Evaluation of a parametric approach for estimating potential evapotranspiration across different climates

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Abstract

Potential evapotranspiration (PET) is key input in water resources, agricultural and environmental modelling. For many decades, numerous approaches have been proposed for the consistent estimation of PET at several time scales of interest. The most recognized is the Penman-Monteith formula, which is yet difficult to apply in data-scarce areas, since it requires simultaneous observations of four meteorological variables (temperature, sunshine duration, humidity, wind velocity). For this reason, parsimonious models with minimum input data requirements are strongly preferred. Typically, these have been developed and tested for specific hydroclimatic conditions, but when they are applied in different regimes they provide much less reliable (and in some cases misleading) estimates. Therefore, it is essential to develop generic methods that remain parsimonious, in terms of input data and parameterization, yet they also allow for some kind of local adjustment of their parameters, through calibration. In this study we present a recent parametric formula, based on a simplified formulation of the original Penman-Monteith expression, which only requires mean daily or monthly temperature data. The method is evaluated using meteorological records from different areas worldwide, at both the daily and monthly time scales. The outcomes of this extended analysis are very encouraging, as indicated by the substantially high validation scores of the proposed approach across all examined data sets. In general, the parametric model outperforms well-established methods of the everyday practice, since it ensures optimal approximation of PET.
In the literature are referred ~50 PET models, which can be grouped into 7 classes (cf. review by McMahon et al., 2013).

Penman-Monteith formula (Penman, 1948; Monteith, 1965):
- Analytical approach, where evaporation is viewed both as energy (heat) exchange and aerodynamic process.
- Proposed by FAO (Allen et al., 1998) as the standard method for computing PET, with numerous applications in hydrology and agrometeorology.
- Requires data for four meteorological variables (temperature, wind speed, relative humidity, sunshine).

Radiation-based methods:
- Simplified approaches accounting for the two main sources of variability in evapotranspiration, namely temperature and net solar radiation.
- Further simplification by substituting net solar radiation by extraterrestrial radiation, which is only function of latitude and time.
- Many researchers emphasize the need for further model calibration (i.e. fitting of model parameters against “real” PET data), especially in the energy term of radiation, to improve the overall performance of such methods.
## PET methods used in comparison tests

<table>
<thead>
<tr>
<th>Method</th>
<th>PET expression</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thornthwaite (1948)</td>
<td>$16 \left( \frac{10 \ T_a}{I} \right)^a \left( \frac{d}{12} \right) \left( \frac{N}{30} \right)$</td>
<td>Temperature-based</td>
</tr>
<tr>
<td>Blaney &amp; Criddle (1950)</td>
<td>$0.254 \ p (32 + 1.8 \ T_a)$</td>
<td>Temperature-based</td>
</tr>
<tr>
<td>Jensen &amp; Haise (1963)</td>
<td>$\frac{R_a \ T_a}{40 \ \lambda \ \rho}$</td>
<td>Radiation-based</td>
</tr>
<tr>
<td>McGuiness &amp; Bordne (1972)</td>
<td>$\frac{R_a (T_a + 5)}{68 \ \lambda \ \rho}$</td>
<td>Radiation-based</td>
</tr>
<tr>
<td>Hargreaves &amp; Zamani (1982)</td>
<td>$0.0023 \frac{R_a}{\lambda} \ (T_a + 17.8) \ (T_{max} - T_{min})$</td>
<td>Radiation-based</td>
</tr>
<tr>
<td>Oudin <em>et al.</em> (2005)</td>
<td>$\frac{R_a (T_a + 5)}{100 \ \lambda \ \rho}$</td>
<td>Radiation-based</td>
</tr>
</tbody>
</table>

**Notation:**
- $T_a$: average monthly temperature ($^\circ$C)
- $d$: average number of daylight hours per day for each month
- $N$: number of days in the month
- $I$: annual heat index, which is function of monthly $T_a$
- $a$: parameter, which is function of $I$
- $p$: mean daily percentage of annual daytime hours
- $R_a$: extraterrestrial radiation (kJ/m$^2$/d)
- $\lambda$: latent heat of vaporization (=2460 kJ/kg)
- $\rho$: water density (=1000 kg/m$^3$)
- $T_{max}$, $T_{min}$: maximum and minimum monthly temperature ($^\circ$C)
The parametric formula

- In the Penman-Monteith formula, the numerator is the sum of a term related to net solar radiation, $R_n$, and a term related to the rest of meteorological variables, while the denominator is function of temperature, $T_a$, i.e.:

$$E = \frac{1}{\lambda \rho} \frac{R_n + \gamma \lambda F(u) D}{1 + \gamma' / \Delta}$$

- In a parametric simplification of the Penman-Monteith formula, the numerator is approximated by a linear function of extraterrestrial solar radiation, $R_a$, while the denominator is approximated by a linear descending function of temperature, i.e.:

$$E = a R_a + b \frac{1}{1 - c T_a}$$

- Physical interpretation of model parameters, $a$ (kg/kJ), $b$ (kg/m2) and $c$ (°C-1):
  - Dimensionless term $a / \lambda \rho$ represents the average percentage of the energy provided by the sun (in terms of $R_a$) and, after reaching the Earth’s terrain, is transformed to latent heat, thus driving the evapotranspiration process.
  - Parameter $b$ lumps the missing information associated with aerodynamic processes, driven by the wind and the vapour deficit in the atmosphere.
  - Term $1 - c T_a$ approximates $1 + \gamma' / \Delta$; $\gamma'$ is function of surface and aerodynamic resistance and $\Delta$ is the slope vapour pressure curve, which is function of $T_a$. 

Tegos et al.: Evaluation of a parametric approach for estimating potential evapotranspiration across different climates
Evaluation of parametric formula in Greece

- Estimation of monthly potential evapotranspiration during period 1968-1989 at 37 meteorological stations distributed over Greece, using the Penman-Monteith formula, assumed as reference model for the evaluation of simplified methodologies.

- Fitting of parameters $a$, $b$ and $c$ by optimizing the coefficient of efficiency (CE) against reference data of years 1968-1983 (period 1984-1989 is considered for validation):
  - CE values greater than 95% are achieved at all locations (90% for validation).
  - Obvious superiority over three of the most common empirical methods in Greece (Thornthwaite, Blaney-Criddle, Hargreaves) and the globally used radiation-based approaches by McGuiness et al. (1972) and Oudin et al. (2005; cf. Table).

- Alternative parameterizations were also examined, i.e. (a) by omitting parameter $b$, and (b) by omitting $b$ and substituting $c$ by its average value over Greece; in formulation (a) the reduction of CE was negligible.

<table>
<thead>
<tr>
<th>CE (%)</th>
<th>Parametric McGuiness et al.</th>
<th>Oudin et al.</th>
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<tbody>
<tr>
<td>&gt;0.95</td>
<td>37</td>
<td>30</td>
</tr>
<tr>
<td>0.90-0.95</td>
<td>0</td>
<td>6</td>
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<tr>
<td>0.70-0.90</td>
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<td>1</td>
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<tr>
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<td>0</td>
</tr>
<tr>
<td>&lt;0.50</td>
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Characteristics of data set:
- 37 stations, 21 hydrological years
- Range of latitudes: 35.0° to 41.5°
- Range of elevations: +2 to +663 m
- Range of mean annual PET: 912 mm (Florina) to 1628 mm (Ierapetra).
Assuming the simplified parameterization, in which $b$ is omitted, we re-calibrated the local values of $a$ and $c$, and mapped them over Greece, using typical interpolation tools.

Parameter $a$ exhibits a systematic geographical pattern, since it increases from SE to NW Greece, following the increase of sunshine duration and wind velocity as moving from the continental to insular Greece, while parameter $c$ is site-specific.

For details please refer to Tegos et al. (2013)
Evaluation of parametric formula in California, USA

- Comparison of parametric approach against Hargreaves, Jensen-Heinse, McGuiness and Oudin et al. methods.
- Obvious superiority of parametric and Hargreaves methods, very poor performance of Jensen-Heinse and Oudin et al. models.

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<td>Val</td>
<td>Cal</td>
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<tr>
<td>&lt;0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>12</td>
</tr>
</tbody>
</table>

Comparison of mean annual PET estimated via the Penman-Monteith formula against the parametric model and the four other methods.

For details please refer to Tegos et al. (2014)
Spatial interpolation of model parameters over California

- Implementation of four interpolation methods, including the recently proposed Bilinear Surface Smoothing (Malamos and Koutsoyiannis, 2014).
- Spatial validation at 11 stations (right map), in which the Inverse Weighting Distance, i.e. the simplest of interpolation methods, provides the more accurate point estimations of model parameters.
- The values of $a$ and $c$ present an increasing North to South gradient, while the opposite occurs for $b$. 

Tegos et al.: Evaluation of a parametric approach for estimating potential evapotranspiration across different climates
Conclusions

The proposed parametric model can be considered as simplification of the Penman-Monteith formula, in an attempt to compromise parsimony, in terms of model structure and data requirements, and physical consistency.

Model parameters $a$, $b$ and $c$ have some physical background, since they substitute, to some extent, the three missing meteorological variables.

The model ensures excellent predictive capacity (in terms of reproducing monthly PET estimations through the Penman-Monteith) in all examined locations in Greece and California, as well as in Germany and Spain (full results shown in Tegos et al. 2014).

In Greece, even simpler parameterizations (i.e. the formulation with two parameters, $a$ and $c$) provide similarly good results.

The appropriateness of the method is further revealed through extensive comparisons with other empirical approaches, most of which exhibit poor performance.

Comparisons across different climates reveal the great advantage of parametric approaches against empirical ones, since calibration allows the coefficients that are involved in the mathematical formulas to be fitted to local climatic conditions.

Reliable estimations of PET, both at point basis as well as over extended areas of interest (i.e. river basins), can be obtained by interpolating the known (i.e., locally optimized) parameter values and next employing the parametric formula.
References