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## Application of stochastic methods to double cyclostationary processes for hourly wind speed simulation

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### Abstract

In this paper, we present a methodology to analyze processes of double cyclostationarity (e.g. daily and seasonal). This method preserves the marginal characteristics as well as the dependence structure of a process (through the use of climacogram). It consists of a normalization scheme with two periodicities. Furthermore, we apply it to a meteorological station in Greece and construct a stochastic model capable of preserving the Hurst-Kolmogorov behaviour. Finally, we produce synthetic time-series (based on aggregated Markovian processes) for the purpose of wind speed and energy production simulation (based on a proposed industrial wind turbine).

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### 1. Introduction

Several methods exist for dealing with processes of single periodicity, with most of them preserving the marginal characteristics of the process and assuming a short-range dependence structure (cf. [1]). However, neglecting a possible long-range dependence, i.e. Hurst-Kolmogorov (HK) behaviour, could lead to unrealistic predictions and wind load situations, causing some impact on the energy production and management of renewable sources. Here,

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we focus on the stochastic nature of wind speed in an hourly scale. The most challenging problem of wind speed simulation is the internal periodicities (e.g. daily and seasonal cycle), a common characteristic of hydrometeorological processes. In this paper, we apply the methodology presented in [1], which involves the analysis of a monthly-scale process, but with preserving both daily and seasonal periodicity. Particularly, assuming that the process has a double cyclostationarity, we first normalize each cyclostationary variable, using a scheme of double periodicity with three parameters. Then, we analyze the stochastic structure of the wind process and we construct a model based on the climacogram, a stochastic tool with many advantages in stochastic interpretation and model building [2,3]. Additionally, we produce synthetic time-series for the purpose of wind speed and energy production simulation (based on a proposed industrial wind turbine). Finally, we apply the methodology to the meteorological station of Larissa ([www.hnms.gr](http://www.hnms.gr)) in the area of Thessaly (Greece), with latitude  $22.417^\circ$ , longitude  $39.633^\circ$  and elevation +74 m. This is one of the older stations in Greece and includes up to 75 years of measurements in an hourly scale. Its marginal mean wind speed is estimated as 1.7 m/s and its standard deviation as 2.71 m/s (for more information see in [2]).

In the next section, we describe the normalization method, we show how to analyze the stochastic structure of a normalized process and how to generate synthetic time-series based on aggregated Markovian processes. Finally, we produce a one week hourly wind speed time-series (that preserves the marginal characteristics as well as the dependence structure of the examined process) and we estimate the hypothetically produced energy from a wind turbine. Note that underlined symbols denote random variables and the overline symbol ( $\hat{\phantom{x}}$ ) denotes estimation.

## 2. Stochastic analysis of the wind speed process

### 2.1. Cyclostationarity

One of the most common characteristics of hydrometeorological processes (in a sub-climatic scale) is the double periodicity, i.e. the continuous change of the process' statistical properties in both daily and seasonal scales. Several techniques have been developed to model this behaviour (a brief description can be seen in [1]). However, most of them can capture the marginal characteristics of the process assuming a short-range dependence structure between daily and seasonal variables. A method to model a single periodicity with any type of internal dependence structure is presented in [1], where the process is assumed to be cyclostationary in seasonal scale (e.g. monthly scale). The main feature of this method is the application of a normalization scheme (derived from the principle of maximum entropy) to all seasonal variables, capturing in this way both the marginal properties as well as the dependence structure of the process (zero values are excluded from the analysis since the wind process cannot exhibit zero speeds). Here, we apply this scheme but with also including the daily periodicity since we are interested in sub-daily (e.g. hourly) scale simulation. The normalization scheme is the following:

$$\underline{Z} = \text{sign}\left(\frac{\underline{X} - \mu_c}{\sigma_c}\right) \sqrt{\left(1 + \frac{1}{g_c}\right) \ln\left(1 + g_c \left(\frac{\underline{X} - \mu_c}{\sigma_c}\right)^2\right)} \quad (1)$$

where  $\underline{Z} \sim N(0,1)$  is the transformed process of  $\underline{X}$ ,  $\mu_c$  and  $\sigma_c$  are the mean and standard deviation for each cyclostationary variable (i.e. one for each hour and month), and  $g_c$  is a parameter related to the distribution tail of the cyclostationary process.

From Fig. 1, we observe that the cyclostationary mean value of the process can be well described by a periodic exponential function for the daily scale and with a simple cosine function for the monthly scale (performance of these models to the Larissa station can be also seen in [2]). Also, we observe that the standard deviation can be well modeled by two simple periodic functions and that  $g_c$  significantly varies only within the daily scale and thus, can be described by a single cosine function:

$$\mu_c = \left( a_1 \cos\left(\frac{2\pi t}{T_h}\right) + a_2 \right) e^{-\cos\left(\frac{2\pi t}{T_d}\right)} + a_3 \mu_h \tag{2}$$

$$\sigma_c = \left( a_4 \cos\left(\frac{2\pi t}{T_h}\right) + a_5 \right) \sin\left(\frac{2\pi t}{T_d}\right) + a_6 \sigma_h \tag{3}$$

$$g_c = a_7 \cos\left(\frac{2\pi t}{T_d}\right) + a_8 \tag{4}$$

where  $t$  denotes time (h),  $a_i$  are dimensionless coefficients,  $T_h$  equals the annual time duration in hours and  $T_d=24$  h. For the Larissa station the coefficients  $a_i$  are calculated (with fitting  $R^2$  coefficient around 95% for all cases) as:  $\alpha_1=0.463$ ,  $\alpha_2=0.177$ ,  $\alpha_3=0.6$ ,  $\alpha_4=0.07$ ,  $\alpha_5=-0.1$ ,  $\alpha_6=0.738$ ,  $\alpha_7=0.217$  and  $\alpha_8=0.541$ .

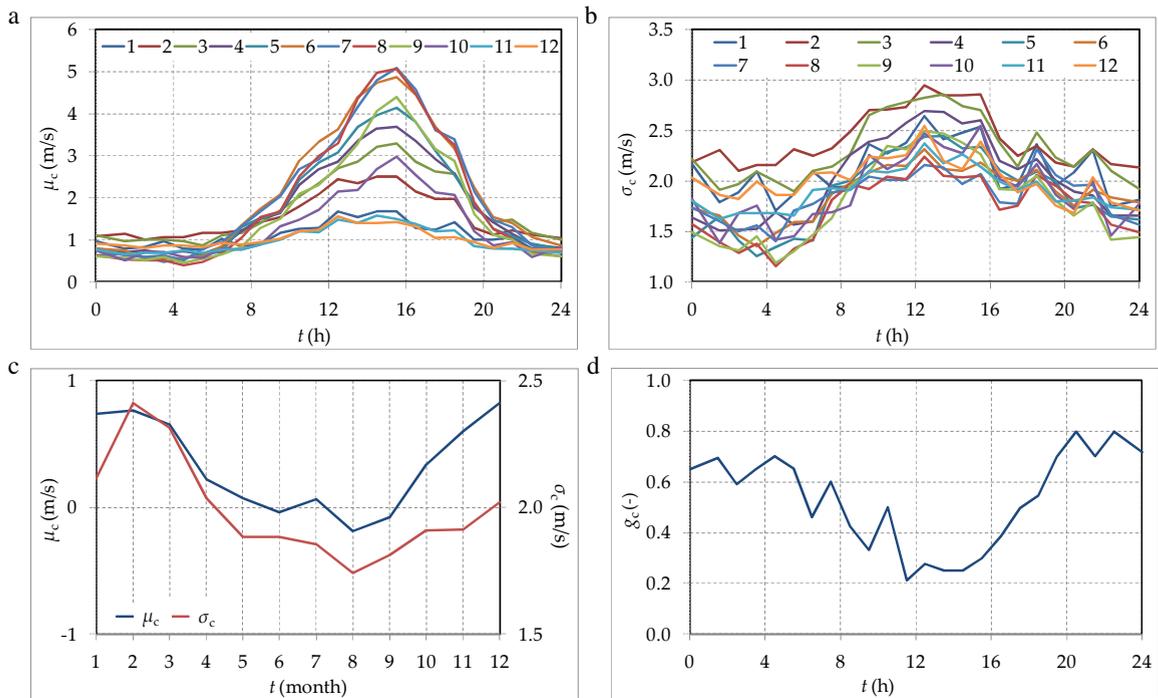


Fig. 1. (a) fluctuation of hourly mean wind speed for each month; (b) fluctuation of hourly wind speed standard deviation for each month; (c) fluctuation in a monthly scale of both mean and standard deviation of hourly wind speed (hourly-averaged); (d) fluctuation in a hourly scale of parameter  $g_c$  (monthly-averaged).

### 2.2. Stochastic structure

By normalizing the process, we have no longer effects of the internal periodicities to the stochastic structure of the process and thus, we can now proceed to the estimation of the latter. There are several stochastic tools available for the analysis of the dependence structure of a process (e.g. autocovariance, power spectrum, variogram). Based on the analysis of [3], we choose to use the climacogram (i.e. plot of variance of the mean aggregated process vs. scale,

cf. [4]). It has been shown that for simple processes, such as Markovian, HK and combinations thereof, the latter stochastic tool often outperforms the aforementioned tools in terms of smaller statistical uncertainty. Furthermore, it has a plethora of advantages in terms of stochastic analysis (e.g. in determining the Hurst coefficient) and model building (e.g. it has simple and analytical expressions for the expected value of the process). The climacogram definition, classical estimator and expected value are shown in the equations below.

$$\gamma(m) = \text{Var} \left[ \int_0^m \underline{X}(\xi) d\xi \right] / m^2 \quad (5)$$

$$\hat{\underline{\gamma}}(\Delta k) = \frac{1}{n-1} \sum_{i=1}^n \left( \frac{1}{k-1} \sum_{l=k(i-1)+1}^{ki} \underline{X}_l^{(\Delta)} - \frac{1}{n} \sum_{l=1}^n \underline{X}_l^{(\Delta)} \right)^2 \quad (6)$$

$$\text{E} \left[ \hat{\underline{\gamma}}(\Delta k) \right] = \frac{1 - \gamma(\Delta n) / \gamma(\Delta k)}{1 - k/n} \gamma(\Delta k) \quad (7)$$

where  $\gamma$  is the continuous-time climacogram (in  $\text{m}^2/\text{s}^2$ ),  $m$  is the continuous-time scale (in h),  $\Delta$  is the sampling time interval (in our analysis equals 1 h),  $n$  is the total number of observations and  $k$  is the discrete-time scale (dimensionless).

In Fig. 2, we observe that the empirical (from the normalized process) climacogram exhibits a Markovian decay at small scales and an HK behaviour at large ones (similar observations in the wind process are derived in [3]). Here, we choose to fit a Markovian model (to control the small scales) and an HK one for the larger scales (shown in the equation below), by assuming that the empirical climacogram represents the expected value of the process. The best fitted parameters are estimated as:  $\lambda_M=6 \text{ m}^2/\text{s}^2$ ,  $q=0.05 \text{ h}$ ,  $\lambda_{\text{HK}}=0.1 \text{ m}^2/\text{s}^2$  and  $H=0.75$ :

$$\gamma(k\Delta) = \frac{2\lambda_M}{(k\Delta/q)^2} (k\Delta/q + e^{-k\Delta/q} - 1) + \frac{\lambda_{\text{HK}}}{(k\Delta)^{2-2H}} \quad (8)$$

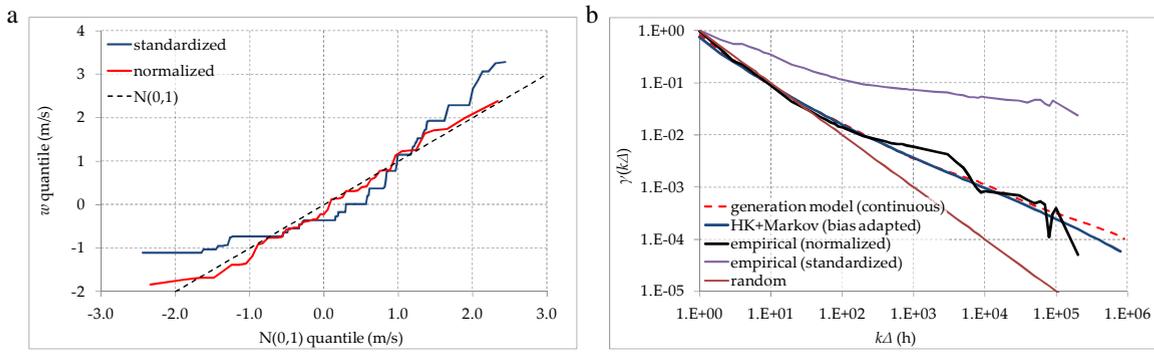


Fig. 2. (a) qq-plot of standardized and normalized time-series of the 1<sup>st</sup> hour of the day of the 1<sup>st</sup> month (where  $w$  denotes wind speed); (b) continuous-time climacograms for a random ( $H=0.5$ ) process, empirical (standardized and normalized) climacograms from the analysis of the Larissa station, the adapted for bias climacogram of the HK and Markovian fitting model to the empirical normalized climacogram as well as the continuous-time model used for the stochastic generation based on the aggregated Markovian process (described in section 2.3).

### 2.3. Stochastic generation and application in energy production simulation

For the stochastic generation we choose the methodology presented in [3]. We produce synthetic HK Gaussian distributed time series based on an aggregation of Markovian processes:

$$\gamma_i(k\Delta) = \frac{2\lambda_i}{(k\Delta/q_i)^2} (k\Delta/q_i + e^{-k\Delta/q_i} - 1) \tag{9}$$

whose parameters  $q_i$  are connected to each other in a pre-defined way (parameters  $\lambda_i$  can be calculated analytically following the analysis of [3]), particularly:

$$q_i = p_1 p_2^{i-1} \tag{10}$$

where  $p_1$  and  $p_2$  are parameters, which can be calculated by minimizing the residouble between the modeled and aggregated-Markovian processes. For the chosen HK process and for  $n \approx 10^6$ , we choose to generate four Markovian processes, with the best fit corresponding to  $p_1=0.113$  and  $p_2=0.099$  (Fig. 2).

Hence, we can generate a N(0,1) process with the desired stochastic structure and then, by applying the inverse normalization scheme described in section 2.1, we can produce a time-series with the same statistical characteristics as the original one, for the purpose of simulation (note that we set all negative synthetic values to zero). In Fig. 3, we illustrate a weekly time-window of generated hourly wind speed with the same stochastic structure and seasonality properties of the Larissa station. Furthermore and for illustration purposes, we assume a reference wind speed (i.e. 10 min mean wind speed at hub height with a 50-year return period) equal to 42.5 m/s and a larger annual average wind speed of 10 m/s. Based on the latter specifications and on the IEC-61400 standards [5], we can install a wind turbine generator of class II, with an industrial solution of ENERCON E-82 (cf. [2]). Finally, we show in Fig. 3 the simulation of the energy production based on the turbine’s power curve.

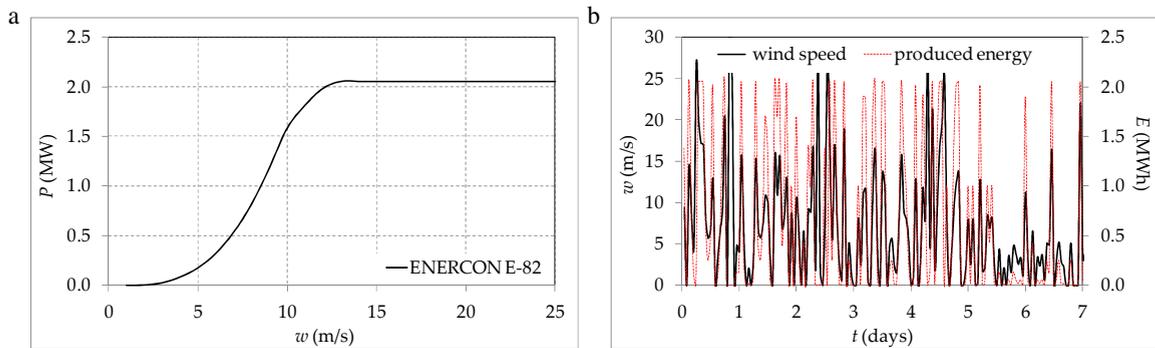


Fig. 3. (a) wind turbine power curve of ENERCON E-82 (enercon.de); (b) a weekly-window of hourly wind speed simulation and the corresponding energy production from the installed wind turbine (where  $w$  denotes wind speed).

### 3. Conclusions

In this paper, we present a methodology for dealing with processes of double cyclostationarity (e.g. daily and seasonal). Most existing methodologies preserve the marginal characteristics and assume a process with a short-range dependence structure. The present method is based on a normalization scheme with two periodicities and it is more appropriate for the wind speed process. Furthermore, we describe how to analyze the stochastic structure of a normalized process with the use of climacogram, a stochastic tool with many advantages in stochastic interpretation and model building. Also, we construct a stochastic model capable of preserving an HK behaviour and we produce synthetic time-series (based on aggregated Markovian processes) for the purpose of simulation. Finally, we apply the above to a meteorological station in Greece and we illustrate an example of simulation of wind speed and energy production (based on a proposed industrial wind turbine).

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