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Temporal disaggregation of rainfall

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ABSTRACT

Temporal disaggregation models of rainfall aim at generating finer scale time series of rainfall that are fully consistent with any given coarse-scale totals. In this work, we present a disaggregation method that initially retains the formalism, the parameter set, and the generation routine of the downscaling model described by Lombardo et al (2012), which generates time series with Hurst-Kolmogorov (HK) dependence structure. Then it uses an adjusting procedure to achieve the full consistency of lower-level and higher-level variables without affecting the stochastic structure implied by the original downscaling model. Furthermore, we investigate how our simple and parsimonious model may account for rainfall intermittency, because the capability of disaggregation models to reproduce rainfall intermittency is a fundamental requirement in simulation. Intermittency is quantified by the probability that a time interval is dry. Here we focus on a modelling approach of a mixed type, with a discrete description of intermittency and a continuous description of rainfall. In other words, we model the intermittent rainfall process as the product of the following two stochastic processes: (i) The rainfall occurrence process, which is described by a binary valued stochastic process, with the values 0 and 1 representing dry and wet conditions, respectively; (ii) The non-zero rainfall process, which is given by our disaggregation model. We study the rainfall process as intermittent with both independent (Bernoullian) and dependent (Markovian) occurrences, where dependence is quantified by the probability that two consecutive time intervals are dry. In either case, we provide the analytical formulations of the main statistics of our mixed-type disaggregation model and show their clear accordance with Monte Carlo simulations.

HK downscaling model (Lombardo et al., 2012)

Let $\underline{z}_1^{(\Delta)}$ be a rainfall amount at a time interval Δ, which is to be downscaled to a certain time scale of interest. We assume that $\underline{z}_1^{(\Delta)}$ is a RV *log-normally* distributed with mean μ_0 and variance σ_0^2 of a stationary stochastic process. Let us now introduce an *auxiliary* Gaussian random variable of the HK process (i.e. fractional Gaussian noise, fGn) at the time scale Δ: $\ln \underline{z}_1^{(\Delta)} = \underline{\tilde{z}}_1^{(\Delta)} = \underline{\tilde{z}}_{1,0}$ with mean $\tilde{\mu}_0 = \mathrm{E}[\underline{\tilde{z}}_{1,0}]$ and variance $\tilde{\sigma}_0^2 = \mathrm{Var}[\underline{\tilde{z}}_{1,0}]$. We disaggregate $\underline{\tilde{z}}_{1,0}$ by a dyadic additive cascade, i.e. $\underline{\tilde{z}}_{1,0}$ is partitioned into two Gaussian random variables at the time scale $\Delta/2$ (first cascade level, k = 1):

$$\underline{\tilde{z}}_{1,0} = \underline{\tilde{z}}_{1,1} + \underline{\tilde{z}}_{2,1} \quad (1)$$

This procedure is applied progressively until we obtain lower-level variables at the time scale of interest. At the generic k-level, corresponding to the scale of aggregation $\Delta/2^k$, we have:

$$\underline{\tilde{z}}_{j,k-1} = \underline{\tilde{z}}_{2j-1,k} + \underline{\tilde{z}}_{2j,k} \quad (2)$$

Thus, in general it suffices to generate the variable of the first subinterval and that of the second is then the remainder (eq. 2). We consider the following linear generation scheme:

$$\underline{\tilde{z}}_{2j-1,k} = \boldsymbol{\theta}^{\mathrm{T}} \underline{\boldsymbol{Y}} + \underline{v} \quad (3)$$

where $\underline{\boldsymbol{Y}} = [\underline{\tilde{z}}_{2j-5,k}, \underline{\tilde{z}}_{2j-4,k}, \underline{\tilde{z}}_{2j-3,k}, \underline{\tilde{z}}_{2j-2,k}, \underline{\tilde{z}}_{j,k-1}, \underline{\tilde{z}}_{j+1,k-1}, \underline{\tilde{z}}_{j+2,k-1}]^{\mathrm{T}}$, $\boldsymbol{\theta}$ is a vector of parameters, and $\underline{v}$ is a Gaussian white noise that represents an innovation term. All the parameters in $\boldsymbol{\theta}$ and the variance of $\underline{v}$ are estimated just in terms of the Hurst coefficient H. Eq. (3) allows the generated lower-level variable to preserve autocorrelations with four earlier lower-level variables (level k) and two later higher-level variable (level $k-1$).

It can be shown that the above stepwise disaggregation approach effectively generates fGn. Then, we apply a specific exponential transformation to obtain the actual log-normal process with scaling properties similar to those of fGn.

Summary statistics at the generic k-level can be expressed as:

$$\mu_k = \mathrm{E}[\underline{z}_{j,k}] = \mu_0/2^k \quad (4)$$

$$\sigma_k^2 = \mathrm{Var}[\underline{z}_{j,k}] = \sigma_0^2/2^{2Hk} \quad (5)$$

$$\rho_k(i) = \mathrm{Corr}[\underline{z}_{j,k}, \underline{z}_{j+i,k}] = \frac{\exp\left(\tilde{\sigma}_k^2 \tilde{\rho}(i)\right) - 1}{\exp\left(\tilde{\sigma}_k^2\right) - 1} \quad (6)$$

where $\tilde{\rho}(i) = |i+1|^{2H}/2 + |i-1|^{2H}/2 - |i|^{2H}$ is the autocorrelation function of the fGn.

Adjusting procedure

When we apply our specific exponentiation to the HK process (auxiliary process) to make it log-normal (actual process), we introduce an error in the additive property. To overcome this problem, we use the power adjusting procedure introduced by Koutsoyiannis and Manetas (1996) in order to restore consistency without affecting the stochastic structure implied by our model. It allocates the error in the additive property among the lower-level variables $\underline{z}_{j,k}$.

Intermittency

The concepts expressed above are enriched by accounting for rainfall intermittency in the modelling framework. Indeed, the rainfall process features an intermittent character at fine timescales, and thus the probability that a time interval is dry is generally greater than zero. Therefore, we aim to obtain downscaled time series of rainfall with a given probability dry $p_{0,k}$ (a new model parameter). We assume the intermittent rainfall at the cascade level k and discrete time j (= 1, …, 2^k) $\underline{x}_{j,k}$ as:

$$\underline{x}_{j,k} = \underline{z}_{j,k} \cdot \underline{y}_{j,k} \quad (7)$$

where $\underline{z}_{j,k}$ denotes the continuous random variable generated by our downscaling model, which represents the non-zero rainfall process. While the rainfall occurrence process is represented by $\underline{y}_{j,k}$ that is a binary-valued random variable taking values 0 (dry condition) and 1 (wet condition), respectively with probability $p_{0,k}$ and $1 - p_{0,k}$.

From the above considerations, it can be easily shown that:

$$\mathrm{E}[\underline{x}_{j,k}] = (1 - p_{0,k})\mathrm{E}[\underline{z}_{j,k}] \quad (8)$$

$$\mathrm{Var}[\underline{x}_{j,k}] = (1 - p_{0,k})\left(\mathrm{Var}[\underline{z}_{j,k}] + p_{0,k}\left(\mathrm{E}[\underline{z}_{j,k}]\right)^2\right) \quad (9)$$

We need to investigate now the dependence structure of this particular stochastic process, i.e. the pairwise dependence of $\underline{x}_{j,k}$ and $\underline{x}_{j+i,k}$, where i is the time lag. Hence, we should derive the formulation of the autocovariance function Cov[$\underline{x}_{j,k}$, $\underline{x}_{j+i,k}$]. To this aim, we assume the following two dependence structures of the rainfall occurrence process:

1. Purely random model
2. Markov chain model

Random occurrences

In the first case, rainfall occurrences $\underline{y}_{j,k}$ are modelled as a Bernoulli process in discrete time, which is characterized by only one parameter (i.e., probability dry $p_{0,k}$). It can be shown that:

$$\mathrm{Cov}[\underline{x}_{j,k}, \underline{x}_{j+i,k}] = \left(1 - p_{0,k}\right)^2 \mathrm{Cov}[\underline{z}_{j,k}, \underline{z}_{j+i,k}] \quad (10)$$

If we put it in terms of the autocorrelation function, then we have:

$$\mathrm{Corr}[\underline{x}_{j,k}, \underline{x}_{j+i,k}] = \left(1 - p_{0,k}\right)\rho_k(i)\frac{\mathrm{Var}[\underline{z}_{j,k}]}{\mathrm{Var}[\underline{z}_{j,k}] + p_{0,k}\left(\mathrm{E}[\underline{z}_{j,k}]\right)^2} \quad (11)$$

Markovian occurrences

As a second example, we assume the simplest possible occurrence process $\underline{y}_{j,k}$ with some correlation, i.e. dependence of the current value on previous one. In other words, we assume that the state (dry or wet) in a time interval depends on the state in the previous interval. This is a process with Markovian dependence, which is completely determined by lag-one autocorrelation coefficient $\rho_{y,k}$ = Corr[$\underline{y}_{j,k}$, $\underline{y}_{j-1,k}$]. Therefore, we assume $\rho_{y,k}$ as an additional parameter, which can be expressed as a function of the probability dry $p_{0,k}$ as follows (see eq. (13) by Koutsoyiannis, 2006):

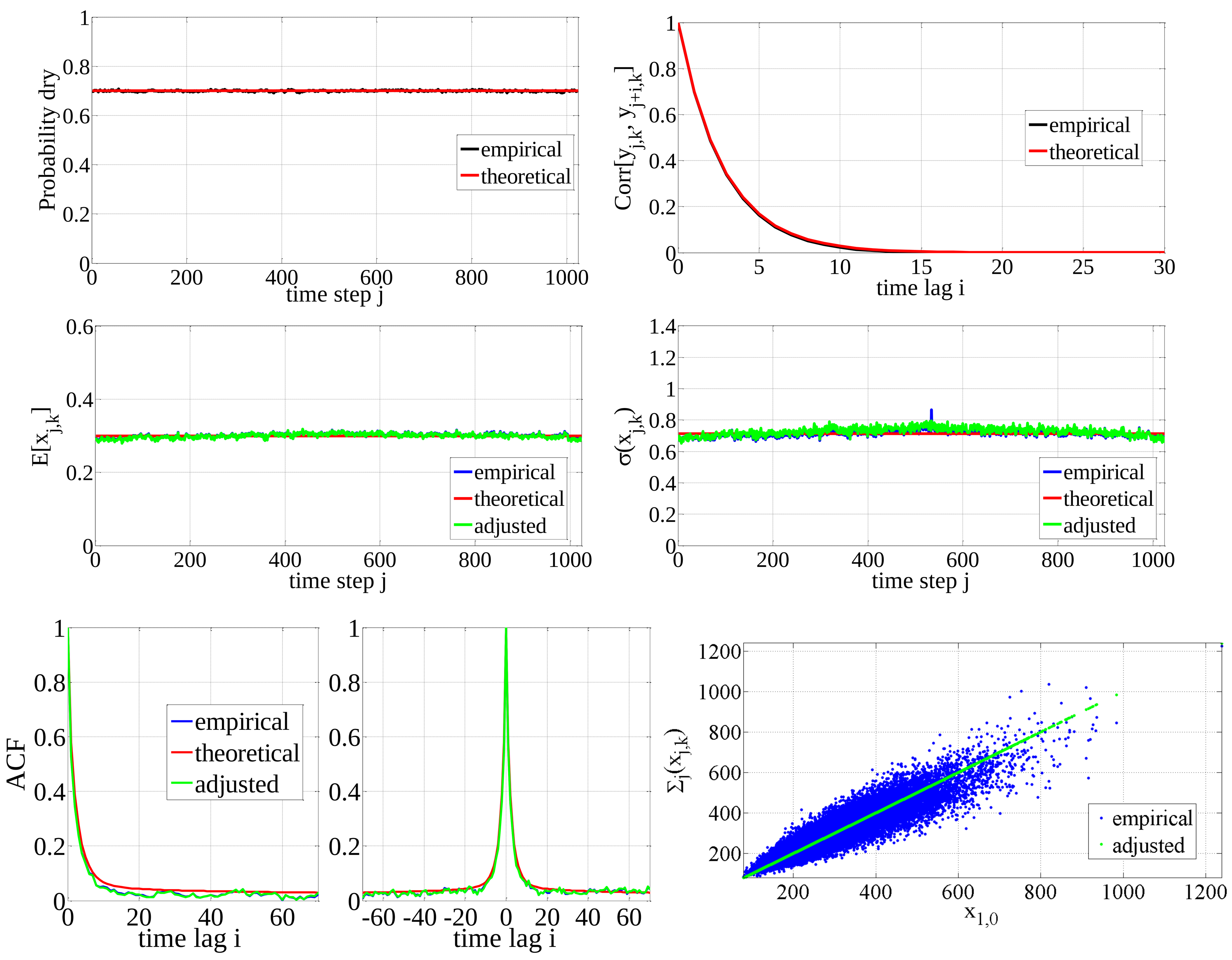
$$\rho_{y,k} = \frac{p_{00,k} - p_{0,k}^2}{p_{0,k} - p_{0,k}^2} \quad (12)$$

where $p_{00,k} = \Pr\{\underline{y}_{j,k} = 0, \underline{y}_{j-1,k} = 0\}$. Under these assumptions, $\mathrm{Corr}\left[\underline{y}_{j,k}, \underline{y}_{j+i,k}\right] = \rho_{y,k}^{|i|}$ holds true for any time lag i. Then, the autocorrelation function of the intermittent rainfall process $\underline{x}_{j,k}$ is given by:

$$\mathrm{Corr}[\underline{x}_{j,k}, \underline{x}_{j+i,k}] = \frac{\left(1 - p_{0,k} + \rho_y^{|i|} p_{0,k}\right)\rho_k(i)\mathrm{Var}[\underline{z}_{j,k}] + \rho_y^{|i|} p_{0,k}\left(\mathrm{E}[\underline{z}_{j,k}]\right)^2}{\mathrm{Var}[\underline{z}_{j,k}] + p_{0,k}\left(\mathrm{E}[\underline{z}_{j,k}]\right)^2} \quad (13)$$

Numerical simulations

We generate 30000 time series with sample size $n = 2^{10} = 1024$, unit mean and variance (i.e. E[$\underline{z}_{j,k}$] = Var[$\underline{z}_{j,k}$] = 1), H = 0.85, $p_{0,k}$ = 0.7, $\rho_{y,k}$ = 0.7 for Markovian occurrences.



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