez(2)}$ и β_{d1}.

Далее приведены:
случаи при которых описаный основной метод упрощается и уточняется (рис. 8, 9, 10, 11), и модификация метода для любой календарной границы года и для произвольно сложного режима внутригодового распределения стока — рис. 12 (основной метод разработан для так называемого водохозяйственного года с началом в конце межени и простого гидрологического режима с ясно выявленными половодьем и меженью).

В последней части дается практический пример расчета: по данным 30-летнего ряда средних месячных расходов определяется с использованием изложенного выше метода общая полезная емкость водохранилища для коэффициента регулирования $\alpha = 0{,}8$ и обеспеченности изменяющейся в диапазоне $O < P < 100\,\%$ — рис. 13, 14, 15, 16 (многолетняя составляющая емкости определена методом Крицкого-Менкеля).

15. 4. 1963

VÍT KLEMEŠ

SUB-ANNUAL RESERVOIR STORAGE CAPACITY IN STATISTICAL METHODS OF STREAM FLOW CONTROL

(Part I and II)

Nearly up to this time, the statistical methods of stream flow control computations were examined mainly from the point of view of long term storage capacity.

As for the effect of sub-annual fluctuations on the storage capacity of reservoirs, there was comparatively a little interest taken in it. Calculations of this effect often are based on an excessive schematisation of the annual stream flow distribution and on a great simplification of relations between the sub-annual and long-term components of the storage capacity. This fact depreciates in many cases the results obtained by statistical methods and makes difficult their applications, especially, if the degree of flow regulation is too low (in this case the sub-annual storage capacity represents a large part of the total available storage).

In this paper, a method is presented by means of which the sub-annual storage capacity for the constant value of the guaranteed discharge can be fixed. The method represents a generalisation of the V. G. Andrajanov's method (fig. 1), which flows from it when only the seasonal (annual) regulation of stream flow is realised, that's when the given draft α is lower than the minimal value of the mean annual runoff K_{min}.

On the base of stream gauging rank the frequency curves of the sub-annual storage capacity are composed, separately for the interval $\langle O, p_\alpha \rangle$ of the mean annual runoff frequency curve, and separately for the interval $\langle p_\alpha, 100\% \rangle$ of this curve. In the first case, the sub-annual storage is equal to the whole storage capacity, needed for the guarantee of the controlled discharge α (in this paper it is called „the seasonal storage" and is denoted with the symbol $\beta_{sez(1)}$ — see equation No 8); in the second case, the sub-annual storage represents only a part of the storage capacity which must be added to the long-term storage (in this paper it is called „the seasonal component of the long-term storage" and is denoted with the symbol $\beta_{sez(2)}$ — see equation No 12).

This second case is analysed particularly and it is shown that:

1. the component $\beta_{sez(2)}$ is influenced — in contradiction to the component $\beta_{sez(1)}$ — by all years having whatever value of the mean runoff, and not only by the years with the runoff $K \geqq \alpha$ (fig. 2);

2. the component $\beta_{sez(2)}$ is influenced not only by the years at the boundary of the critical period — (i), (j), but also by the years standing in a greater distance from it — $(i-1)$, $(j+1)$, $(i-2)$, $(j+2)$ etc. (fig. 3).

In the next part of the article a graphical probability method is suggested that makes possible to take both previous conclusions into consideration (fig. 4, 5, 6).

On the figure 7, all the components taking place in the determination of the resulting function $\beta = F(\alpha, P)$ (here $\alpha =$ const.) are shown:

$\beta_{sez(1)}$ — the seasonal storage,
$\beta_{sez(2)}$ — the seasonal component of the long-term storage,
β_{d1} — the long-term storage,
$\beta_{(2)}$ — the composition of components $\beta_{sez(2)}$ and β_{d1}.

Further an abstract relative to simplification and precisioning possibilities of the basic variant of the method (fig. 8, 9, 10, 11) follows including the modification of this method for any position of the beginning of the yearly period and for any kind of the regime of the annual stream flow distribution — fig. 12 (the basic variant of the method deals with so called „hydraulic years" beginning at the end of the low-water period, and it supposes a simple hydrological regime with an expressive low-water period and with an expressive high-water period).

In the last part, a practical example is solved. On the basis of the mean monthly discharge rank of 30 years, the total available storage capacity for the given draft $\alpha = 0{,}8$ and the guarantee P varying from zero up to one hundred per cent is determined by means of the described method — fig. 13, 14, 15, 16 (the long-term storage capacity is fixed by the method of Krickij - Menkel).

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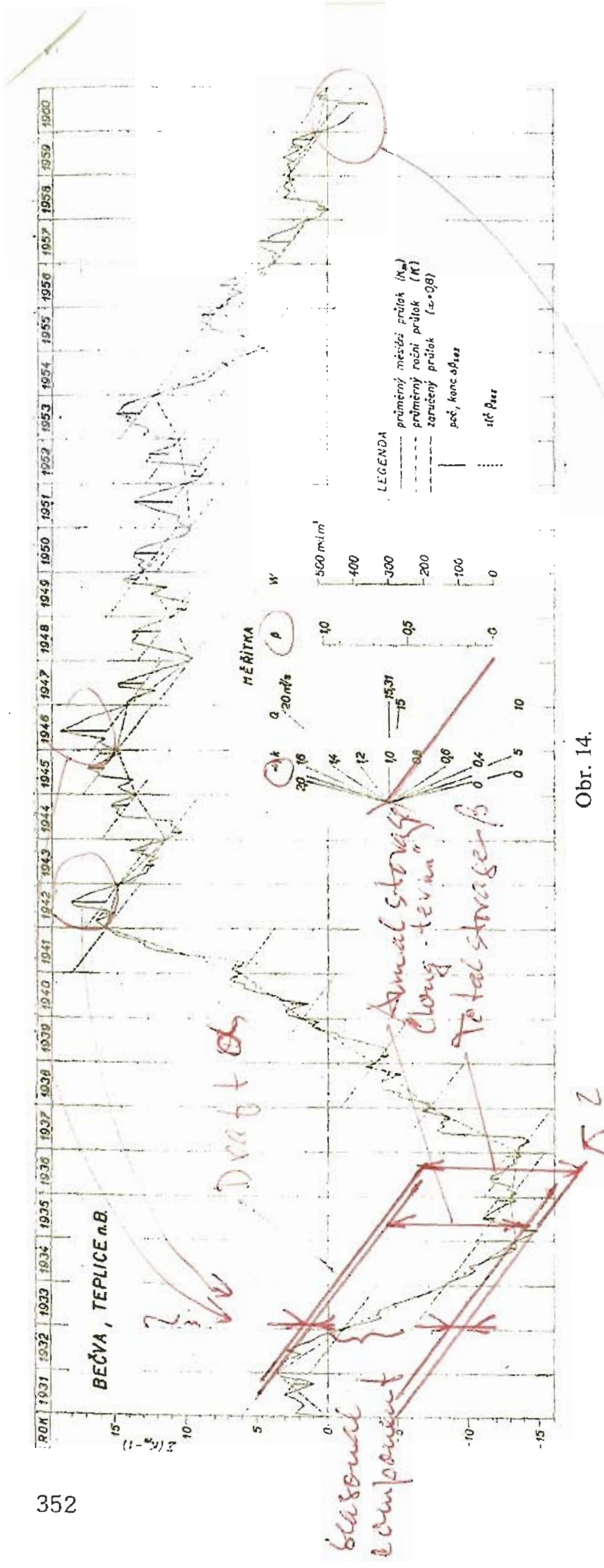


Obr. 14.

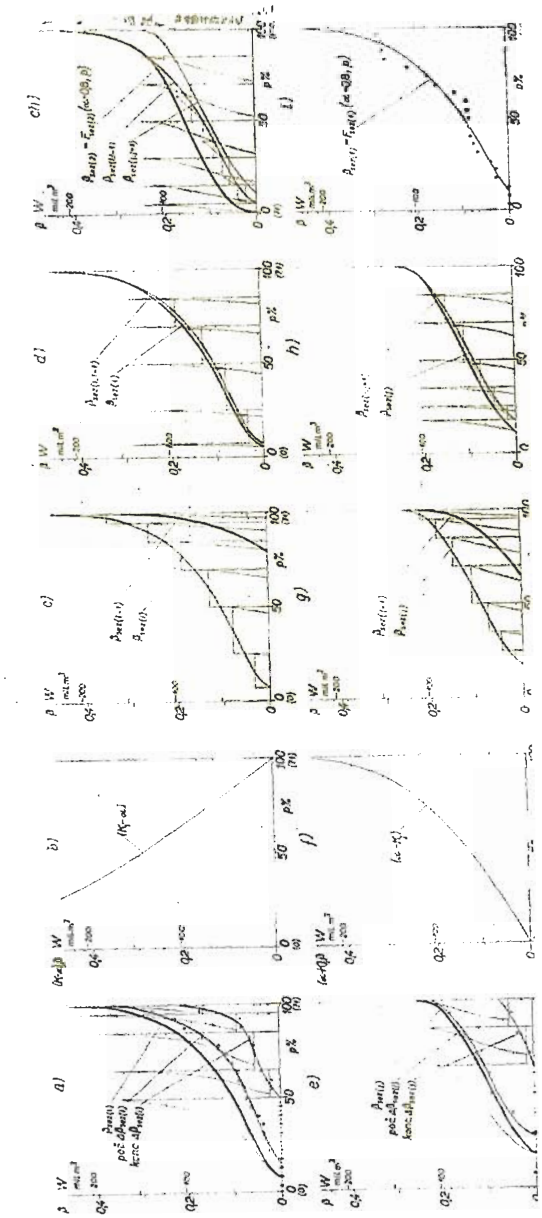
roku $m + 1$, resp. nejbližšího dalšího roku, nastupuje-li za rokem m kritické období. Všechny hodnoty uspořádáme ve vzestupném pořadí a určíme pro ně pravděpodobnosti překročení (obě skupiny roků pokládáme za samostatné).

e) Sestrojujeme čáru zabezpečenosti sezónních složek dlouhodobého objemu $\beta_{sez(2)}$ (funkce 12). Její hodnoty jsou vybrány ze sezónních složek určených čarou $\beta_{sez(2)(K>\alpha)}$ a čarou $\beta_{sez(2)(K<\alpha)}$.

1. Nejprve sestrojíme čáru $\beta_{sez(2)(K>\alpha)}$. Sezónní složka v prvém roce skupiny $K > \alpha$, v roce i, je součtem počátečních a koncových částí, které pokládáme za nezávislé. Na základě hodnot ze sloupce 9 a 10 tab. 3 sestavíme empirické čáry zabezpečenosti částí poč. $\Delta\beta_{sez(i)}$ a konc. $\Delta\beta_{sez(i)}$ tak, že řady bodů vyrovnáme plynulými křivkami. Obě křivky sečteme tak, že jednu z nich nahradíme stupňovitou čarou, ke každému stupni (jehož šířku pokládáme za 100%) připočteme druhou křivku a provedeme opětné sloučení do jedné čáry. Její pořadnice udávají velikost sezónní složky, která je výsledkem společného působení roků předcházejícího a následujícího za kritickým obdobím, tedy hodnoty $\beta_{sez(i)}$ (obr. 15a). Nyní zjistíme vliv dalšího z předcházejících, resp. následujících roků (roku $i-1$) na průběh této křivky. Především sestrojíme čáru $\beta_{sez(i-1)}$, a to tak, že od čáry $\beta_{sez(i)}$ odečteme rozdíl $K - \alpha$, vynesený v obr. 15b. Vlastní odečtení obou křivek je provedeno v obr. 15c. Z čar $\beta_{sez(i)}$ a $\beta_{sez(i-1)}$ sestrojíme vylučovací metodou čáru $\beta_{sez(i, i-1)}$ (obr. 15d). Vliv roku $i - 2$ a dalších je zřejmě malý a nebudeme jej uvažovat; bude tedy $\beta_{sez(2)(K>\alpha)} \equiv \beta_{sez(i, i-1)}$.

2. Stejným způsobem sestrojíme na základě hodnot sloupce 17 a 18 tab. 3 křivku $\beta_{sez(2)(K<\alpha)}$. Postup od čar poč. $\Delta\beta_{sez(j)}$ a konc. $\Delta\beta_{sez(j)}$ k čáře $\beta_{sez(j, j+1)}$ je na obr. 15 e, f, g, h.

3. Obě výsledné čáry přeneseme do jednoho obrazce a vylučovací metodou z nich sestrojíme čáru zabezpečenosti sezónních složek dlouhodobého objemu $\beta_{sez(2)}$, která bude platit v intervalu $\langle p_\alpha = 71\%$, $100\% \rangle$ křivky překročení průměrných ročních průtoků (obr. 15ch).

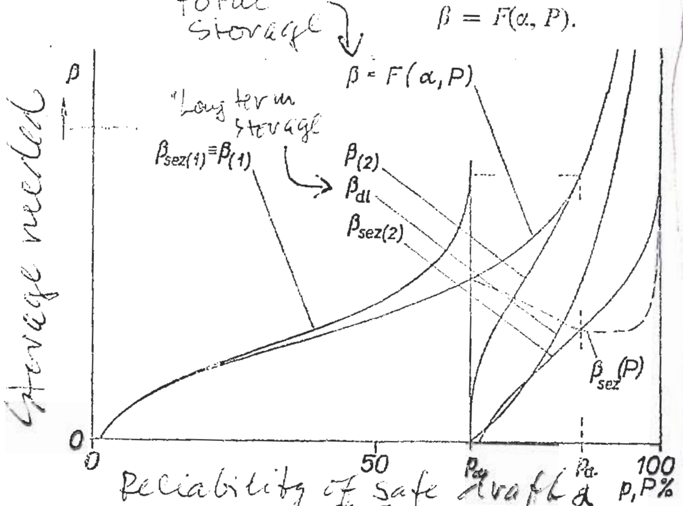


f) Sestrojíme čáru zabezpečenosti sezónních objemů $\beta_{sez(1)}$ (funkce 8) že vyneseme hodnoty sloupce 11 tab. 3 a vyrovnáme je plynulou (obr. 15i).

From Klemeš (1963)

Final theoretical result

$\beta = F(\alpha, P)$.

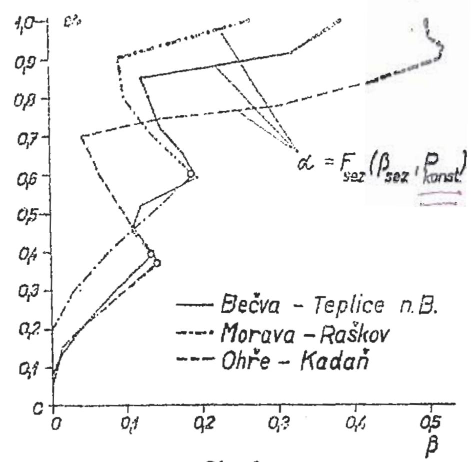


Obr. 7.

(14)

Obr. 8. "Seasonal component" (Total minus long-term (annual))

Empirical results for 3 rivers

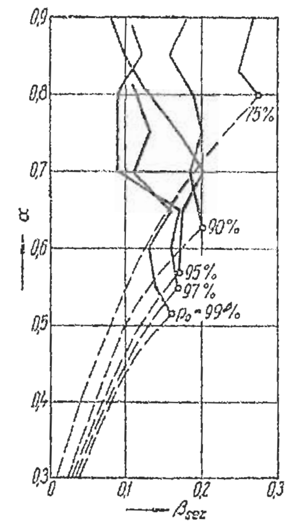


Obr. 9.

From Votruba & Broža 1966

z průběhu čáry β_v podle reálných r
tedy výrazně v rozsahu sledovanýcl
V. K l e m e š e (1963). Podle 110let
spojnicí bodu na ose α pro $\beta_{sez} =$ (
a dále svislicí tímto bodem až do α
lze tuto prostou představu o sezón

Představu o velikosti sezónní sl
odtoku dává *obr. 156*, na němž jsc
hodnot $p_o =$ konst, odvozené z př
pro $p_o = 75$ až 90 % procházejí s
pro $p_o = 95$ až 100 % byly vyrov
vidět, že sezónní složka klesá pod
čili p
kou



Obr. 155. Vztah $\beta_{sez} = f(\alpha)$ při $p_o =$ konst pro přirozenou řadu Labe v Děčíně (1851—1960)

Empirical results for different values of P

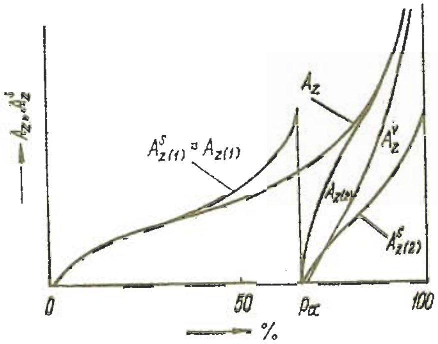
Textbook by Votruba & Broža, 1966

prostoru nádrže při viceletém řízení odtoku, je-li požadovaný nalepšený odtok (odběr) z nádrže O_p = konst.

Postup řešení je naznačen na *obr. 98.*

Uvedené řešení bylo vysvětleno za předpokladu zjednodušeného hydrologického režimu při uvážení vodohospodářského roku.

V. Klemeš zobecnil řešení i pro obecnou polohu začátku roku a pro složitější hydrologické poměry.



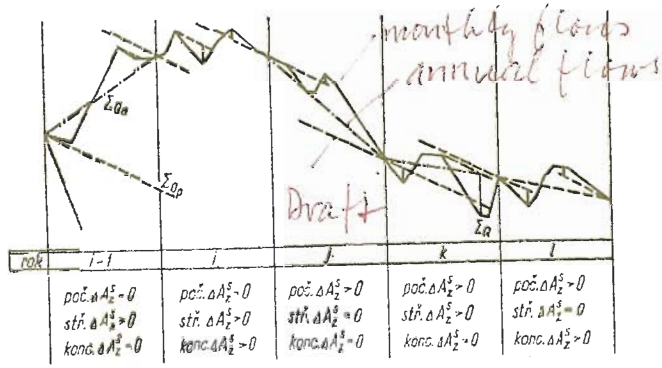
Obr. 98. Postupné sloučení čar zabezpečení víceletých a sezónních objemů nádrže s víceletým řízením odtoku

Při obecné poloze hranic roku se sezónní složka skládá zpravidla ze dvou částí, *poč.* ΔA_z^s a *konc.* ΔA_z^s, a kromě toho se uvnitř roků mohou vyskytnout uzavřené málovodné sezóny, v nichž k nalepšení průtoku na O_p je třeba sezónní objem v nádrži, který je dále označován *stř.* ΔA_z^s. V konkrétních případech může být některá z uvedených částí rovna nule, popř. některý rok nemusí klást nároky na sezónní složku zásobního objemu nádrže vůbec. Na *obr. 99* je úsek součtové čáry, v němž jsou v každém roce vyznačeny velikosti jednotlivých částí sezónní složky.

V postupu řešení dochází k následujícím změnám:

1. V intervalu $< 0;\ p_\alpha >$ se nahradí čára zabezpečení $A^s_{z(1)}$ čarou $A^s_{z(i)}$ (neuvažuje se zatím vliv uzavřených málovodných sezón, tj. *stř.* ΔA_z^s).

V letech s $Q_a > O_p$ se stanoví velikosti *poč.* ΔA_z^s a *konc.* ΔA_z^s a hodnoty $A^s_{z(i)}$ se určí součtem *konc.* ΔA_z^s v jednom roce a *poč.* ΔA_z^s v roce následujícím, pokud jeho střední roční průtok $Q_a \geqq O_p$. Jestliže následuje málovodné období, přičte se *poč.* ΔA_z^s prvního roku po tomto období, jehož $Q_a > O_p$. Z takto zjištěných hodnot se sestaví čára zabezpečení $A^s_{z(i)} = f(p)$.



Obr. 99. Stanovení jednotlivých částí sezónní složky (*poč.* ΔA_z^s, *konc.* ΔA_z^s, *stř.* ΔA_z^s) při obecných hranicích roků

Je možno postupovat tak, že se sestrojí zvlášť čára zabezpečení části *poč.* $\Delta A^s_{z(i)}$ a zvlášť části *konc.* $\Delta A^s_{z(i)}$. Za předpokladu vzájemné nezávislosti obou částí se provede prostá skladba čar zabezpečení ve výslednou čáru $A^s_{z(i)}$. Je-li mezi částmi sezónních korelační závislost, stanoví se pro různé hodnoty počátečních částí koncových částí sezónních složek a skladbou příslušných čar se zí čení $A^s_{z(i)}$.

2. V intervalu $< p_\alpha;\ 100\ \% >$ se sezónní složka $Q_a > O_p$, skládá z *konc.* ΔA_z^s roku, který předchází kri ΔA_z^s roku, který následuje po kritickém období. Obě pokládat za nezávislé veličiny (leží mezi nimi vždy ale k výsledné čáře zabezpečení $A^s_{z(i)}$ se dospěje složením $\Delta A^s_{z(i)}$ a *konc.* $\Delta A^s_{z(i)}$. Přitom předpokládáme, že se r málovodného období může se stejnou pravděpodobností rok s $Q_a > O_p$. Čára $A^s_{z(i)}$ v intervalu $< p_\alpha;\ 100\ \% >$ s čarou $A^s_{z(i)}$ v intervalu $< 0;\ p_\alpha >$, není-li však mezi v intervalu $< 0;\ p_\alpha >$ korelační závislost, jsou v obou totožné.

Stejným postupem se sestaví čára zabezpečení $A^s_{z(j}$ Započtení vlivu let $i - 1, \ldots i - x$ a $j + 1, \ldots, j +$ vysvětleno za zjednodušujících předpokladů, rovněž da vení výsledné čáry zabezpečení $A_z = f(p)$ je stejný.

3. Provede se korekce výsledné čáry zabezpečení uzavřených málovodných sezón, který až dosud neby *stř.* ΔA_z^s je rozhodující, je-li větší než celkový objem r zabezpečení *stř.* ΔA_z^s, jejíž platnost uvažujeme v int a výběrem směrodatných hodnot z čar A_z a *stř.* ΔA_z^s z zabezpečení $A_z = F(p)$, která je konečným výsledkem

Čára zabezpečení sezónních složek nemá při řešení zás je však průběh sezónní složky $A_z^s = f(O_p)$ při konstan hodnotě zabezpečenosti p, který je uveden na *obr.* Pozoruhodný je lom čáry při hodnotě $O_p = Q_a(p)$ v případě, kdy je velikost nalepšeného odtoku rovna st nímu ročnímu průtoku, jehož přirozená pravděpodob překročení je rovna návrhové zabezpečenosti p.

Obr. 100. Teoretická závislost sezónní složky zásobního prostoru na lepšeném odtoku $A_z^s = f(O_p)$

Popsané řešení je poměrně složité. Při praktický možná podstatná zjednodušení.

Jde-li pouze o vysoké stupně zabezpečení (což bý případ), je možné zanedbat vliv let vzdálenějších od období, protože korigují čáry $A^s_{z(i,\, i-1, \ldots,\, i-x)}$ a A něji jen v oblasti nízkých zabezpečeností. Čára zal

24 October 2004

Dear Demetris,

In addition to your Opinion paper, it was the fact that you contrasted (in HSJ 2002) the 'scaling analysis' of a time series with Hurst's analysis, which has inspired this 'second instalment' of my comments (by the way, in both papers, you have a misprint in the reference to Hurst's paper: its starting page is 770, not 776). Strictly speaking, these are not so much comments on your papers: rather, they have crystalized into an 'opinion' essay which could well go under the title "Scaling versus Hurst". I wrote it as if it were intended for the 'uninitiated' so that I could still understant it six months later; hence you may find much of it redundant.

You of course are meant as the 'somebody' in my proposition at the end of the writeup - if you have not already done what I propose.

With regards,

Vít

Encl.