Chapter 74
Extreme Rainfall: Global Perspective

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Abstract The study of rainfall extremes is important for design purposes of flood protection works, in the development of flood risk management plans and in assessing the severity of occurring storm and flood events. Such study unavoidably relies on observational data, which, given the enormous variability of the precipitation process in space and in time, should be local, of the area of interest. While general statistical laws or patterns apply over the globe, the parameters of those laws vary substantially and need local data to be estimated. Because of their global coverage, satellite data can be insightful to show the behavior of precipitation over the globe. However, only ground data (observations from raingages) are reliable enough for rainfall extremes and also have adequate length of archive that allows reliable statistical fitting. The study of the record rainfalls throughout the globe provides some useful information on the behavior of rainfall worldwide. While most of these record events have been registered at tropical areas (with a tendency for grouping in time with highest occurrence frequency in the period 1960-1980), there are record events that have occurred in extratropical areas and exceed, for certain time scales, those that occurred in tropical areas. The record values for different time scales allow the fitting of a curve which indicates that the record rainfall depth increases approximately proportionally to the square root of the time scale. Clearly, however, these record values do not suggest an upper limit of rainfall and are destined to be exceeded, as past record values have already been exceeded. In addition, the very concept of the probable maximum precipitation, which assumes a physical upper limit to precipitation at a site, is demonstrated to be fallacious. The only scientific approach to quantify extreme rainfall is provided by the probability theory. Theoretical arguments and general empirical evidence from many rainfall records worldwide suggest power-law distribution tail of extreme rainfall and favor the Extreme Value type II (EV2) distribution of maxima. The shape parameter of the EV2 distribution appears to vary in a narrow range worldwide. This facilitates fitting of the EV2 distribution and allows its easy implementation in typical engineering tasks such as estimation and prediction of design parameters, including the construction of theoretically consistent ombrian (also known as IDF) curves, which constitute a very important tool for hydrological design and flood severity assessment.

1. Introduction: The importance of studying extreme rainfall and the related difficulties

The design and management of flood protection works and measures requires reliable estimation of flood probability and risk. A solid empirical basis for this estimation can be offered by flow observation records with appropriate length, sufficient to include a sample of representative floods. In practice, however, flow measurements are never enough to support flood modelling. In both rural and urban catchments, the flood control points are numerous but the flow gage sites are scarce or non-existing. The obvious alternative is the use of hydrological models with rainfall input data and the substitution of rainfall for streamflow empirical information. Notably, even when flow records exist, yet information on rainfall extremes has still a major role in hydrologic practice; for instance in major hydraulic structures, the design floods are generally estimated from appropriately synthesized design storms (e.g. U.S. Department of the Interior, Bureau of Reclamation, 1977, 1987; Sutcliffe, 1978).

The study of rainfall and flow extremes on an event basis usually suffices for design purposes, while in other applications, such as monitoring of flood risk in real time and development of flood warning systems, continuous flow simulation is usually necessary. Even in these latter more demanding applications, the study of extremes is again a prerequisite as it provides the basis for assessing the severity of occurring storm and flood events in real time or a posteriori. Furthermore, the development of flood risk management plans (c.f. the European Flood Directive; European Commission, 2007) strongly relies on the study of precipitation extremes.

Such study cannot be based merely on principles of physics and, particularly, thermodynamics, even though the latter effectively describes aspects of the atmospheric and hydrological processes related to the occurrence of water vapor in the atmosphere (in particular, quantification of the upper limit of the vapor that the atmosphere can contain; Koutsoyiannis, 2012) and the generation of precipitation (Koutsoyiannis and Langousis, 2011). The difficulties of modelling precipitation based on merely physical principles is illustrated by the fact that, despite of tremendous efforts to simulate it through Global Circulation Models (GCMs), the resulting simulations have been irrelevant to reality at monthly to multi-year (climatic) time scales (Koutsoyiannis et al., 2008, 2011; Anagnostopoulos et al., 2010). Furthermore, at short time scales, models produce more frequent and less intense precipitation (cf. Stephens et al., 2010, who use the expression "*dreary state of precipitation in global models*") while the extremes are underestimated even by an order of magnitude (Tsaknias et al., 2011).

Therefore any serious attempt to model extreme rainfall should unavoidably rely on observational data. In addition, because of the enormous variability of the precipitation process in space and in time, at all spatial and temporal scales, the data should be local, of the area of interest. This does not mean that no general statistical laws or patterns apply; rather it means that the parameters of such laws vary substantially and need local data to be estimated. Only ground data, i.e. observations from raingages, are reliable enough for rainfall extremes and also have adequate length of archive, which can allow reliable statistical fitting. However, radar and satellite data calibrated against raingage data can be helpful (Endreny and Imbeah, 2009; Overeem et al., 2010; AghaKouchak et al., 2011; Gourley et al., 2011; Berne et al., 2013; Stampoulis et al., 2013; Villarini et al., 2014), particularly when the spatial dimension of rainfall is of interest (see section 6.3).

Because of their global coverage, satellite data can be insightful to show the behavior of precipitation over the globe, i.e. the global precipitation climatology. Fig. 1 shows the spatial variability of precipitation over the globe in mm/d at a climatic scale (average for the 35-year period 1979-2014), based mostly on satellite data (see figure caption). While the average precipitation rate over the globe for the specified 35-year period is 2.67 mm/d or 977 mm/year, we observe huge differences in different areas of the globe. In some areas, mostly in tropical seas and in equatorial areas of South America and Indonesia, this rate exceeds 10 mm/d or 3.65 m/year. On the other hand, in large areas in the subtropics, where climate is dominated by semi-permanent anticyclones, precipitation is lower than 1 mm/d or 365 mm/year. Significant portions of these areas in Africa, Australia, and America are deserts, where the average precipitation is much lower than 1 mm/d. In addition, in polar regions, where the available atmospheric moisture content is very low (due to the low amount of vapor that the atmosphere can contain, as a result of the low temperatures and the Clausius-Clapeyron law) the amounts of precipitation are very small or even zero. For example, it is believed that certain dry valleys in the interior of Antarctica have not received any precipitation during the last two million years (Uijlenhoet, 2008).

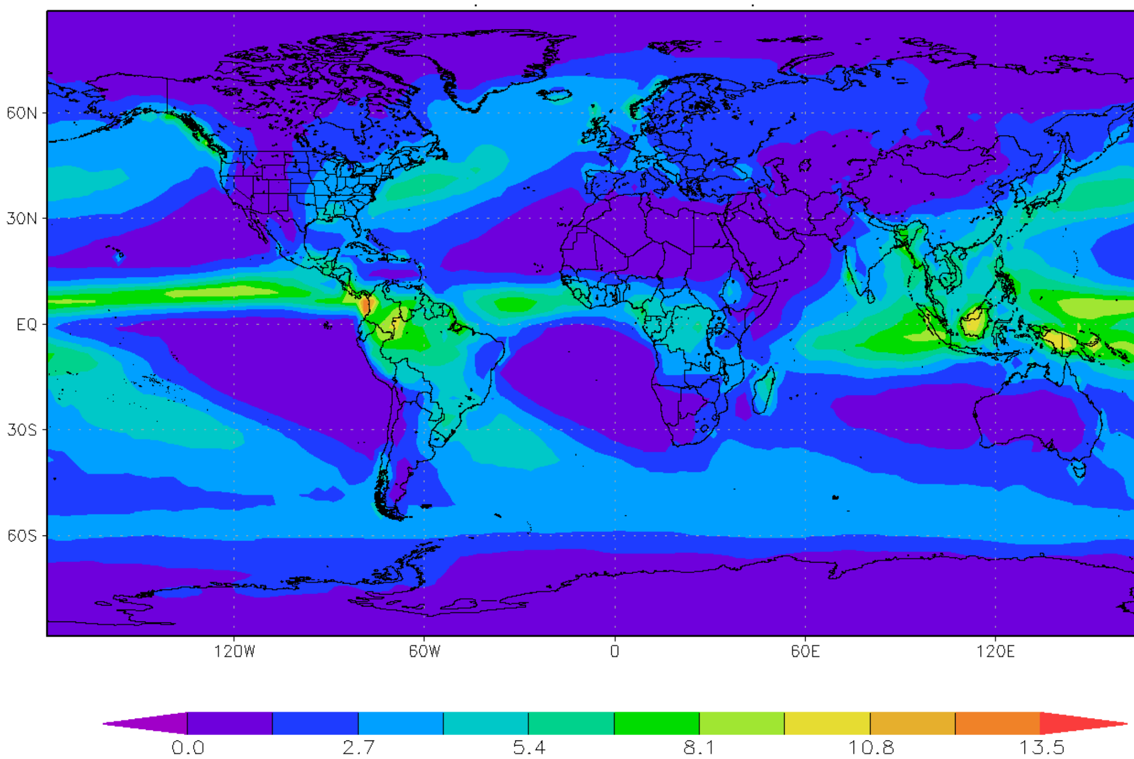

Fig. 1 Precipitation distribution over the globe in mm/d (average for the 35-year period 1979-2014). Data availability and image generation due to the Global Precipitation Climatology Project (GPCP_RAIN.2.2) provided by NASA (http://disc.gsfc.nasa.gov/precipitation/tovas; http://gdata1.sci.gsfc.nasa.gov/daac-bin/G3/gui.cgi?instance_id=GPCP_Monthly); resolution 2.5°×2.5°.

Fig. 2 depicts the zonal precipitation profile and shows that the climatic precipitation rate at an annual basis is highest at a latitude of 5°N, exceeding 2000 mm/year, and has a second peak of about 1500 mm/year at 5°S. Around the Tropics of Cancer and Capricorn, at 23.4°N and S, respectively, the rainfall rate displays troughs of about 600 mm/year whereas at mid latitudes, between 35° and 60° both N and S, rainfall increases again and remains fairly constant, close to the global average of 977 mm/year. Then, toward the poles, it decreases to about 150 mm/year. Fig. 2 also shows the monthly minimum and maximum values observed over the 35-year period 1979-2014, whose profiles roughly follow that of the temporally average climatic profile, but indicate a huge temporal variability. Obviously such variability becomes much huger as the time scale decreases from monthly to minute, as the spatial scale is replaced by a point in space, and as the period of observations increases.

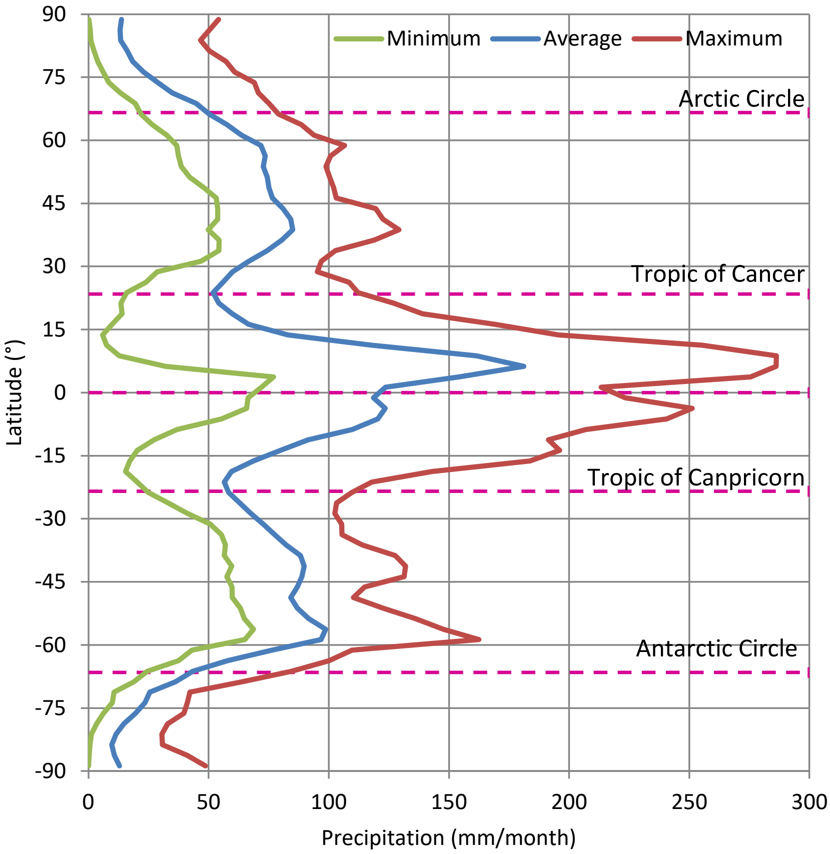

Fig. 2 Zonal precipitation profile: precipitation spatially averaged over all longitudes for latitude varying from 90°N to 90°S (−90°N). Temporal averages over the 35-year period 1979-2014, as well as observed monthly minima and maxima are shown (data from GPCP as in Fig. 1).

While the spatial variability over the globe (shown in Fig. 1 and Fig. 2) as well the seasonal variability (not shown in figures) are well comprehended and roughly explainable in terms of basic physical and astronomical knowledge (i.e. solar radiation, relationship of temperature and atmospheric moisture content, motion of Earth), in other words they are 'regular', there is also temporal variability from year to year, which is irregular and difficult or even impossible to predict. Fig. 3 focuses on the irregular interannual variability of precipitation. It depicts the annual variation of the globally averaged precipitation according to three different data sets, where the first is the GPCP data set, mostly based on satellite data as discussed above. The other two data sets, 'g55wld0098' (Hulme et al., 1998) and the Global Historical Climatology Network (GHCN), are from ground data for global land areas, gridded at 5° resolution, and cover much longer periods, starting from 1900. For the 'g55wld0098' data set only years with more than 600 grid points were considered (and thus the years 1995-98 which had fewer grid points were excluded), while a correction for missing grid points was applied, based on the ratio of zonal precipitation of the existing (not missing) grid points to the global average. The values calculated from the GHCN data set, which is originally given in terms of "*anomalies*" (differences from some unspecified average) were converted into precipitation values by assuming that the unspecified average is 1015 mm, so that the temporal average over the period 1900-94 matches that of the 'g55wld0098' data set.

We can see that the annual variability is remarkable. Thus, the satellite-based annual precipitation in the last 35 years has varied between 956 and 997 mm; the corresponding figures for the precipitation over land are 847 and 942 mm. The variation from the ground data for a longer period (beginning in 1900) is much higher, 912 to 1085 mm. Remarkable are also the differences between the satellite and ground data sets for land, which raises doubts on the accuracy on the data (particularly the satellite ones).

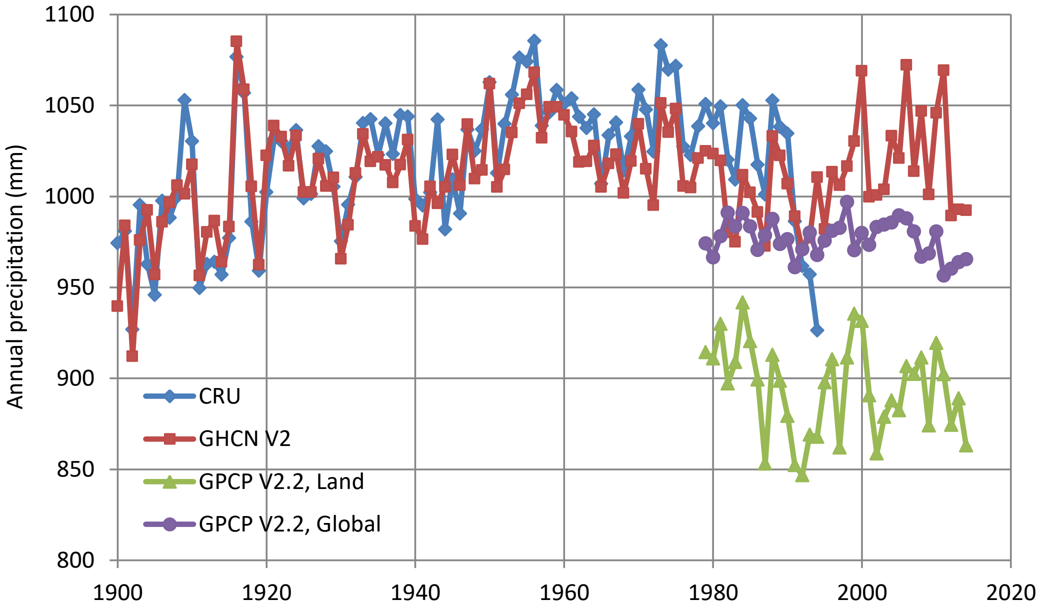

Fig. 3 Evolution of the globally averaged annual precipitation according to several data sets of global precipitation, i.e., (a) GPCP for the 35-year period 1979-2014 (see caption of Fig. 1) for the entire globe and over land (the data over land were retrieved using the option "*land points*" from http://climexp.knmi.nl/select.cgi?id=someone@somewhere&field=gpcp_22); (b) the 'g55wld0098' data set (Version 1.0, March 1999) gridded at 5° resolution for global land areas for the period 1900-98, available via the Climatic Research Unit (CRU; http://www.cru.uea.ac.uk/cru/data/precip/; see also Hulme et al., 1998); (c) gridded precipitation calculated from GHCN (version 2) monthly precipitation data set, consisting of 2 592 gridded data points at 5° resolution for the period 1900-2014 (http://www.ncdc.noaa.gov/temp-and-precip/ghcn-gridded-products/).

An impressive behavior in the plots of Fig. 3 is the appearance of long term patterns, which could be characterized as "*trends*" (e.g. an increasing trend from 1900 to 1950s) or else tendency of high (or low) values to group in time. Such patterns, omnipresent in natural (and human) processes, are regarded in most part of the literature as "*deterministic components*", while attempts to attribute them to some physical or anthropogenic mechanisms abound. Undoubtedly, such long-term changes are driven by some physical mechanisms, but the complexity of the processes responsible for the rainfall generation does not allow easy deterministic explanations and attributions. Besides, such patterns are better described in stochastic terms, as the Hurst-Kolmogorov behavior (e.g. Koutsoyiannis, 2013). Simple quantification of this behavior is provided by the Hurst coefficient, H, where values of H higher than ½ (a value indicating random noise) and approaching 1 reflect greater intensity of patterns. Indeed, all four series plotted in Fig. 3 yield H higher than 0.75, with three of them exceeding H = 0.85.

2. A global survey of record rainfall depths

As already mentioned, only ground data can be accurate enough to support reliable estimation of extreme precipitation and give quantitative information on "how extreme is extreme" (cf. Papalexiou et al., 2013). In this respect, Table 1 registers all record rainfall values which have ever been recorded at raingages for varying time scales (sometimes called durations), from 1 min to 2 years. The entries of the table originate from the publications indicated in the footnote of this table, as well as from the inspection of several thousands of stations made in this study. Specifically for the latter, the GHCN daily database (version 3.02, www.ncdc.noaa.gov/oa/climate/ghcn-daily), comprising several thousands of daily rainfall records from all over the world, was used. However, the database includes many records that are too short, have a large percentage of missing values, or include data of disputable quality (assigned with quality flags, as detailed in the above website). Consequently, to ensure the quality of the data, a subset of the database was used with stations satisfying the following criteria: (a) record length > 50 years, (b) percentage of missing values < 20% and, (c) percentage of values assigned with quality flags < 1%. Moreover, to further ensure the data quality, each of the records was explored in detail for specific flagged values, i.e., values assigned with the flags 'G' (failed gap check) and 'X' (failed bounds check). These values are essentially unrealistically large values and were excluded from the analysis.

The resulting subset contains 17 490 stations. These were separated in four classes, corresponding to northern (NH) or southern (SH) hemisphere, and to tropical or extratropical areas; the numbers of stations and station-years in each zone are shown in Table 2. The time scales for which maxima were extracted from the GHCN data set are shown in Fig. 4 (horizontal axis), which also depicts the huge geographical variability of these maxima in terms of box plots (notice the logarithmic vertical axis).

The investigation of the GHCN data resulted in record rainfall values which are somewhat lower than the values of Table 1. These have been registered in Table 3 and correspond to time scales from 1 to 730 days (2 years). By comparing Table 1 and Table 3 for time scales ≥ 1 d, it is seen that the only entry appearing in both tables is that of Koumac for the 2-day scale. This means that this entry had been neglected in earlier publications related to record rainfall events, such as those in the footnote of Table 1. It also means that the GHCN data set does not include some of the stations or time periods that have given the most extreme records. Interestingly, while the Cherrapunji station in India is contained in the GHCN data set and has given a lot of record values in Table 3, those values of the same station contained in Table 1 (constituting the records for time scales from 1 month to 2 years), are missing from Table 3. The reason is that the record values of Table 1 for Cherrapunji are from the 19th century (1860-61), while the GHCN data for Cherrapunji start at 1902.

Table 1 World record point precipitation measurements.

Time scale	Amount (mm)	Location	Lat. (°)	Lon.(°)	Start date	Event#	Ref.§
1 min	38	Barot, Guadeloupe	16.25	-61.45	26 Nov 1970	N	1,2
3	44	Haughton Grove, Jamaica	18.33	-77.98	30 Sep 1925	G	3
5	63	Porto Bello, Panama	9.55	-79.65	29 Nov 1911	D	3
8	126	Fussen, Bavaria, Germany	47.87	12.17	25 May 1920	F	1
15	198	Plumb Point, Jamaica	17.93	-76.78	12 May 1916	E	1
20	206	Curtea-de-Arges, Romania	45.12	-24.42	7 Jul 1889	B	1
30	280	Sikeshugou, Hebei, China	41.78	117.93	03 Jul 1974	P	3
42	305	Holt, Missouri, USA	39.45	-94.33	22 Jun 1947	J	1,4
60	401	Shangdi, Nei Monggol, China	42.27	119.13	03 Jul 1975	Q	1
72	440	Gaoj, Gansu, China	34.85	104.67	12 Aug 1985	V	3
2 h	489	Yujiawanzi, Nei Monggol, China	41.50	118.93	19 Jul 1975	R	3
2.17	483	Rockport, USA	42.58	-70.92	18 Jul 1889	C	1
2.5	550	Bainaobao, Hebei, China	41.58	114.3	25 Jun 1972	O	3
2.75	559	D'Hanis, Texas, USA	29.33	-99.28	31 May 1935	H	1
3	724	Smethport, Pennsylvania, USA	41.80	-78.43	18 Jul 1942	I	3
4.5	782	Smethport, Pennsylvania, USA	41.80	-78.43	18 Jul 1942	I	1
6	840	Muduocaidang,Nei Monggol, China	32.98	113.59	01 Aug 1977	T	1
9	1087	Belouve, La Réunion	-21.00	55.50	28 Feb 1964	L	1
10	1400	Muduocaidang,Nei Monggol, China	32.98	113.59	01 Aug 1977	T	1
18	1589	Foc-Foc, La Réunion	-21.23	55.68	07 Jan 1966	M	3
18.5	1689	Belouve, La Réunion	-21.00	55.50	28 Feb 1964	L	1
20	1697	Foc-Foc, La Réunion	-21.23	55.68	07 Jan 1966	M	3
22	1780	Foc-Foc, La Réunion	-21.23	55.68	07 Jan 1966	M	3
1 d	1870	Cilaos, La Réunion	-21.13	55.47	15 Mar 1952	K	1
2	2774	Koumac, New Caledonia	-20.57	164.28	16 Jan 1976	S	*
3	3637	Commerson, La Réunion	-21.20	55.65	24 Feb 2007	W	3,5
4	4869	Commerson, La Réunion	-21.20	55.65	24 Feb 2007	W	3,5
5	4979	Commerson, La Réunion	-21.20	55.65	24 Feb 2007	W	3
6	5075	Commerson, La Réunion	-21.20	55.65	24 Feb 2007	W	3
7	5400	Commerson, La Réunion	-21.20	55.65	24 Feb 2007	W	3
8	5510	Commerson, La Réunion	-21.20	55.65	24 Feb 2007	W	3
9	5692	Commerson, La Réunion	-21.20	55.65	19 Jan 1980	T	1
10	6028	Commerson, La Réunion	-21.20	55.65	18 Jan 1980	T	1
11	6299	Commerson, La Réunion	-21.20	55.65	17 Jan 1980	T	1
12	6401	Commerson, La Réunion	-21.20	55.65	16 Jan 1980	T	1
13	6422	Commerson, La Réunion	-21.20	55.65	15 Jan 1980	T	1
14	6432	Commerson, La Réunion	-21.20	55.65	14 Jan 1980	T	1
15	6433	Commerson, La Réunion	-21.20	55.65	14 Jan 1980	T	1
1 month	9300	Cherrapunji, Meghalaya, India	25.30	91.70	1 Jul 1861	A	1
2	12767	Cherrapunji, Meghalaya, India	25.30	91.70	1 Jun 1861	A	1
3	16369	Cherrapunji, Meghalaya, India	25.30	91.70	1 May 1861	A	1
4	18738	Cherrapunji, Meghalaya, India	25.30	91.70	1 Apr 1861	A	1
5	20412	Cherrapunji, Meghalaya, India	25.30	91.70	1 Apr 1861	A	1
6	22454	Cherrapunji, Meghalaya, India	25.30	91.70	1 Apr 1861	A	1
1 year	26461	Cherrapunji, Meghalaya, India	25.30	91.70	1 Aug 1860	A	1
2	40768	Cherrapunji, Meghalaya, India	25.30	91.70	1 Jan 1860	A	1

#Events are labeled A to W in time order.

§Ref.: *Current study; 1. World Meteorological Organization, 1994; 2. Klein, 1971 ; 3. NOAA, 2015; 4. Lott, 1954; 5. Quetelard, et al., 2009, 2015.

Table 2 Numbers of GHCN stations and station-years per zone.

Zone	No. of stations	Station-years
NH, Tropical	1 332	90 921
NH, extratropical	9 282	726 852
SH, Tropical	1 353	95 091
SH, extratropical	5 523	481 729
Total	17 490	1 394 593

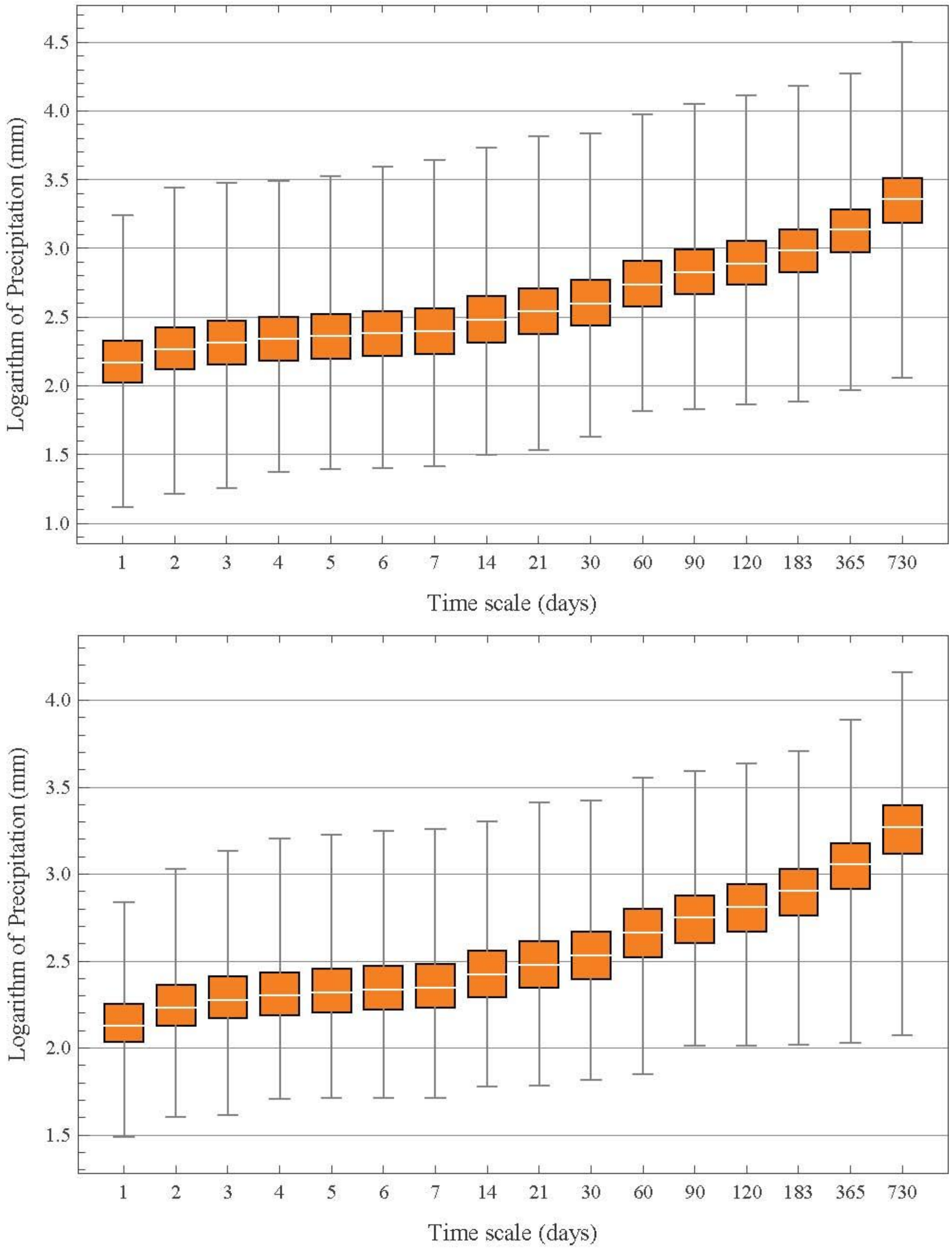

Fig. 4 Box plots of maximum recorded rainfall depths of GHCN stations per time scale: (upper) global; (lower) SH extratropical zone, where the extreme rainfall regime is different from all other zones (see text). The central mark inside each box is the median, the box edges are the 25th and 75th percentiles and the ends of the whiskers represent the observed minimum and maximum values.

Table 3 Record point precipitation measurements for four different zones, extracted from the GHCN database.

Time scale (d)	Amount (mm)	Corresponding K_m	Station ID	Location	Lat. (°)	Lon. (°)	Elev. (m)	Start date
NH, tropical								
1	909.3	17.77	IN010050600	Quilandi, India	11.45	75.70	8	28/05/1961
2	1143.3	5.13	IN012121100	Matheran, India	18.98	73.28	756	23/07/1921
3	1473.2	12.48	IN010050600	Quilandi, India	11.45	75.70	8	20/05/1961
4	1857	13.99	IN010050600	Quilandi, India	11.45	75.70	8	25/05/1961
5	2158.6	14.21	IN010050600	Quilandi, India	11.45	75.70	8	21/05/1961
6	2595.8	14.78	IN010050600	Quilandi, India	11.45	75.70	8	20/05/1961
7	3105.1	16.18	IN010050600	Quilandi, India	11.45	75.70	8	22/05/1961
14	4215.7	15.92	IN010050600	Quilandi, India	11.45	75.70	8	19/05/1961
21	6503.2	18.31	IN010050600	Quilandi, India	11.45	75.70	8	20/05/1961
30	6851.7	14.75	IN010050600	Quilandi, India	11.45	75.70	8	19/05/1961
60	9444.9	6.76	IN012121100	Matheran, India	18.98	73.28	756	23/07/1921
90	10882.6	6.12	IN012121100	Matheran, India	18.98	73.28	756	03/07/1921
120	11327.1	11.80	IN010050600	Quilandi, India	11.45	75.70	8	20/05/1961
183	13169.3	10.31	IN010050600	Quilandi, India	11.45	75.70	8	15/05/1961
365	13550.4	6.45	IN010050600	Quilandi, India	11.45	75.70	8	02/11/1960
730	17906.9	2.45	IN009082100	Bhagamandala, India	12.38	75.52	876	03/04/1961
SH, tropical								
1	1750	1.93	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	17/01/1976
2	2774	2.84	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	16/01/1976
3	2983	2.76	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	14/01/1972
4	3094	2.72	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	14/01/1972
5	3172	2.65	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	13/01/1972
6	3198	2.45	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	13/01/1972
7	3278	2.42	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	14/01/1972
14	5406	3.72	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	14/01/1976
21	5782	3.04	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	14/01/1976
30	5955	2.28	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	06/01/1976
60	9124	2.31	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	14/01/1976
90	11039	2.16	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	14/12/1975
120	12030	2.08	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	12/12/1975
183	13808	1.93	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	22/11/1998
365	18838	1.65	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	31/12/1975
730	30243	1.34	NC000091577	Koumac, New Caledonia	-20.57	164.28	18	11/02/1975
NH, extratropical								
1	973.8	2.38	IN014020800	Cherrapunji, India	25.25	91.73	1313	05/06/1956
2	1655.5	2.56	IN014020800	Cherrapunji, India	25.25	91.73	1313	09/06/1966
3	2239.8	2.74	IN014020800	Cherrapunji, India	25.25	91.73	1313	08/06/1966
4	2840.2	3.28	IN014020800	Cherrapunji, India	25.25	91.73	1313	07/06/1966
5	3346.2	3.63	IN014020800	Cherrapunji, India	25.25	91.73	1313	08/06/1966
6	3946.6	4.37	IN014020800	Cherrapunji, India	25.25	91.73	1313	07/06/1966
7	4397	4.77	IN014020800	Cherrapunji, India	25.25	91.73	1313	06/06/1966
14	4880.7	3.87	IN014020800	Cherrapunji, India	25.25	91.73	1313	01/06/1966
21	5372.1	3.05	IN014020800	Cherrapunji, India	25.25	91.73	1313	03/06/1956
30	6533	3.15	IN014020800	Cherrapunji, India	25.25	91.73	1313	04/06/1966
60	8674.6	2.05	IN014020800	Cherrapunji, India	25.25	91.73	1313	15/06/1970
90	11217.8	2.22	IN014020800	Cherrapunji, India	25.25	91.73	1313	05/05/1954
120	13011.4	2.23	IN014020800	Cherrapunji, India	25.25	91.73	1313	27/04/1954
183	15354.5	2.36	IN014020800	Cherrapunji, India	25.25	91.73	1313	27/04/1970
365	17517.8	2.13	IN014020800	Cherrapunji, India	25.25	91.73	1313	26/06/1955
730	31678.9	2.30	IN014020800	Cherrapunji, India	25.25	91.73	1313	25/06/1954

Time scale (d)	Amount (mm)	Corre-sponding K_m	Station ID	Location	Lat. (°)	Lon. (°)	Elev. (m)	Start date
SH, extratropical								
1	687.6	3.33	ASN00040192	Springbrook Forestry, Australia	-28.23	153.28	806	24/01/1947
2	1068.2	7.61	ASN00040197	Mt Tamborine Fern St, Australia	-27.97	153.20	515	26/01/1974
3	1357.3	7.29	ASN00040257	Yandina Post Office, Australia	-26.56	152.96	10	01/02/1893
4	1597.8	8.24	ASN00040257	Yandina Post Office, Australia	-26.56	152.96	10	31/01/1893
5	1694.3	8.36	ASN00040257	Yandina Post Office, Australia	-26.56	152.96	10	30/01/1893
6	1767.2	8.56	ASN00040257	Yandina Post Office, Australia	-26.56	152.96	10	30/01/1893
7	1808	5.54	ASN00040550	Numinbah, Australia	-26.56	152.96	355	01/05/1996
14	2007	7.17	ASN00040257	Yandina Post Office, Australia	-26.56	152.96	10	30/01/1893
21	2578.2	8.72	ASN00040257	Yandina Post Office, Australia	-26.56	152.96	10	27/01/1893
30	2653.7	7.88	ASN00040257	Yandina Post Office, Australia	-26.56	152.96	10	20/01/1893
60	3576.6	5.44	BR00G4-0010	Cananeia, Brazil	-25.02	-47.93	5	10/09/1935
90	3925.8	4.84	BR00G4-0010	Cananeia, Brazil	-25.02	-47.93	5	02/09/1935
120	4307.5	4.74	BR00G4-0010	Cananeia, Brazil	-25.02	-47.93	5	16/07/1935
183	5118.9	2.91	ASN00040192	Springbrook Forestry, Australia	-28.23	153.28	806	05/12/1973
365	7708.4	4.51	BR00G4-0010	Cananeia, Brazil	-25.02	-47.93	5	25/11/1934
730	14494	5.01	BR00G4-0010	Cananeia, Brazil	-25.02	-47.93	5	25/11/1933

Graphical depiction of the record values of both Table 1 and Table 3 is given in Fig. 5. To the empirical data of Table 1 the following equations, characteristic of rainfall maxima (see below) are fitted:

$$i = \frac{a}{(1 + d/\theta)^{\eta}}, \qquad h = \frac{ad}{(1 + d/\theta)^{\eta}} \tag{1}$$

where i is the temporally averaged rainfall intensity over time scale d and h is the corresponding rainfall depth, whereas the parameter values are a = 1615 mm/h, θ = 0.07 h and η = 0.52. It is noted that the equation is not actually an envelope of record rainfall data but it is fitted by least squares to them. For d > 1 h, the latter equation can be simplified as $h \approx 405\ d^{0.48}$ (h in mm, d in h), which indicates that the record rainfall depth is roughly proportional to the square root of time scale. For time scales > 1 d and for the GHCN data set, Fig. 6 depicts also information related to the geographical zone, NH or SH and tropical or extratropical.

Interestingly, no substantial differences appear among these zones, except that the SH extratropical zone seems to give lower values than those of the other three zones. The numbers of stations and station-years (see Table 2) certainly have an effect on the results. In either of the tropical zones the number of stations is about one seventh of those in the NH extratropical zone and hence the extreme events per se are rather underrepresented in the former, compared to the latter. Likewise, the number of stations in the SH extratropical zone is smaller than in NH extratropical zone, which justifies somewhat lower record rainfall values. On the other hand, the extreme rainfall regime in the extratropical zone of the SH is clearly lower than in all other zones, including the two tropical areas, even though the number of station years in each of the two is five times smaller.

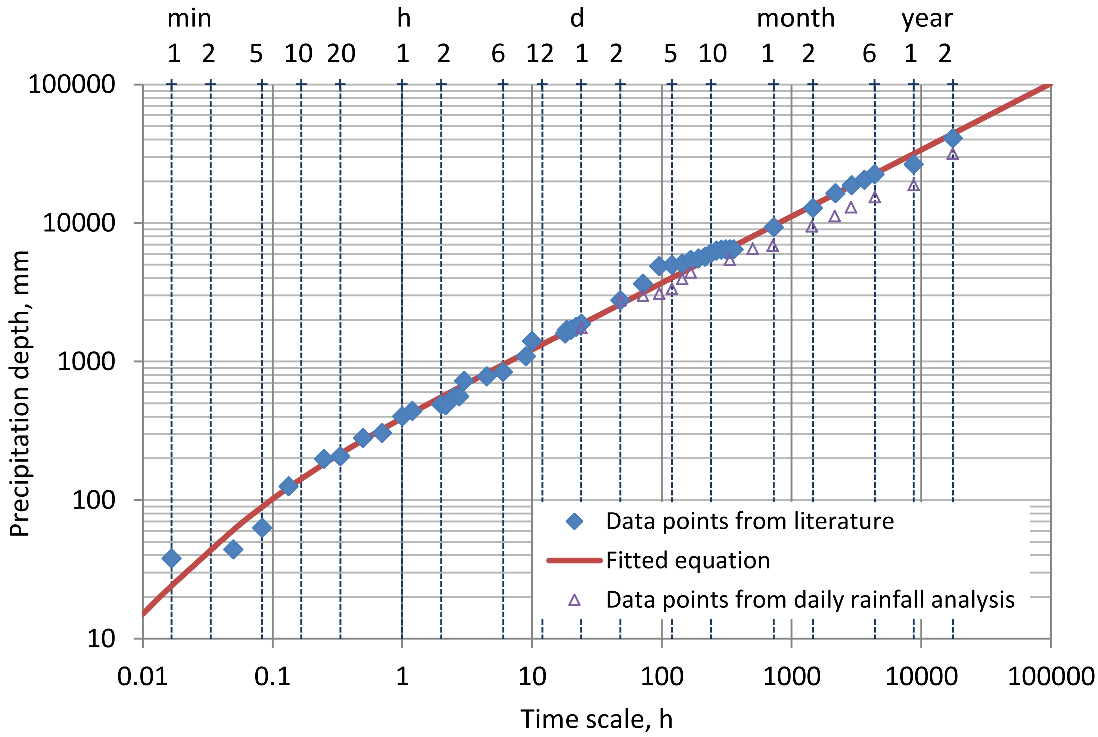

Fig. 5 Depiction of the record rainfall values over the globe from both Table 1 (data points from literature, to which equation (1) is fitted) and Table 3 (data points from daily rainfall analysis).

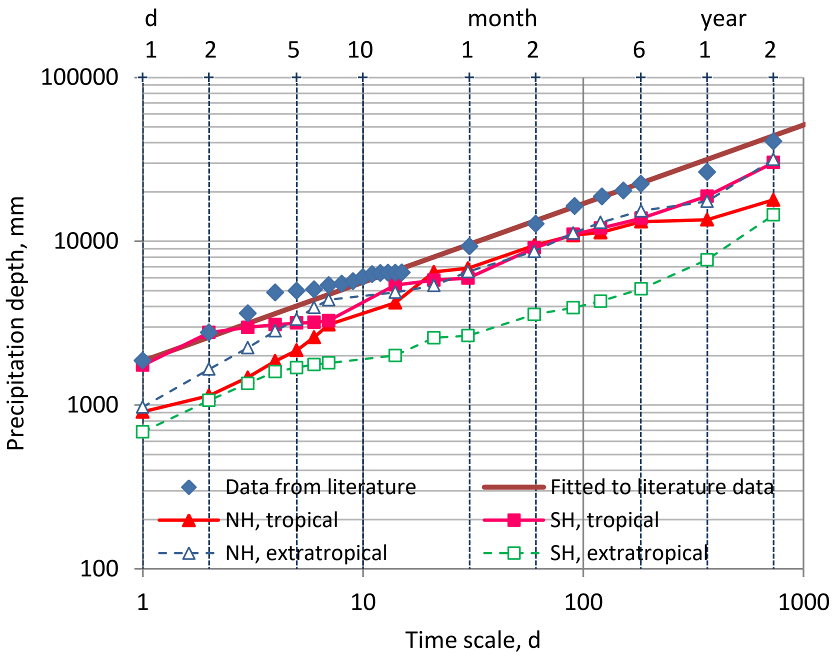

Fig. 6 As Fig. 5 with separation of the GHCN data of Table 3 into zones.

The geographical and temporal distribution of the record rainfall events listed in Table 1 and Table 3 are depicted in Fig. 7 and Fig. 8, respectively. Interestingly, while Fig. 8 shows that almost all records correspond to tropical areas (or very near), Fig. 7, which reflects broader information and also extends to short time scales, contains events in higher latitudes, up to 47.87°N (event F; Bavaria, Germany). This indicates that, even though high rainfall rates are more frequent in tropical areas, extratropical ones are not safe too as some of the records in rainfall have been observed out of the tropics.

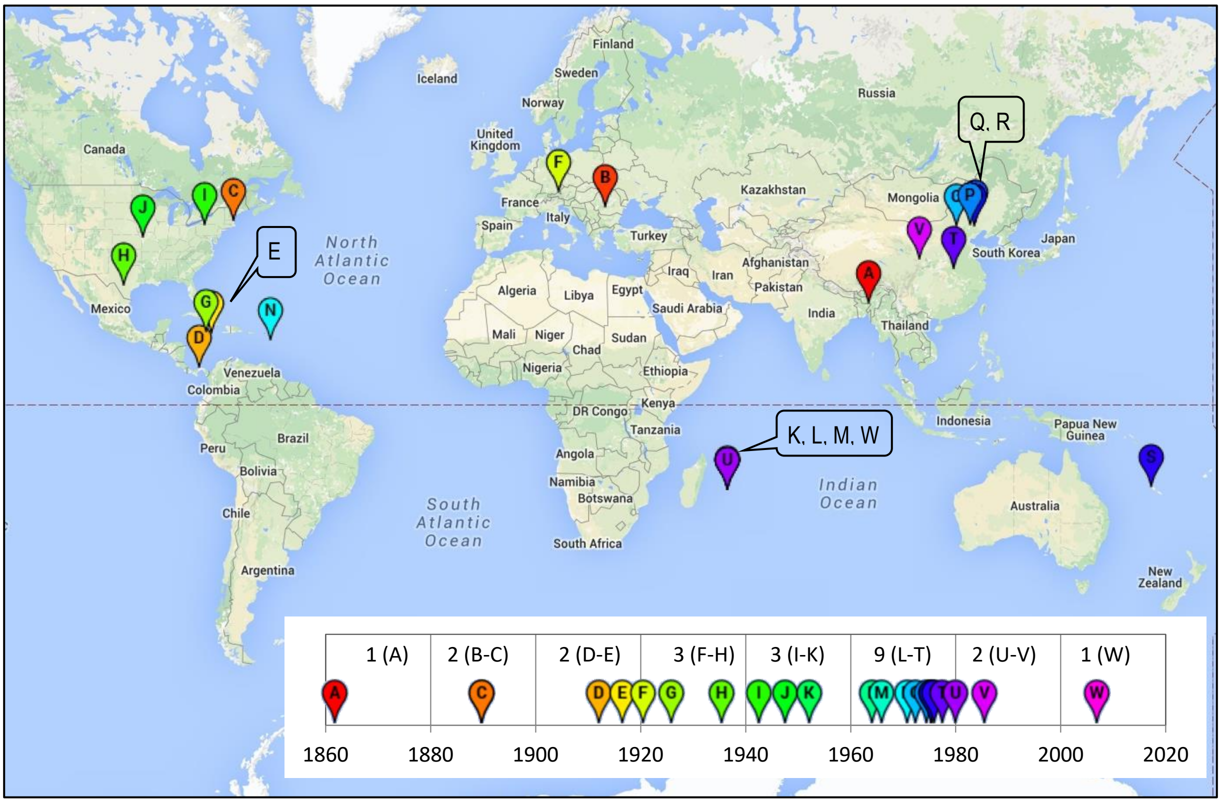

Fig. 7 Geographical and temporal distribution of the record rainfall events listed in Table 1.

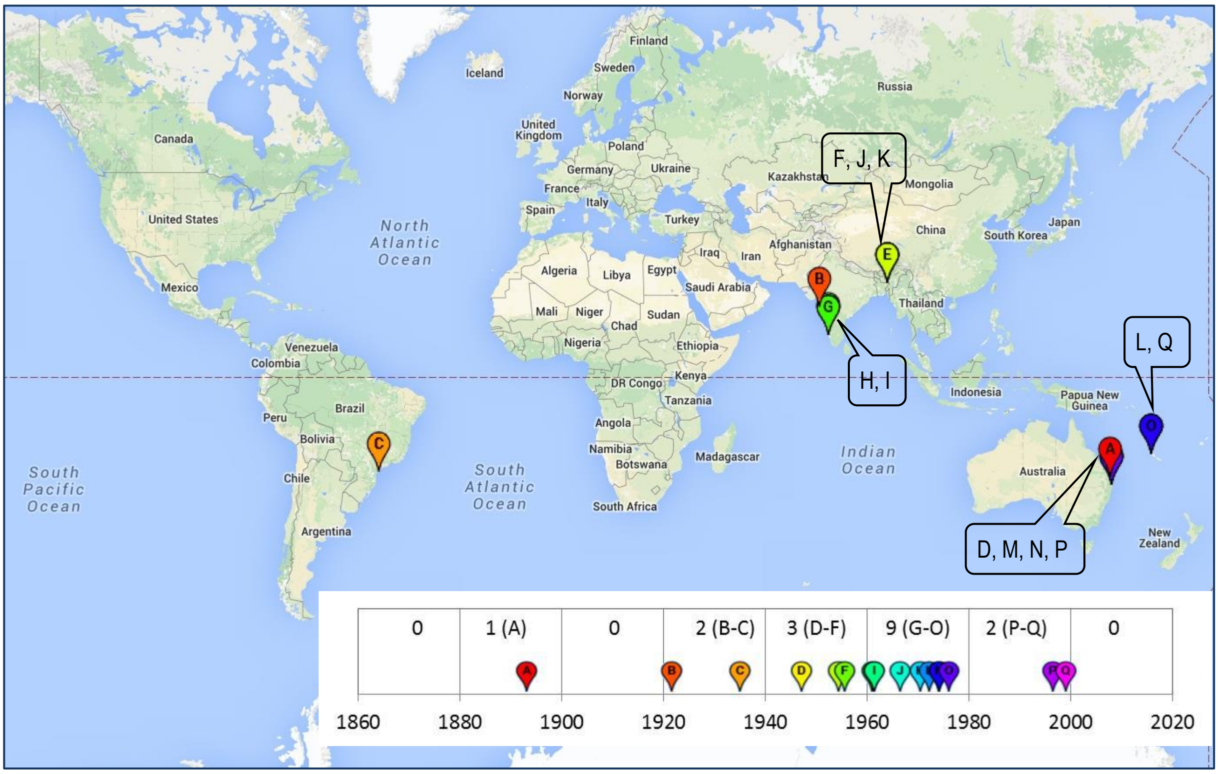

Fig. 8 Geographical and temporal distribution of the record rainfall events listed in Table 3.

Time stamps of the record rainfall events are also shown in both Fig. 7 and Fig. 8. It is most interesting that the 20-year period 1960-80 contains the largest, among all 20-year periods, number of record rainfall events for both data sets (9 events in each one), followed by the period 1940-60 (3 events in each data set), while in the most recent years such extreme events have become less frequent. The grouping of extreme events in specific time periods (mostly in 1960-80) is consistent with the Hurst-Kolmogorov stochastic dynamics, as described above.

Apparently the values contained in Table 1 or Table 3 do not represent any physical upper bound of precipitation rate. They just represent what was observed as record rainfall. Certainly, higher rates have occurred in places where no raingages exist or in longer periods of history. Furthermore, values registered in older publications as record values no longer represent record values. Obsolete record values from older publications, which now have been exceeded, are shown in Table 4 and graphically in Fig. 9. Logically, we can be confident that the records presented here will surely be broken in the future.

Table 4 Older world record point precipitation data that are not records any more.

Time scale		Amount (mm)	Location	Start date	Ref.*
2	d	2467	Aurere, La Réunion	07/04/1958	1
3	d	3130	Aurere, La Réunion	06/04/1958	1
4	d	3721	Cherrapunji, India	12/09/1974	1
5	d	4301	Commerson, La Réunion	23/01/1980	1
6	d	4653	Commerson, La Réunion	22/01/1980	1
7	d	5003	Commerson, La Réunion	21/01/1980	1
8	d	5286	Commerson, La Réunion	20/01/1980	1
12	h	1340	Belouve, La Réunion	28/02/1964	2
3	d	3240	Cilaos, La Réunion	15/03/1952	2
4	d	3504	Cilaos, La Réunion	15/03/1952	2
5	d	3854	Cilaos, La Réunion	15/03/1952	2
6	d	4055	Cilaos, La Réunion	15/03/1952	2
7	d	4110	Cilaos, La Réunion	15/03/1952	2
8	d	4130	Cilaos, La Réunion	15/03/1952	2
2	d	2259	Hsin-Liao, Taiwan	17/10/1967	3
3	d	2759	Cherrapunji, Meghalaya, India	12/09/1974	3
4	d	3721	Cherrapunji, Meghalaya, India	12/09/1974	3
8	d	3847	Bellenden Ker, Queensland	01/01/1979	3

*Ref.: 1. World Meteorological Organization, 1994; 2. Linsley et al. (1975); 3. Smith (1993)

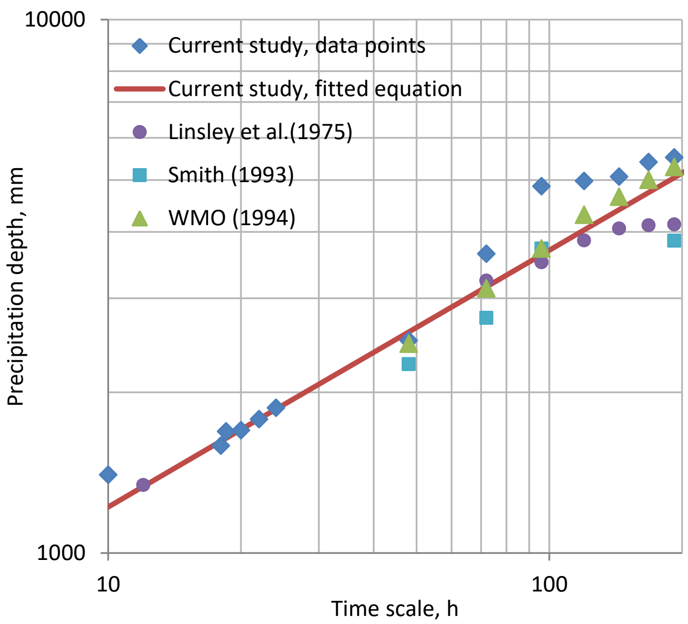

Fig. 9 Graphical depiction of the broken rainfall records of Table 4.

3. Approaches in estimating extreme rainfall

As already mentioned, only rainfall observations can provide a sound basis for quantification of extreme rainfall. Such quantification needs also to be founded on some theoretical considerations. Two different lines of thought have been quite common in hydrology and in engineering practice, one deterministic and one probabilistic.

Deterministic thinking in science is strong enough and has been dominant even in extreme rainfall, despite strong resistance of the rainfall process to comply with deterministic descriptions. Despite spectacular failures in adequate modelling of rainfall based on first principles (see section 1), attempts keep on. Perhaps the oldest of the attempts, yet very popular even today, aims to determine physical upper bounds to precipitation that could be used to design risk-free constructions or practices. The resulting concept of probable maximum precipitation (PMP), that is, an upper bound of precipitation that is physically feasible (World Meteorological Organization, 1986, 2009), is perhaps one of the biggest failures in hydrology but it is still in wide use: A Google Scholar search reveals that about 1000 recent publications that appeared since 2010 include the term "probable maximum precipitation". More than 50 of them contain this term in their title, while some examine the PMP concept in the context of climate change (e.g. Kunkel et al., 2013). In addition, the method is still quite popular in engineering studies.

Using elementary logic we can understand that even the terminology is self-contradictory, and thus not scientific. Namely, the word "*probable*" contradicts the existence of a deterministic limit. Note that the "probable maximum" concept began as "maximum possible" and was later renamed in an attempt to salvage the failed concept (Benson, 1973). Furthermore, as the method makes, in fact, inference from data rather than from physical principles, in essence it is not deterministic but statistical.

According to the probabilistic approach, any nonnegative value, including any estimated PMP, has a certain probability of exceedence. This approach is logically consistent, purely probabilistic and relies on local rainfall observations, while theoretical concepts such as the principle of maximum entropy (Koutsoyiannis, 2005a, 2014; Papalexiou and Koutsoyiannis, 2012) and analysis of global rainfall behaviors (Koutsoyiannis, 2004a,b; Papalexiou and Koutsoyiannis, 2013) assist in formulating the probability distribution function.

One typical argument against the use of probabilistic approaches, in favor of PMP, has been stated by Horton (1931; from Klemes, 2000): "*It is, however, important to recognize the nature of the physical processes involved and their limitations in connection with the use of statistical methods. … Rock Creek cannot produce a Mississippi River flood any more than a barnyard fowl can lay an ostrich egg*". However, this argument reveals an incorrect perception of probability and statistics. In a probability theoretic context no logical inconsistency arises, as illustrated by the following example, adapted from Koutsoyiannis and Langousis (2011). Let us assume that the annual peak flood of the Mississippi river and that of a certain small creek are both distributed according to the extreme value type II (EV2) distribution (equation (13)) with scale parameters λ_M and λ_C for the Mississippi and the small creek, respectively, and the same shape parameter κ, which can be assumed of the order of 0.1 (according to the investigation that follows and having in mind the theoretical reasons which equate the shape parameters of rainfall and flood; Koutsoyiannis, 2005a,b, 2007). For small enough probability of exceedence, say $\Phi(x) < 0.1$, the probability of exceedence can be approximated as a power-law function, i.e. $\Phi_M(x) = (\lambda_M / \kappa x)^{1/\kappa}$, $\Phi_C(x) = (\lambda_C / \kappa x)^{1/\kappa}$. Hence, for large enough x, the ratio of the probabilities that both rivers have the same peak flood x is $\Phi_C(x)/ \Phi_M(x) = (\lambda_C / \lambda_M)^{1/\kappa}$. Assuming that, as an order of magnitude, the Mississippi floods are, a million times larger than those of the small creek, the ratio of the scale parameters will be $\lambda_C / \lambda_M = 10^{-6}$ and the ratio of the probabilities that the floods in the two cases are equal $\Phi_C(x)/\Phi_M(x) = 10^{-60}$. That is, according to the probabilistic approach, the return period of the event that the small creek flood matches or exceeds a specified flood of the Mississippi is 10^{60} years. Assuming that the age of the universe is of the order of 10^{10} years, one would wait, on the average, 10^{50} times the age of the universe to see such an event happen—if one foolishly hoped that the creek, the Mississippi and the Earth would exist for such a long time. Evidently, such a low probability could be regarded as synonymous to impossibility, which shows that the probabilistic approach does not regard the floods of Mississippi equivalent to those of a small creek (see also an example about the age of a person by Feller, 1950).

For completeness, in the following sessions both approaches are outlined, even though the deterministic approach is not scientific and ought to be abandoned.

4. The concept of probable maximum precipitation

4.1 Theoretical analysis of the concept

Several methods to determine PMP exist in literature and are described by World Meteorological Organization (1986, 2009). However, all suffer logically, as they are based on the fallacious concept of an upper limit. Thorough examination of each of the specific methods separately will reveal that each one is affected by additional logical inconsistencies. While they all assume the existence of a deterministic upper limit, they determine this limit statistically. This is obvious in the so-called "*statistical approach*" by Hershfield (1961, 1965), who used about 95 000 station-years of annual maximum daily rainfall belonging to 2645 stations (see below), standardized each record and found the maximum over the 95 000 standardized values. Naturally, one of the 95 000 standardized values would be the greatest of all others, but this is not a deterministic limit to call PMP (Koutsoyiannis, 1999). If we examine additional measurements we will find even higher values (see below). Thus the logical problem here is the incorrect interpretation that an observed maximum in precipitation is a physical upper limit.

The situation is perhaps even worse with the so-called moisture maximization approach of PMP estimation (World Meteorological Organization, 1986, 2009), which seemingly is more physically (hydrometeorologically) based than the statistical approach of Hershfield. In fact, however, it suffers twice by the incorrect interpretation that an observed maximum is a physical upper limit, as will be detailed below.

Rational thinking and fundamental philosophical and scientific principles can help identify and dispel such fallacies. In particular, the Aristotelian notions of *potentia* (potentiality; Greek 'δύναμις') and of potential infinity (Greek 'ἄπειρον'; Aristotle, Physics, 3.7, 206b16) that "*exists in no other way, but ... potentially or by reduction*" (and is different from mathematical complete infinity) would help us to avoid the PMP concept. In fact, this does not need a great deal of philosophical penetration. The same thing is more practically expressed as "*conceptually, we can always imagine that a few more molecules of water could fall beyond any specified limit*" (Dingman, 1994). Yet the linkage to the Aristotelian notions of *potentia* and potential infinity may make us more sensitive in seeing the logical inconsistencies (see also Koutsoyiannis, 2007).

According to Popper (1982) the extension of the Aristotelian idea of *potentia* in modern terms is the notion of probability. Indeed, probability provides a different way to perceive the intense rainfall and flood, and to assign to each value a certain probability of exceedence (see next session) avoiding the delusion of an upper bound of precipitation and the fooling of decision makers that they can build risk-free constructions. In this respect, the criticism of the PMP and the probable maximum flood (PMF) involves logical, technical, philosophical and ethical issues (Benson, 1973).

4.2 Estimation based on mere rainfall data

The so-called "*statistical approach*" to PMP is based on the work of Hershfield (1961, 1965) and its application requires only the mean and standard deviation of a rainfall record. No other meteorological data are needed. Mathematically, it is expressed by the simple equation

$$h_\mathrm{m} = \overline{h}^* + K_\mathrm{m} s^* \tag{2}$$

where h_m is the rainfall depth that corresponds to PMP, $\overline{h}^*$ and s^* are the mean and standard deviation estimated from a rainfall record of a specified time scale d as detailed below, and K_m is a "frequency factor".

To evaluate this factor, Hershfield (1961) initially analyzed a total of 95 000 station-years of annual maximum rainfall belonging to 2645 stations, of which about 90% were in the USA, and found that the maximum observed value of K_m was 15. Then, he concluded that an estimate of the PMP amount can be determined by setting K_m = 15 in (2) and substituting h_m for the PMP value. Subsequently, Hershfield (1965), proposed that K_m varies with the time scale d and the mean $\overline{h}^*$. More specifically, he found that the value of K_m = 15 is too high for areas with heavy rainfall and too low for arid areas, whereas it is too high for rain durations shorter than 24 hours. Therefore, he constructed an empirical nomograph indicating that K_m varies between 5 and 20 depending on the time scale d and the mean $\overline{h}^*$.

This nomograph along with equation (2) constitute the basis of the statistical method for estimating PMP, which was standardized by World Meteorological Organization (1986, 2009). Koutsoyiannis and Xanthopoulos (1999, p. 161) gave the following analytical approximation of the nomograph:

$$K_m = 20 - 8.6 \ln\left(1 + \frac{\overline{h}^*}{130\ \mathrm{mm}}\right)\left(\frac{24\ \mathrm{h}}{d}\right)^{0.4} \tag{3}$$

Hershfield (1961) proposed also some adjustments of the mean and standard deviation, to make the estimation of K_m more unbiased by reducing the effect of the sample size and maximum observed event. These were adopted by the World Meteorological Organization (1986, pp. 97–107) as a standard procedure for applying the method and have been expressed again in the form of nomographs. Koutsoyiannis (2000) has converted the nomographs into equations. Specifically, if n is the sample size, $\overline{h}$ and s are the sample mean and standard deviation, and $\overline{h'}$ and s' are the mean and standard deviation of the sample after removing the largest value contained in it, then the adjusted quantities are:

$$\overline{h}^* = \overline{h}\varphi_1(n)\,\psi_1(\overline{h}, \overline{h'}, n) \tag{4}$$

$$s^* = s\varphi_2(n)\,\psi_2(s, s', n) \tag{5}$$

where

$$\varphi_1(n) := \frac{1}{1 - 0.96\exp(-n^{0.47})} \tag{6}$$

$$\varphi_2(n) := \frac{1}{1 - 4.2\exp(-n^{0.47})} \tag{7}$$

$$\psi_1(\overline{h}, \overline{h'}, n) := 1 + 1.04\left(\frac{\overline{h'}}{\overline{h}} - 1\right) + \frac{0.42}{n^{0.75}} \tag{8}$$

$$\psi_2(s, s', n) := 1 + \frac{1.37}{n^{0.06}}\left(\frac{s'}{s} - 1\right) + \frac{0.65}{n^{0.5}} \tag{9}$$

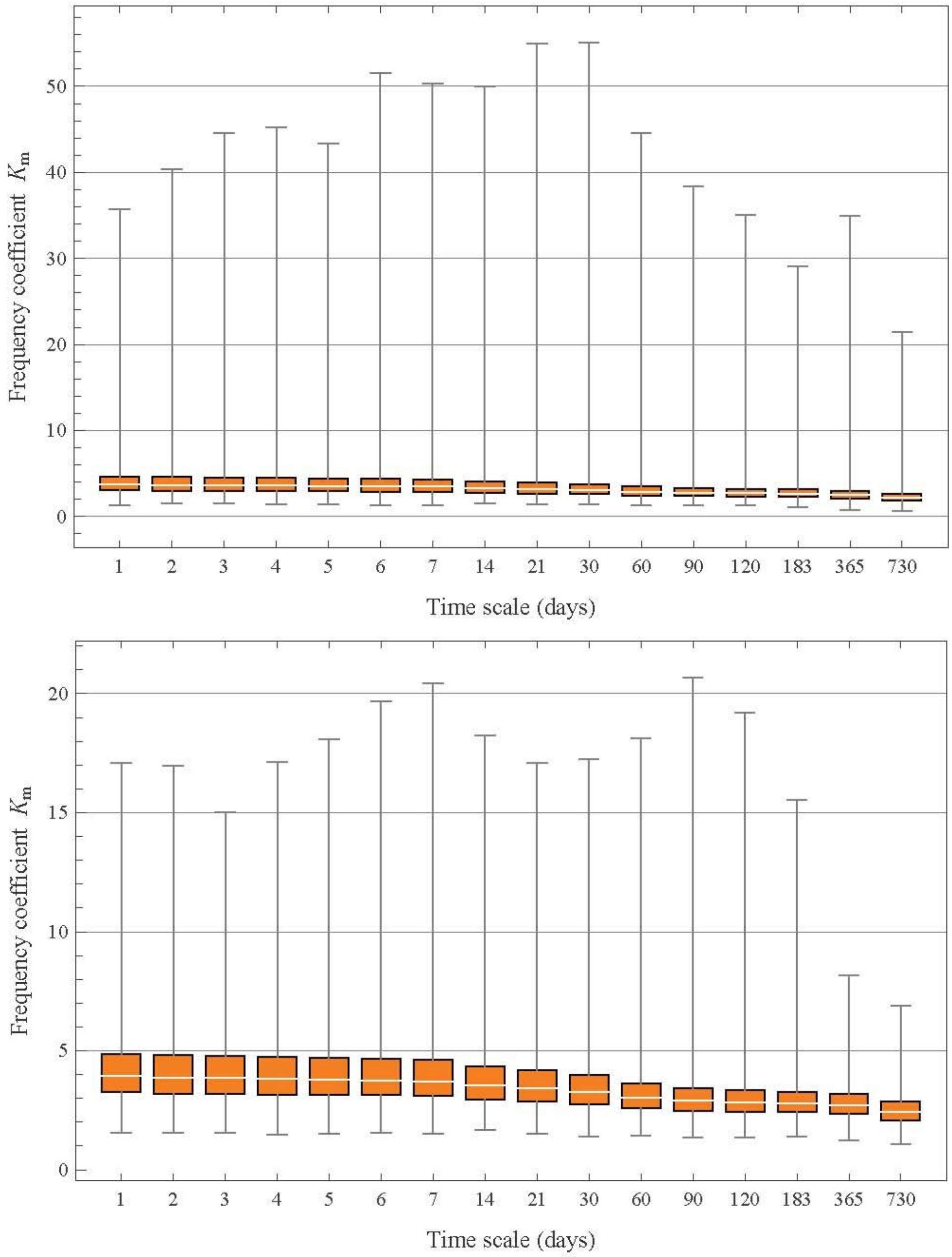

Fig. 10 Box plots of maximum standardized rainfall depths K_m of GHCN stations per time scale: (upper) global; (lower) SH extratropical. The central mark inside each box is the median, the box edges are the 25th and 75th percentiles and the ends of the whiskers represent the observed minimum and maximum values.

Koutsoyiannis (1999) has shown that the huge (for its time) amount of rainfall data used by Hershfield does not suggest the existence of a deterministic upper limit of precipitation.

Rather the maximum value of K_m given by Hersfhield (15 or 20) is just the maximum value of a sample of standardized maxima and would be greater if a bigger sample were available. Subsequently, Koutsoyiannis reformulated Hershfield's results in a purely probabilistic basis (see below), without considering an upper limit for precipitation.

Here we have repeated Hershfield's exercise for the GHCN database, which is by an order of magnitude bigger than Hershfield's (namely ~7 times bigger in terms of stations and ~15 times bigger in terms of station years). The variation of K_{m}'s of different GHCN stations globally and for the SH is shown in the form of box plots per time scale in Fig. 10. The maximum K_{m} for each zone and time scale is shown in Fig. 11. As expected, K_{m} can be much higher than the values 15 or 20 reported by Hershfield. For the NH, they exceed 35 for the daily scale and 55 for time scale close to monthly. Hence, there is no scientific meaning in using Hershfield's method any more.

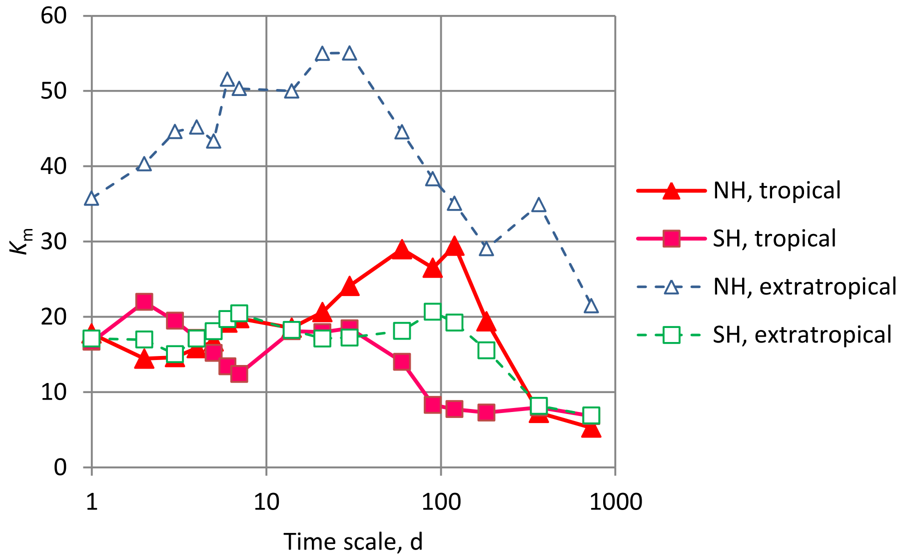

Fig. 11 Calculated values of maximum K_{m} per zone and per time scale for all GHCN stations investigated here using original Hershfield's method.

4.3 Estimation based on additional hydrometeorological data

As already mentioned, the other PMP estimation methods suggested by World Meteorological Organization (1986, 2009) are more problematic than Hershfield's statistical approach, even though they seem more physically (hydrometeorologically) based. As the most representative, we discuss here the so-called moisture maximization approach, which is also the most popular. The method is based on the simple formula

$$h_{\mathrm{m}} = \frac{W_{\mathrm{m}}}{W} h \qquad (10)$$

where h_{m} is the maximized rainfall depth, h is the observed precipitation, W is the precipitable water in the atmosphere during the day of rain, estimated by the corresponding dew point T_{d}, and W_{m} is the maximized precipitable water. The latter is estimated from the maximum dew point for the corresponding month, which is either the maximum recorded value from a sample of at least 50 years length, or the value corresponding to a 100-year return period, for samples smaller than 50 years (World Meteorological Organization, 1986).

As shown by Papalexiou and Koutsoyiannis (2006), the method suffers twice by the incorrect interpretation that an observed maximum is a physical upper limit. It uses a record of observed dew point temperatures to determine an upper limit, which is the maximum observed value. Then it uses this "limit" for the so called "maximization" of an observed sample of storms, and asserts the largest value among them as PMP. Clearly, this is a questionable statistical approach, because (a) it does not assign any probability to the value determined and (b) it is based only on one observed value (known in statistics as the largest order statistic), rather than on the whole sample, and thus it is enormously sensitive to one particular observation of the entire sample (Papalexiou and Koutsoyiannis, 2006; Koutsoyiannis, 2007).

Thus, not only does the determination of PMP use a statistical approach (rather than deterministic physics), but it uses bad statistics. The arbitrary assumptions of the approach extend beyond the confusion of maximum observed quantities with physical limits. For example, the logic of moisture maximization at a particular location is unsupported given that a large storm at this location depends on the convergence of atmospheric moisture from much greater areas. In other words, the PMP concept should be abandoned in its entirety.

5. Probabilistic approach to extreme rainfall

5.1 Basic concepts of extreme value distributions

Having got rid of the concept of an upper limit to precipitation, the obvious alternative is to adopt a probabilistic approach. The real scientific problems are then the determination of the marginal distribution function of extreme rainfall (e.g. maximum for a specified period such as a year) and the dependence of maxima in time. With respect to the latter problem, a popular assumption is that extreme rainfall events can be assumed independent in time. It is noted, though, that, from a global perspective, several indications disfavor this assumption, including the above observation (section 2) about the grouping of record rainfall events in time. Also the lately growing body of publications examining "nonstationarity" in rainfall extremes may au fond reflect time dependence of extremes, as time dependence is quite often misinterpreted as nonstationarity (Koutsoyiannis and Montanari, 2015). However, in what follows we deal only with the first problem, the marginal distribution, for which a lot of evidence has been gathered in the last decade and which is more important from a practical engineering point of view.

The major question in this regard is how the rainfall intensity grows as the probability of exceedence decreases. This question is again related to the notion of infinity. Clearly, as the probability of exceedence tends to zero, the intensity tends to infinity. There exists a mathematically proven lower limit to the rate of this growth, which is represented by an exponential decay of the probability of exceedence with intensity. The alternative is a power-law decay. The two options may lead to substantial differences in design quantities for high return periods.

Accordingly, the distribution tails are important to know in engineering design. However, the study of the tails is difficult and uncertain because the tails refer to infrequent events that require very long records to appear. Traditionally, rainfall records are analyzed in two ways. The most frequent is to choose the highest of all recorded precipitation intensities (for a given averaging time scale) at each year and form a statistical sample (commonly referred to as "block maxima") with size equal to the number of years of the record. The other is to form a sample (sometimes referred to as "peaks-over-threshold"—POT) with all recorded intensities over a certain threshold irrespectively of the year they occurred. Usually the threshold is chosen high enough, so that the sample size is again equal to the number of years of the record. This however is not necessary: it can well be set equal to zero, so that all recorded intensities are included in the sample. However, the threshold simplifies the study and helps focus the attention on the distribution tail.

If $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n$ are random variables (hence denoted with underlined symbols, following the so-called Dutch convention) representing the recorded average intensities within a year at nonoverlapping time periods equal to a chosen time scale d, then the maximum among them $\underline{y} := \max(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n)$ has a distribution function $H_n(y)$ fully dependent on the joint distribution function of $\underline{x}_i$. If we assume that $\underline{x}_i$ are independent and identically distributed (IID) with common distribution function $F(x)$, then $H_n(x) = (F(x))^n$. If n is not constant but rather can be regarded as a realization of a random variable (corresponding to the fact that the number of rainfall events is not the same in each year) with Poisson distribution with mean ν, then the distribution function becomes (e.g. Todorovic and Zelenhasic, 1970; Rossi et al., 1984),

$$H(x) = \exp\left(-\nu\left(1 - F(x)\right)\right) \tag{11}$$

where $H(y)$ (without a subscript n) denotes the distribution function of $\underline{y}$ for randomly varying, Poisson distributed $\underline{n}$. In particular, if the threshold has been chosen with the above rule (to make the sample size equal to the number of years of the record) then obviously $\nu = 1$. Equation (11) expresses in a satisfactory approximation the relationship between the above two methodologies and the respective distributions F and H. The two options discussed above are then represented as follows:

1. Exponential tail

$$F(x) = 1 - \exp(-x/\lambda + \psi)\,, \qquad H(x) = \exp(-\exp(-x/\lambda + \psi))\,, \qquad x \geq \lambda\psi \tag{12}$$

where $\lambda > 0$ and $\psi > 0$ are parameters, so that $\lambda\psi$ represents the specified threshold. Here F is the exponential distribution and H is the Gumbel distribution, also known as extreme value type I (EV1) distribution.

2. Power tail

$$F(x) = 1 - \left(1 + \kappa\left(\frac{x}{\lambda} - \psi\right)\right)^{-1/\kappa}, \qquad H(x) = \exp\left(-\left(1 + \kappa\left(\frac{x}{\lambda} - \psi\right)\right)^{-1/\kappa}\right), \qquad x \geq \lambda\left(\psi - \frac{1}{\kappa}\right) \tag{13}$$

where $\lambda > 0$, $\psi > 0$ and $\kappa > 0$ are parameters, with the shape parameter κ being the most characteristic as it determines the heaviness of the distribution tail. Here F is the generalized Pareto distribution and H, for the case $\kappa > 0$ considered here, is the extreme value type II (EV2) distribution (or Fréchet distribution).

The case $\kappa < 0$ in equation (13) is mathematically possible and is called the extreme value type III (EV3) distribution (or reversed Weibull law). However, this is inappropriate for rainfall as it puts an upper bound ($\lambda\psi - \lambda/\kappa$) for x, which is inconsistent. The case $\kappa = 0$, corresponds precisely to the exponential tail (exponential and Gumbel distributions, equation (12)). All three cases of $H(x)$ are collectively referred to as the generalized extreme value (GEV) distribution.

It is further noted that, while here the correspondence between $F(x)$ and $H(x)$ has been founded on equation (11), assuming a randomly varying number of events $\underline{n}$ with mean $\nu = 1$, the same correspondence is also found if we assume a constant n and examine the asymptotic behavior of $H_n(x) = (F(x))^n$ as $n \to \infty$. Indeed, for a specified value of $H_n(x)$ and large n, $F(x)$ should be large (or $F(x) \to 1$ as $n \to \infty$); since $\ln(F(x))^n = n \ln(1 - (1 - F(x))) = n(-(1 - F(x)) - (1 - F(x))^2 - \ldots) \approx -n(1 - F(x))$, it turns out that for large $F(x)$, $H_n(x) \approx \exp\left(-n\left(1 - F(x)\right)\right)$, which is similar to $H(x)$ in (11) (see additional information in Koutsoyiannis, 2004a).

Further important topics of extreme value theory on rainfall can be found in Salvadori and De Michele, 2001; Koutsoyiannis, 2004a; Papalexiou and Koutsoyiannis, 2013; Marani and Ignaccolo, 2015; and many other studies. While most of the above mathematical arguments have assumed independent random variables, the results can be approximately valid even in case of variables dependent in time. Specifically, Leadbetter (1983) demonstrated that maxima of dependent series follow the same distributional limit laws as those of independent series, provided the series has limited long-term persistence at extreme levels.

5.2 Distribution type: Gumbel, Fréchet or Weibull?

For years, the exponential tail and the Gumbel distribution have been the prevailing models for rainfall extremes, despite the fact that they yield unsafe (too small) design rainfall values for large return periods. Recently, however, their appropriateness for rainfall has been questioned. Koutsoyiannis (2004a, 2005a, 2007) discussed several theoretical reasons that favor the power-law/EV2/Fréchet distribution over the exponential/EV1/Gumbel case. By now, several studies have provided empirical evidence supporting the power-law case. Some of them, based on empirical evidence from daily rainfall records worldwide, are explicitly mentioned below:

1. The data set compiled by Hershfield (1961) with 95 000 station-years, which he used to formulated his PMP method, in the already mentioned study by Koutsoyiannis (1999), was found consistent with the EV2 distribution with shape parameter κ = 0.13, which corresponds to Hershfield's assumption of a fixed K_{m} = 15. Accordingly, if a varying K_{m} is assumed, then Hershfield's equation (3), for a time scale of 24 h, is consistent with a shape parameter varying with $\overline{h}^*$ (in mm) as

$$\kappa = 0.183 - 0.00049\,\overline{h}^* \tag{14}$$

2. Koutsoyiannis (2004b, 2005a) compiled an ensemble of annual maximum daily rainfall series from 169 stations in the Northern Hemisphere (28 from Europe and 141 from the USA) roughly belonging to six major climatic zones, all having lengths from 100 to 154 years, and comprising a total of 18 065 station-years. The analysis provided sufficient support for the general applicability of the EV2 (Fréchet) distribution model worldwide. Furthermore, the ensemble of all samples was analyzed and supported the estimation of a unique shape parameter κ for all stations. The estimated value of κ varied for different methods of estimation and was found κ = 0.09 for the maximum likelihood method, κ = 0.10 for the L-moments method, κ = 0.13 for the method of moments and κ = 0.15 for a weighted least squares method. The latter method, by assuming weights equal to the empirical quantiles, gave higher importance to the high values and, as the resulting value leads to more conservative design (gives higher maximum rainfall depths), the value κ = 0.15 was suggested as the preferred one.
3. Papalexiou and Koutsoyiannis (2013) analyzed the annual maximum daily rainfall of 15 137 records from the GHCN daily database, with lengths varying from 40 to 163 years. Using the L-moments method, they fitted to all stations the GEV distribution, which comprises the three types of H described above as special cases. The results clearly suggested that the EV3 distribution (a GEV distribution bounded from above, with negative shape parameter) is completely inappropriate for rainfall, while the EV2/Fréchet law (the GEV law with positive shape parameter), prevails over the EV1/Gumbel law. The mean value of the shape parameter κ for all stations was found to be 0.114. However, this value was not found to be representative for all parts of the world.

4. Cavanaugh et al. (2015) analyzed again a subset of the GHCN daily database, selecting over 22 000 high quality stations across the globe, which pass certain quality control and temporal completeness criteria. They utilized an advanced test for differentiating between exponential- and heavy-tailed distributions of precipitation, and their results indicated that the majority of precipitation exceedance probabilities are of Pareto type and, therefore, most precipitation records have power-law distributed tails, not exponential.

Additionally, Veneziano et al. (2009) used multifractal analysis to show that the annual rainfall maximum for time scale d can be approximated by a GEV distribution and that typical values of κ lie in the range 0.09 to 0.15 with the larger values being associated with more arid climates. Similar results were provided by Chaouche (2001) and Chaouche et al. (2002). Chaouche (2001) exploited a data base of 200 rainfall series of various time steps (month, day, hour, minute) from the five continents, each including more than 100 years of data. Using multifractal analyses it was found that (a) an EV2/Pareto type law describes the rainfall amounts for large return periods; (b) the exponent of this law is scale invariant over scales greater than an hour (in fact, this is dictated by theoretical reasons; see below); and (c) this exponent is almost space invariant. Other studies have also expressed skepticism for the appropriateness of the Gumbel distribution for the case of rainfall extremes. Coles et al. (2003) and Coles and Pericchi (2003) concluded that inference based on a Gumbel distribution model fitted to the annual maxima may result in unrealistically high return periods for certain observed events and suggested a number of modifications to standard methods, among which is the replacement of the Gumbel model with the GEV model. Mora et al. (2005) and Bacro and Chaouche (2006) confirmed that rainfall in Marseille (a raingage included in the study by Koutsoyiannis, 2004b) and other raingages in southern France are not in the Gumbel law domain. Sisson et al. (2006) highlighted the fact that standard Gumbel analyses routinely assign near-zero probability to subsequently observed disasters, and that for San Juan, Puerto Rico, standard 100-year predicted rainfall estimates may be routinely underestimated by a factor of two. Schaefer et al. (2006) using the methodology by Hosking and Wallis (1997) for regional precipitation-frequency analysis and spatial mapping for 24-hour and 2-hour durations for the Washington State, USA, found that the distribution of rainfall maxima in this State generally follows the EV2 distribution type.

5.3 Global survey of the distribution shape of maximum rainfall

Once the EV2/Fréchet/Pareto law has been found to be representative for extreme rainfall worldwide, a relevant question is, can the shape parameter κ of this law be assumed constant for the entire globe? This question is very important as the estimation of three parameters of a distribution function is generally problematic if the statistical sample is not very large (Lombardo et al. 2014) and this is particularly the case for the shape parameter of the distribution of extreme rainfall (Papalexiou and Koutsoyiannis, 2013). If the shape parameter can be assumed unique for the entire globe, then the estimation of the remaining two parameters of the three-parameter EV2 law can be supported by the sample.

Samples of daily rainfall offer the most precious basis to study this question, because of the abundance and higher reliability (in comparison to sub-daily rain recorder data) of the daily records. It is noted that if a reliable estimation of the shape parameter is made for a certain time scale (in this case daily), then the same value holds for all other time scales. A simple way to demonstrate this is to recall that a certain κ of the Pareto law $F(x)$ in equation (13) implies that all statistical moments for order > $1/\kappa$ diverge to infinity. Consequently, this divergence is conveyed to all time scales of aggregation.

The above question was implicitly or explicitly considered in three of the already discussed above studies which investigated the global distribution of extreme rainfall. Specifically:

1. Koutsoyiannis (1999), who used Hershfield's (1961) data set, did not study the question explicitly but, in accordance with the original study by Hershfield, assumed that either (a) the distribution function shape is the same for all stations, which allowed merging the entire data set (after standardizing) and estimating a common shape parameter $\kappa = 0.13$, or (b) the distribution function shape depends of the mean of the annual maximum rainfall $\bar{h}^*$, which allowed the extraction of equation (14).
2. Koutsoyiannis (2004b) examined that question and concluded that the variability of the κ estimates for each of the 169 stations can be attributed to statistical sampling effects; this again enabled merging the entire data set (after standardizing) and estimating a common shape parameter, with value varying for different estimation methods from $\kappa = 0.09$ to $\kappa = 0.15$, with the latter value suggested as the preferred one.
3. In Papalexiou and Koutsoyiannis (2013) the vast amount of stations and data allowed a more thorough study of the question. It was found that, when the effect of the record length is corrected, the shape parameter varies in a narrow range yet a single value cannot be representative for the entire globe. As expected, the statistical sampling effect explains a big part of the observed variability of the shape parameter around its mean value $\kappa = 0.114$; however, as seen in Fig. 12, it does not explain the total variability. The authors concluded that the geographical location on the globe may affect the value of the shape parameter. Furthermore, they constructed a map of the geographical distribution of the GEV shape parameter, reproduced here in Fig. 13, which shows that large areas of the world share approximately the same GEV shape parameter, yet different areas of the world exhibit different behavior in extremes (for example, the eastern Brazil and the western USA seem to have lower κ than other parts of the world). As a final remark, the authors suggested not to follow blindly the statistical estimate of κ based on whatever statistical method. In particular, they proposed that in the case where data suggest a GEV distribution with negative shape parameter (distribution bounded from above), this should not be used. Instead it is more reasonable to use in this case a Gumbel or, for additional safety, a GEV distribution with a shape parameter value equal to 0.114.

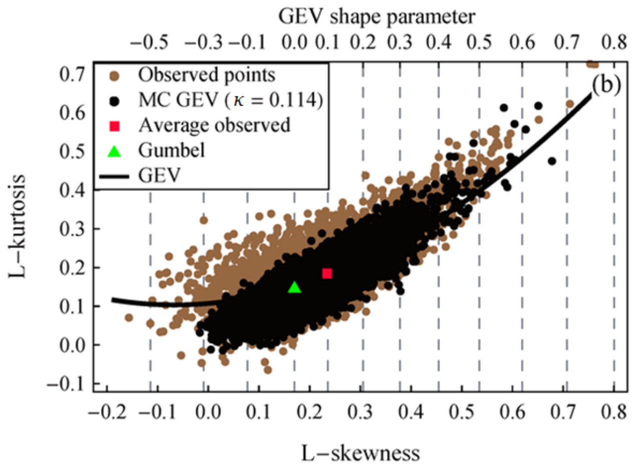

Fig. 12 L-kurtosis vs. L-skewness plots for the observed samples of the 15 137 records from the GHCN database as well as an equal number of Monte Carlo simulated samples assuming GEV distribution with fixed shape parameter $\kappa = 0.114$ (from Papalexiou and Koutsoyiannis, 2013).

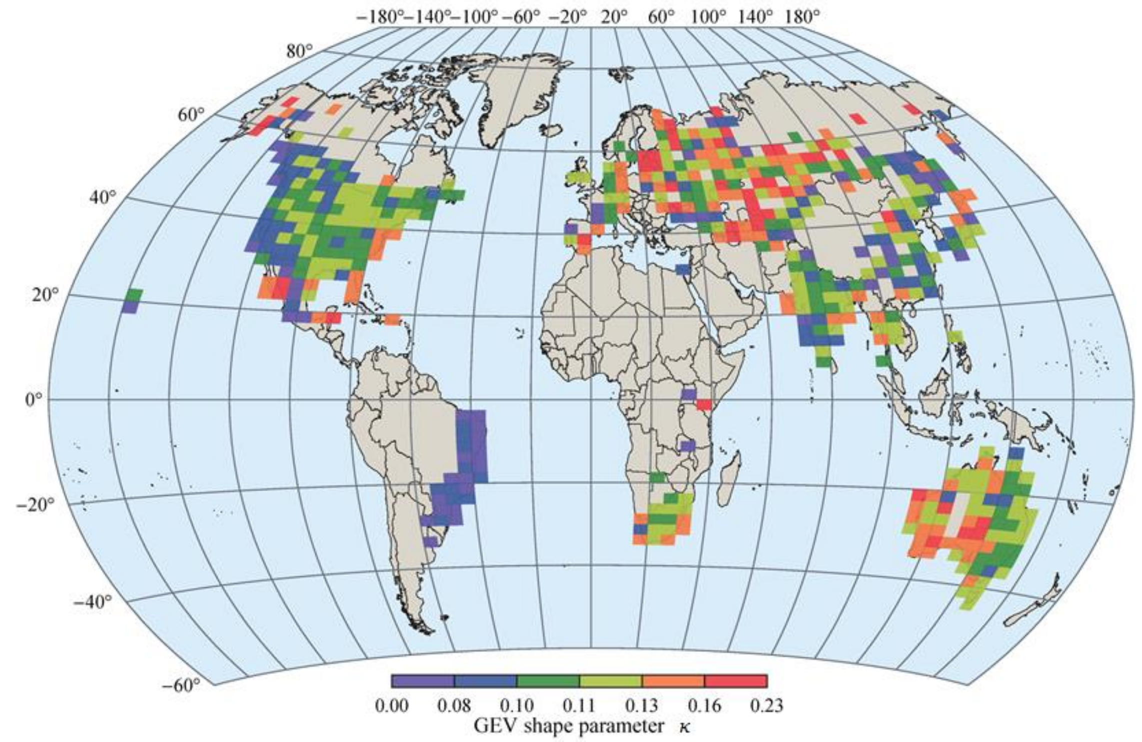

Fig. 13 Geographical distribution of the mean value of the GEV shape parameters in regions of latitude difference $\Delta\varphi$ = 2.5° and longitude difference $\Delta\lambda$ = 5° (from Papalexiou and Koutsoyiannis, 2013).

6. Ombrian (intensity-duration-frequency) curves

6.1 Theoretical basis and mathematical formalism

One of the major tools in hydrologic design is the ombrian relationship, more widely known by the misnomer rainfall intensity-duration-frequency (IDF) curve. An ombrian relationship (from the Greek 'όμβρος', rainfall) is a mathematical relationship estimating the average rainfall intensity i over a given time scale d (sometimes incorrectly referred to as duration) for a given return period T (also commonly referred to as frequency, although frequency is generally understood as reciprocal to period). Several forms of ombrian relationships are found in the literature, most of which have been empirically derived and validated by the long use in hydrologic practice. Attempts to give them a theoretical basis have often used inappropriate assumptions and resulted in oversimplified relationships that are not good for engineering studies.

In fact, as shown in Koutsoyiannis et al. (1998) an ombrian relationship is none other than a family of distribution functions of rainfall intensity for multiple time scales. This is because, the return period is tied to the distribution function, i.e., $T = \Delta/(1 - F(x))$, where Δ is the mean interarrival time of an event that is represented by the variable $\underline{x}$, typically Δ = 1 year. Thus, a distribution function such as one of those described in the previous section, is at the same time an ombrian relationship, once generalized for a multitude of time scales. Koutsoyiannis et al. (1998) showed that the empirical considerations usually involved in the construction of ombrian curves are not at all necessary, and create difficulties and confusion.

However, the direct use in engineering design of a fully consistent multiscale distribution function may be too complicated. Simplifications are possible to provide satisfactory approximations, given that only the distribution tail is of interest and that the range of scales of interest in engineering studies is relatively narrow. Such simplifications, which were tested recently and were found to be reasonable (Papalexiou and Koutsoyiannis, 2009), are:

1. the separability assumption, according to which the influences of return period and time scale are separable (Koutsoyiannis et al., 1998), i.e.,

$$i(d, T) = \frac{a(T)}{b(d)} \tag{15}$$

   where $a(T)$ and $b(d)$ are mathematical expressions to be determined;
2. the adoption of the Pareto distribution for the rainfall intensity over some threshold at any time scale, as discussed in the previous section; this readily provides a simple expression for $a(T)$ of the form $l((T/\Delta)^{\kappa} - c)$, where l and c are constants related to the scale and location parameters of the Pareto distribution, while κ is the shape parameter of that distribution (Koutsoyiannis et al., 1998); and
3. the expression of $b(d)$ by the simple form

$$b(d) = \left(1 + \frac{d}{\theta}\right)^{\eta} \tag{16}$$

   where $\theta > 0$ and $\eta > 0$ are parameters. A justification of this relationship, which is a satisfactory approximation for time scales up to several days, can be found in Koutsoyiannis (2006).

Based on assumptions 1-3, we deduce that the final form of the ombrian relationship is

$$i(d, T) = \lambda' \frac{(T/\Delta)^{\kappa} - \psi'}{(1 + d/\theta)^{\eta}} \tag{17}$$

where $\psi' > 0$, $\lambda' > 0$ and $\kappa > 0$ are parameters. In particular, as discussed earlier, κ is the shape parameter of the EV2/Pareto laws. Equation (17) is dimensionally consistent, provided that θ has units of time (as well as Δ), λ' has units of intensity, and κ and ψ are dimensionless. The numerator of equation (17) differs from a pure power law that has been commonly used in engineering practice, as well as in some multifractal analyses.

Apparently, the parameters ψ', λ' and κ do not depend on the time scale d or the return period, T. These parameters are related to those of the Pareto/EV2 laws of equation (13) by the following relationships:

$$\lambda' = \frac{\lambda}{\kappa}\left(1 + \frac{d}{\theta}\right)^{\eta} \tag{18}$$

$$\psi' = 1 - \kappa\psi \tag{19}$$

This implies that, in order for equation (17) to hold for every time scale d, in addition to the constancy of parameter κ which is guaranteed by theoretical reasons, as explained above, the parameter ψ should also be constant for all time scales. However, the parameter λ in equation (13) should vary with time scale d in a manner that λ' in equation (18) be constant (i.e., $\lambda(d) = \kappa\lambda'(1 + d/\theta)^{-\eta}$).

6.2 Parameter estimation methods

Consistent parameter estimation techniques for ombrian relationships have been discussed in Koutsoyiannis et al. (1998). Parameter estimation does not differ from a typical parameter estimation problem except that samples from several time scales d should be handled simultaneously, in a manner that parameters κ and ψ be constant for all time scales, while λ vary with time scale in such a manner that λ' in equation (18) be constant. Some relevant notes, in terms of data sets that should be used, follow:

1. The parameter θ, which is typically smaller than 1 h, needs sub-hourly data to be estimated. These can be provided by observations from autographic rain recorders with high temporal resolution, or from digital sensors with sub-hourly time step. Without such data the estimated θ tends to approach zero.
2. For the estimation of the parameter η, data for hourly or multi-hour time step can also be quite useful.
3. The parameters of the numerator of equation (17) are better deduced from daily raingage data rather than from autographic rain recorder data, because the latter are more susceptible to measurement errors.
4. In particular the parameter κ of the numerator (which is the shape parameter of the Pareto/EV2 distributions) should be based on multi-station data of the area, or be assumed independently of data, according to the previous subsection.
5. Finally, for the parameters ψ' and λ', daily raingage data of an adequate length (of several decades) usually suffice for a reliable estimation.

If one source of data is used and is deemed reliable for all time scales, then the *one-step least squares method* of Koutsoyiannis et al. (1998) can be readily applied to estimate all parameters of the ombrian relationship simultaneously. Otherwise, if different sources of data are used for different time scales, then a two-step procedure, also described in Koutsoyiannis et al. (1998) is advisable. First the parameters of the denominator of (17) are estimated so that, the quantities $i(d, T)(1 + d/\theta)^{\eta}$ have the same distribution for all time scales d; to this aim an optimization technique in terms of a Kruskal-Wallis statistic involving data of all time scales has been proposed.

6.3 Areal reduction of point ombrian curves

The statistical analysis of rainfall extremes and the construction of ombrian curves refer to a point (i.e., the rainfall station). On the other hand, the transformation of rainfall to runoff occurs at the catchment scale and thus the rainfall intensity should refer to the catchment area. This should require making statistical analysis for the areally averaged rainfall intensity. This however is usually too difficult or impossible, because of the sparse network of raingages as well as synchronization problems among the recordings of different devices. Therefore, a common method for a transformation of point estimates, to account for the spatiotemporal variability of rainfall across the river basin, suggests applying a reduction coefficient, called the areal reduction factor (ARF).

The ARF is defined to be the ratio of the areally averaged precipitation depth over a certain area A for a specified return period T and time scale d to the precipitation depth over any point of the area (assumed to be climatically homogeneous) for the same return period and time scale. Accordingly, to find the ARF we need to determine the distribution functions of both areal and point rainfall and divide the two for several return periods and time scales. A prerequisite for this is to form statistical samples of areal rainfall with sufficient length and for various time scales. Another prerequisite for the definition to apply is the climatic homogeneity of the entire area, so that the same ombrian relationship applies to any point at the given area.

Some studies miss the above definition and determine the ARF empirically, e.g. by averaging precipitation per event and considering the ratio of maximum point precipitation (also known as the center point precipitation) to the areal precipitation; this does not make much sense. In fact, empirical procedures like the latter imply different empirical definitions of ARF. A comprehensive review of empirical procedures and alternative definitions can be found in Svensson and Jones (2010). Despite theoretical inconsistencies, results from empirical studies of ARF have certainly some usefulness. Recent studies which adopt the consistent definition have been made by Lombardo et al. (2006) and Overeem et al. (2010). Both of these studies use radar data to estimate ARF, which certainly provide a great potential for studying the spatial variability of extreme precipitation due to the improved spatial coverage, resulting in good indications of the spatial patterns of rainfall. Major improvements in ARF estimation are anticipated in the near future, as radar data of rainfall will become more reliable and will accumulate in time providing samples with lengths adequate enough to enable reliable investigation of the probability distribution of areal rainfall. It is noted though that the poorer quality of radar data, compared to raingage data, is also expected to affect ARF estimation. Indeed, Allen and DeGaetano (2005) found that radar-based ARF decays at a faster rate (with increasing area) than gage-based ARF.

Current literature typically gives ARF as a function of A and d. Comprehensive investigations were carried out by Natural Environment Research Council (1975) which provided tabulated values of ARF for a wide range of areas (1 to 30 000 km^2) and time scales (1 min to 25 days). Koutsoyiannis and Xanthopoulos (1999, p. 154) fitted the following empirical expression to those data:

$$\varphi = \max\left(0.25, 1 - \frac{0.048A^{0.36-0.01\ln A}}{d^{0.35}}\right) \qquad (20)$$

where A is given in km^2 and d in h. The same relationship was compared with a nomograph constructed for the western USA by the U.S. Weather Bureau (1960; see also World Meteorological Organization, 1986, p. 103), where the differences are visible but not very substantial; this supports applicability of equation (20) in other parts of the world.

7. Summary and conclusions

While satellite rainfall data provide means to study the rainfall climatology, only ground rainfall data from raingages allow reliable quantification of rainfall extremes. The study of the global extremes, i.e., the record rainfalls throughout the globe, provides some useful information on the behavior of rainfall worldwide. While most of these record events have been registered at tropical areas, there are record events that have occurred in extratropical areas and exceed, for certain time scales, those that occurred in tropical areas. Interestingly, those records indicate a tendency for grouping in time, with highest occurrence frequency in the period 1960-80. The record values for different time scales allow the fitting of a curve which indicates that the record rainfall depth increases approximately proportionally to the square root of the time scale. According to this curve, the world record value for the hourly time scale is about 400 mm and that for the daily scale is about 1800 mm.

Clearly, however, these record values do not suggest an upper limit of rainfall and, sooner or later, are destined to be exceeded, as past record values have already been exceeded. In addition, the very concept of the probable maximum precipitation, which assumes a physical upper limit to precipitation at a site, is demonstrated to be fallacious. The only scientific approach to quantify extreme rainfall is provided by the probability theory. Theoretical arguments and general empirical evidence from many rainfall records worldwide suggest power-law distribution tail of extreme rainfall and favor the EV2 distribution of maxima. The shape parameter of the EV2 distribution appears to vary in a narrow range worldwide. This facilitates fitting of the EV2 distribution and allows its easy implementation in typical engineering tasks such as estimation and prediction of design parameters, including the construction of theoretically consistent ombrian (IDF) curves, which constitute a very important tool for hydrological design and flood severity assessment.

Among the outstanding problems related to extreme rainfall from a global perspective, the most important are related to the mapping of the variability of the probability distribution of extreme rainfall over the globe. The construction of maps with parameters of ombrian curves over the globe, or large parts thereof, using a theoretically consistent methodology, is also of major practical importance and usefulness. The combination of gage, radar and satellite data to produce more complete and reliable ARF relationships is an additional future step. The long-term fluctuations of precipitation and particularly rainfall extremes, in connection with the investigation of dependence in time, is another open problem, whose study can be based on the longest available precipitation records and related proxies. Nonetheless, more important than all above potential future studies and research directions is the consistent logical foundation of all concepts related to extreme rainfall. Indeed, the field suffers from several fallacies, as substantiated above. Without removing these fallacies, scientific progress on the field is questionable.

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