Reply to Fahim Ashkar's comments

We thank Fahim Ashkar for his interesting comments, which help us clarify the purpose of our work. To improve readability, we quoted the original Reviewer's comments as Italic text.

1. *In the text following Eq. (16) of the paper, relative to the Weibull distribution, the authors state that: "the case with (compressed exponential function, i.e. a tail lighter than the exponential one) has less practical importance". In my view, this assertion is not true because light-tailed distributions are also very common in hydrology, especially for modeling low stream flows, for example, as well as for modeling other types of hydrological variables. The emphasis on the problem of thick-tailed distributions in hydrology should by no means minimize the importance of thin-tailed distributions that are also widely encountered.*

   We agree that light-tailed distributions are widely encountered in hydrological modelling (indeed we also accounted for the Gaussian distribution in our analysis), but sometimes a mixture of light-tailed distributions can result in heavy-tailed distributions anyhow (see e.g. Koutsoyiannis, 2004). Nevertheless, we clarified in the paper title, abstract and text that our work refers to hydrological processes exhibiting scaling behaviour or more generally subexponential distribution tails, which are ubiquitous in natural and social sciences (see e.g. Newman, 2005). In this context, light-tailed distributions have actually less practical importance, as we clearly explain in the Introduction section of our manuscript.

2. *However, the fact that a statistic (e.g., a sample moment) is highly variable and skewed, does not by itself mean that the statistic should be avoided in the modeling. If, for example (as is often the case), the aim is to estimate the quantiles of a hypothesized distribution, then the primary focus should be on the quantile estimates and not on the sample moments that are used to estimate these quantiles. Therefore, questions such as the following should be asked: Are the quantiles of interest situated in the right tail, in the left tail, or in the central part of the distribution? How far in the tail are these quantiles? Does the high variability and skewness of the moment estimators lead to similar characteristics in the quantile estimators? (The answer to this last question may be Yes or No). What is the degree of correlation between the moments' estimators being used, and how does this correlation affect the quantile estimates? One should keep in mind that sample moments of low order which exhibit low variability and skewness but which are highly correlated, are not necessarily better for the modeling than higher order moments that are more variable, but less correlated.*

   Using the words by Papoulis (1991, p. 245): "A *point estimate* is a function $\hat{\theta} = \mathrm{g}(\mathbf{x})$ of the observation vector $\mathbf{x} = [x_1, \ldots, x_n]$. The corresponding RV $\underline{\hat{\theta}} = \mathrm{g}(\underline{\mathbf{x}})$ is the *point estimator* of θ. Any function of the sample vector $\underline{\mathbf{x}} = [\underline{x}_1, \ldots, \underline{x}_n]$ is called a *statistic*. Thus a point estimator is a statistic". Sample moments are indeed point estimators. We have found that high-order sample moments are likely to give an estimate of the corresponding population parameter θ very different from the true value, even if they are unbiased estimators (see e.g. in the paper Fig. 4, the large difference between the mode and the expected value of the 5th-order moment estimator, which is a result of high variance and skewness). It is unanimously agreed that efficiency is a desirable property of point estimators, hence we should find other unbiased estimators that are also comparatively low in variance. The point estimators called *L*-moments, for example, are linear combinations of the ranked observations; therefore, *L*-moments are much less variable than their conventional counterparts and are nearly normal in distribution (Kottegoda and Rosso, 2008, p. 107). Furthermore, maximum likelihood estimators are also appealing because they have pleasing asymptotic properties (consistency, normality and efficiency). The asymptotic normality can, in turn, be used to construct confidence regions that attain the specified coverage as the sample becomes large (Von Storch and Zwiers, 1999, p. 89).

In terms of the Reviewer's assertion that "*the aim is to estimate the quantiles of a hypothesized distribution*", we fully agree that this is the ultimate aim of a statistical approach and this applies to the original variable for which the study refers to (e.g. rainfall, river flow, etc.). Of course, it is important to estimate the quantiles of this variable as faithfully and reliably as possible. However, our paper is not about this: we refer to statistics, i.e. functions of samples of the original variable, and not the original variable per se. Actually, to estimate correctly the quantiles of the original variable, we need to use as estimators statistics with good properties. The latter constitute the scope of the paper in which we demonstrate that high order moments do not have the required good properties.

Finally, concerning the Reviewer's question about the correlation between the moments' estimators, we performed Monte Carlo simulations and we noticed that the estimators of raw moments are highly correlated even in case of Gaussian distribution. Specifically, we saw that the correlation of moments with consecutive orders actually increases with the order of the moments. Then, this is another argument against the use of high-order moments, but we do not include it in our paper.

3. *It is true that the tail type significantly influences the variability and skewness of moment estimates, but questions such as those raised in the previous paragraph (questions that are by no means exhaustive) need to be considered and analyzed attentively before arriving at a decision on what moment orders to use or to avoid, or on whether the method of moments or some other fitting method is best for the modeling at hand. The tail thickness of distributions is a factor to keep in mind, but other important factors also need to be taken into consideration. Consequently, one needs to be careful about the limitations of the following two statements put forward in the Conclusions section of the study: "Estimators of high moments whose distribution ranges over several orders of magnitude cannot support inference about a natural behaviour nor fitting of models". "In parameter estimation of three-parameter distributions, it is better to avoid the method of moments and use other fitting methods such as maximum likelihood, L-moments, etc."*

   We trust that we addressed all of the Reviewer's questions in the previous point, and explained the importance of the two quotations reported here.

4. *Given the wide variety of data that are encountered in hydrology, problems of choice of frequency distributions and/or fitting methods should be handled with great care, as they are highly complex, especially if the aim is to estimate events in the distribution tails. Numerous studies have dealt with fitting thick and thin tailed frequency distributions to hydrological data. In fact, a number of fitting methods either originated or were broadly*

advocated by researchers working in hydrology (e.g., methods of L-moments, generalized moments, generalized probability weighted moments, etc.). The complexity of fitting distributions to data for the purpose of estimating events in the distribution tails has led to confusion not only among practitioners, but among experienced researchers as well. One observes in fitting frequency distributions to data that some users tend to be overly confident about the use of L-moments, for example; others are not very confident, or overly confident about maximum likelihood estimation; others are confused as to what sample moments to use or not to use in estimations. Recommendations put forward in hydrological studies tend sometimes to be followed blindly by practitioners if the limitations of these recommendations are not studied well enough or if they are not clearly drafted. Therefore, the recommendations and conclusions of the present study need to be as clear as possible to avoid any misuse by practitioners. I recommend that the authors express their recommendations in as precise a manner as possible, and specify the limitations of these recommendations as clearly as possible. More specifically, if these recommendations are restricted to heavy-tailed distributions, then this should be well reflected in the title, in the abstract and in the conclusions section of the paper.

Indeed, we fully agree that fitting methods should be handled with great care. To this aim, our paper provides, via several Monte Carlo experiments, quantitative results supporting our claims against use of high-order moments when analysing processes exhibiting subexponential distribution tails.

References

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