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Stochastic investigation of the Hurst-Kolmogorov behaviour in arts.

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The poster can be downloaded at: http://www.itia.ntua.gr/

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Introduction

Art functions on multiple levels and is important to our world. It is something that is pleasing to the eye, but also something that portrays an important message in the feelings. Although each artist's work contains its own unique message, the whole field of art possesses an underlying message of connection between the artist and the observer.

Since prehistoric art creations, the art was related to natural forms and was inspired by them but it was also related to mathematic rules.

We know that Ancient Egyptians had a mathematic rule of the analogies of human body, Ancient Greeks with Pythagoras connected art with the mathematics but we could say that the view of Aristotle "art takes nature as its model" and "art not only imitates nature, but also completes its deficiencies" guided artists in creation of art withought specific rules of mathematical modeling. [1] Moreover Plato connected his aesthetic theory with the ideas, the soul and the idealistic expression. [2]

Cartesian rationalism, which derived from the French philosopher Rene Descartes, considers that instead of regarding the aesthetic quality as an inherent quality of a physical object, the distinction of mind and nature have paved the way for humans to appreciate the role of their own subjective feelings in determining their aesthetic preferences. Other philosophers, such as Leibniz believed that there is a norm behind every aesthetic feeling which we simply don't know how to measure. [3]

Until modern times art was defined by natural forms but as modern world found a different way of living with machines and new materials in early 1900 such art found a different way of expression far enough of the naturalistic forms. Therefore the artists found the nature "boring": "Without poets, without artists, men would soon weary of nature's monotony". [4]

A great modern sculptor Hans Jean Arp said: "All things and man as well, should be like nature, without measure". Even artists are critics in mathematical thought and modeling in art, in this poster we will try to find a connection between the artworks and nature with mathematical modeling.

The aim is to investigate whether or not some artworks (paintings, music pieces and poems) exhibit a correlation structure similar to the ones emerging in natural process (i.e., an exponential or a power-law decay of the correlation structure).

Uncertainty in scale

• Art is a mix of determinism (e.g., certain rules have to be followed) and stochasticity (e.g., creativity and inspiration). However, in this analysis we treat each artistic work as a natural process, and attempt to identify their degree of intrinsic uncertainty.

• Stochastics help develop a unified perception for all natural phenomena and expel dichotomies like random vs. deterministic. It seems that rather both randomness and predictability coexist and are intrinsic to natural systems which can be deterministic and random at the same time, depending on the prediction horizon and the time scale [5].

• The high complexity and uncertainty of natural processes has been long identified through plain observations as well as extended analyses of hydrometeorological processes such as temperature, humidity, surface wind, precipitation, atmospheric pressure, river discharges etc (see [6] for a review). Particularly, all these processes seem to exhibit high unpredictability due to the clustering of events as it has been first identified in Nile's water levels by Hurst in 1951.

• The mathematical description and analysis of this behavior is attributed to Kolmogorov who developed it earlier while studying turbulence. To give credits to both scientists Koutsoyiannis [7] named this behaviour as Hurst-Kolmogorov (HK) behaviour. The high uncertainty of climate dynamics has been linked to the power-law type of both the marginal distribution and the dependence structure through empirical evidence [8] as well as theoretical justification [9].

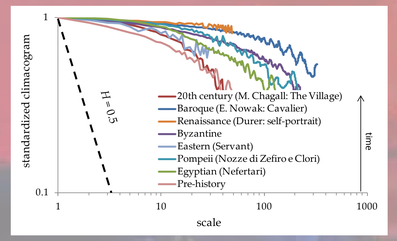
• The dependence structure of a process is represented by the (second-order) climacogram, i.e., the variance of the averaged process vs. scale k, where $k = \kappa D$ is the continuous-time scale in time units and κ the dimensionless discrete one, assuming that D is a time unit that is used for discretization. Although the climacogram is directly linked to the autocovariance and power spectrum (i.e., each of these three stochastic tools contains exactly the same information [10]), it has been shown that the climacogram provides better estimates than the other two [11] and therefore, the measure of uncertainty (i.e., estimation of the Hurst parameter) here is based on that.

Analyzing painting

Basic painting genres
(number of samples based on chronological order)

- Pre-historic (5)
- Egyptian (5)
- Pompeii (5)
- Eastern (5)
- Byzantine (9)
- Renaissance (7)
- Baroque (7)
- 20th century (7)







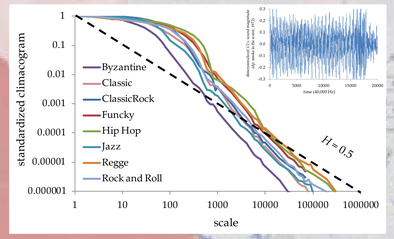

The above Figure depicts a sample of paintings in a chronological order. The dashed black line corresponds to a white noise process.

- Each 2d painting is digitized based on a grayscale colour intensity and the climacogram is estimated based on the geometric [12].
- An HK behaviour is apparent to all the examined samples, with an overall $H \approx 0.9$.
- Patterns can be observed in terms of the dependence structure, with the Egyptian and Pompeii having a moderate structure ($0.85 < H < 0.9$), the Eastern, Byzantine, Renaissance and Baroque exhibiting a strong structure ($H > 0.95$) and the pre-historical and modern corresponding to a weak (but still HK-type) structure ($0.8 < H < 0.85$).

Preliminary analysis. Music

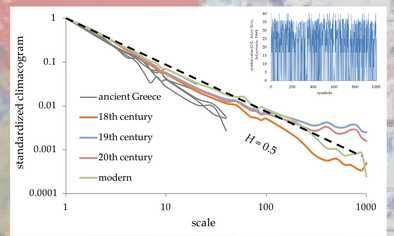
- Each music piece is digitized based on the sound intensity with a very small resolution (for practical reasons a 2 min duration is analyzed from each sample and in total we test 5 sample from each genre).
- An anti-persistent behaviour is apparent to all the examined samples ($0.2 < H < 0.5$).
- An obvious pattern among the examined genres is not observed.

- However, the Byzantine, Classic and Jazz pieces are somehow separated from each other and from the rest, with the latter exhibiting a similar behaviour in both short and long term range of scales.
- All samples exhibit a strong correlation structure in small scales.



Preliminary analysis. Literature

- Each poem is digitized based on a value assigned to each letter in an alphabetic order (24 Greek letters).
- A value is assigned to each common punctuation types (,.!--).
- A white noise behaviour is apparent to all the examined samples ($H \approx 0.5$).
- An obvious pattern among the examined genres is not observed.



Concluding remarks

✓ A power-law type behaviour is apparent in large scales of all the examined samples ($0 < H < 1$). This is indicative of a stochastic similarity among different types of art:

- Painting (in terms of grayscale intensity) exhibits strong correlation structure in both small and large scales ($H \approx 0.9 > 0.5$, i.e. long-term persistent behaviour).
- Music (in terms of sound intensity) exhibits strong correlation structure in small scales, and an anti-persistent behaviour in large scales ($H \approx 0.3 < 0.5$).
- Literature (in terms of letter and punctuation sequence) exhibits a white noise behaviour ($H \approx 0.5$).

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