лаборатория гидрологии речных бассейнов

Российская Академия Наук

Lomonosov Moscow State University

**School for Young Scientists: "Modelling and forecasting of river flows and managing hydrological risks: towards a new generation of methods"**

Moscow, Russia, 22-26 October 2018

# Modelling extreme rainfall in the era of climate change concerns:

# Towards a consistent stochastic methodology

**One Step Forward, Two Steps Back**


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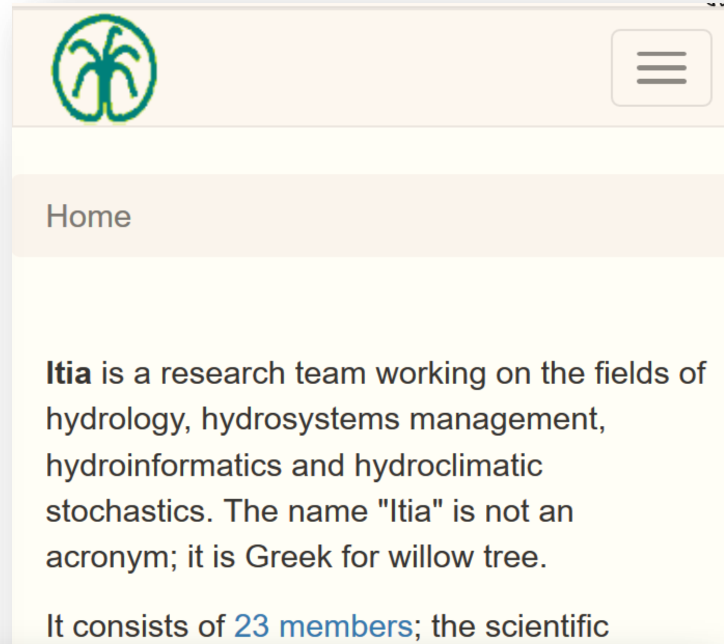
**Presentation available online: http://www.itia.ntua.gr/1897/**

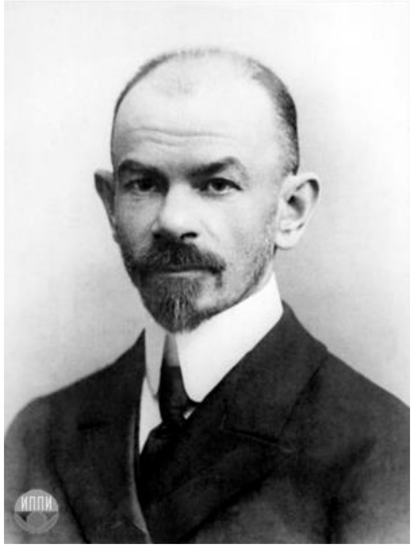
# Greetings from the Itia research team





http://www.itia.ntua.gr/

# Giants of the Moscow School of Mathematics




Dmitri Egorov
(1869 – 1931)


Nikolai Luzin
(1883 – 1950)

Aleksandr Khinchin
(1894 – 1959)

Andrey Kolmogorov
(1903 –1987)

# On names and definitions: A contrast

| | |
|---|---|
| *Ἀρχὴ παιδεύσεως ἡ τῶν ὀνομάτων ἐπίσκεψις*” (*The beginning of education is the inspection of names*)<br>Attributed to Socrates by Epictetus, Discourses, I.17,12 | *What's in a name? That which we call a rose, by any other name would smell as sweet.*<br>William Shakespeare, “Romeo and Juliet” (Act 2, scene 2) |
| *Each definition is a piece of secret ripped from Nature by the human spirit. I insist on this: any complicated thing, being illumined by definitions, being laid out in them, being broken up into pieces, will be separated Into pieces completely transparent even to a child, excluding foggy and dark parts that our intuition whispers to us while acting, separating into logical pieces, then only can we move further, towards new successes due to definitions . . .*<br>Nikolai Luzin (from Graham and Kantor, 2009) | *Let me argue that this situation* [absence of a definition] *ought not create concern and steal time from useful work. Entire fields of mathematics thrive for centuries with a clear but evolving self-image, and nothing resembling a definition*<br>Benoit Mandelbrot (1999, p. 14) |

# Part A
# Premises for a stochastic framework about change

# The general framework: Seeking theoretical consistency in analysis of geophysical data (Using *stochastics*)

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## Seeking Theoretical Consistency in Analysis of Geophysical Data (Using Stochastics)

Demetris Koutsoyiannis · Panayiotis Dimitriadis · ... · Show all 4 collaborators

Goal: Analysis of geophysical data is (explicitly or implicitly) based on stochastics, i.e. the mathematics of random variables and stochastic processes. These are abstract mathematical objects, whose properties distinguish them from typical variables that take on numerical values. It is important to understand these properties before making calculations with data, otherwise the results may be meaningless (not even wrong).

Lab: Laboratory of Hydrology and Water Resources Development

# Why stochastics in geophysics and hydrology?

- **Geophysics** is the branch of physics that **relies most decisively on data**.
- **Geophysical data are numbers** but to treat them we need to use **stochastics, not arithmetic.**
- **Stochastics is the mathematics of random variables and stochastic processes.**
- Random variables and stochastic processes are abstract mathematical objects, whose properties distinguish them from typical variables that take on numerical values.
- It is important to understand these properties before making calculations with data, otherwise the results may be meaningless (not even wrong).
- The numerical data allow us to **estimate** (not to determine precisely) **expectations**.
- Expectations are defined as **integrals** of products of functions. For a continuous random variable $\underline{x}$ with probability density function $f(x)$, the expectation of an arbitrary function $g$ of $\underline{x}$ (where $g(\underline{x})$ is a random variable per se), the **expectation** of $g(\underline{x})$ is defined as $\theta \coloneqq \mathrm{E}\left[g\left(\underline{x}\right)\right] \coloneqq \int_{-\infty}^{\infty} g(x) f(x) \mathrm{d}x$.
- Central among expectations are the **moments**, in which $g(\underline{x})$ is a power of $\underline{x}$ (or a linear expression of $\underline{x}$).
- To estimate true parameters $\theta$ from data we need estimators; the **estimator** $\underline{\hat{\theta}}$ of $\theta$ is a random variable depending on the stochastic process of interest $\underline{x}(t)$ and is a model per se, not a number.
- The **estimate** $\hat{\theta}$ is a number, calculated by using the observations and the estimator.
- Characteristic statistics of the estimator $\underline{\hat{\theta}}$ are its **bias**, $\mathrm{E}\left[\underline{\hat{\theta}}\right] - \theta$, and its **variance** $\mathrm{var}\left[\underline{\hat{\theta}}\right]$. When $\mathrm{E}\left[\underline{\hat{\theta}}\right] = \theta$ the estimator is called unbiased.
- Estimation is made possible thanks to two concepts of stochastics: **stationarity** and **ergodicity**.
- Stationarity and ergodicity are not incompatible with, or contradictory to, **change**.

# Change is crucial in Hydrology: 'Panta Rhei'—The scientific decade of IAHS 2013-2022

PANTA RHEI LIBRARY

## PANTA RHEI

## CHANGE IN HYDROLOGY AND SOCIETY

The new scientific decade 2013–2022 of IAHS, entitled "Panta Rhei – Everything Flows", is dedicated to research activities on change in hydrology and society. The purpose of Panta Rhei is to reach an improved interpretation of the processes governing the water cycle by focusing on their changing dynamics in connection with rapidly changing human systems. Panta Rhei is presented by Montanari et al., Panta Rhei—Everything Flows":Change in hydrology and society—The IAHS Scientific Decade 2013–2022, Hydrological Sciences Journal, 58:6, 1256-1275, DOI:10.1080/02626667.2013.809088. The practical aim is to improve our capability to make predictions of water resources dynamics to support sustainable societal development in a changing environment. The concept implies a focus on hydrological systems as a changing interface between environment and society, whose dynamics are essential to determine water security, human safety and development, and to set priorities for environmental management. The Scientific Decade 2013–2022 will devise innovative theoretical blueprints for the representation of processes including change and will focus on advanced monitoring and data analysis techniques. Interdisciplinarity will be sought by increased efforts to bridge with the socio–economic sciences and geosciences in general.

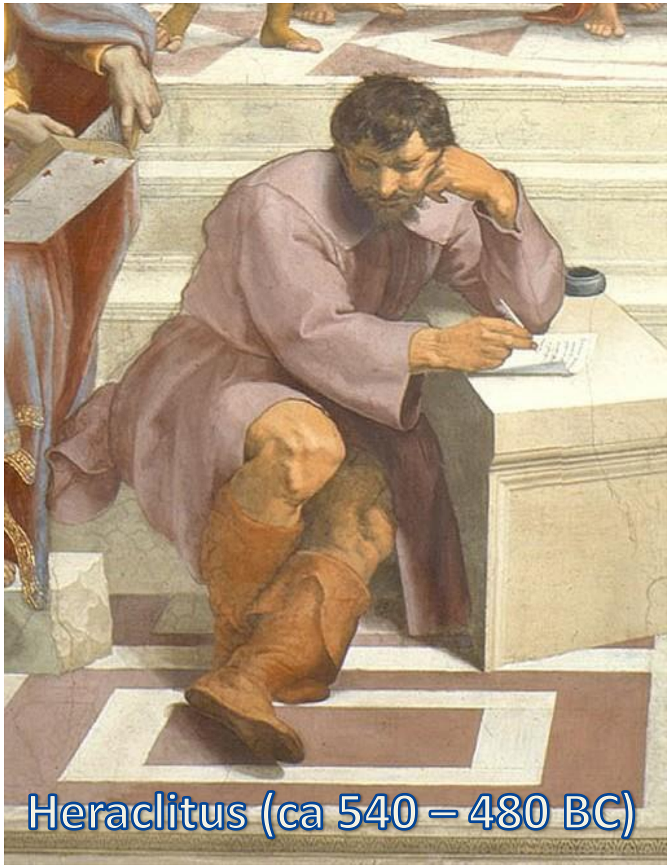
Concepts of Panta Rhei

http://iahs.info/Commissions--W-Groups/Working-Groups/Panta-Rhei.do

# ‘Panta Rhei’: © Heraclitus
# Change and randomness

***Πάντα ῥεῖ***
Everything flows
(Heraclitus; quoted in Plato’s Cratylus, 339-340)

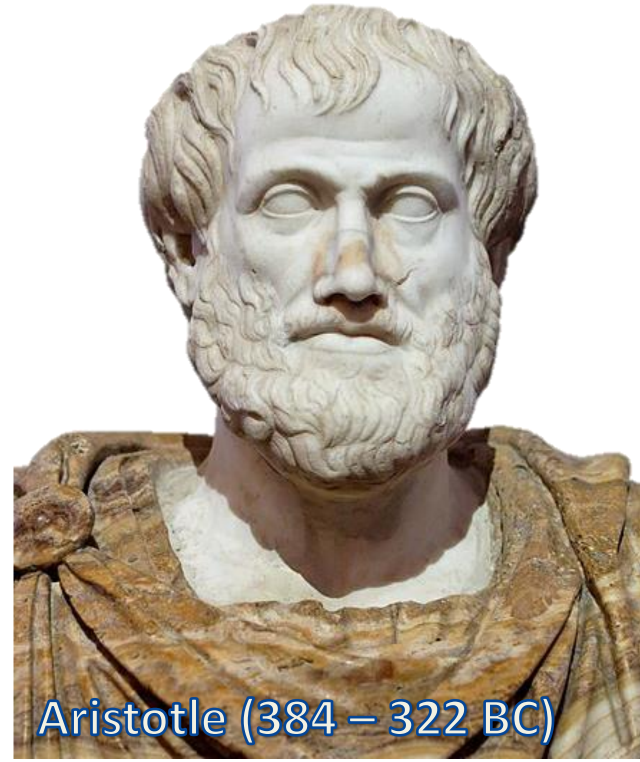
***Αἰών παῖς ἐστι παίζων πεσσεύων***
Time is a child playing, throwing dice
(Heraclitus; Fragment 52)



# Change, logic, precision: © Aristotle

***Μεταβάλλει τῷ χρόνῳ πάντα***
All is changing in the course of time
(Aristotle; Meteorologica, I.14, 353a 16)

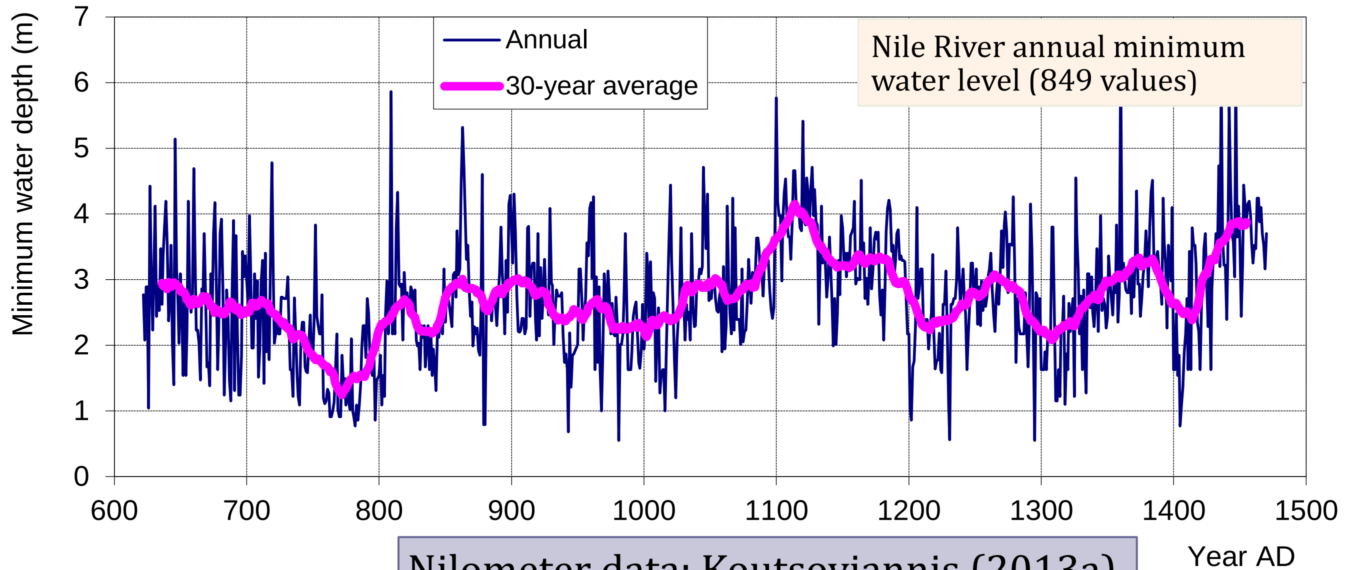
**Λογική, συλλογισμός, επαγωγή, ορθός λόγος**
Logic, deduction, induction,
(right) reason
(Aristotle, Organon & Nicomachean Ethics)




***…τοσοῦτον τάκριβὲς ἐπιζητεῖν καθ᾽ ἕκαστον γένος, ἐφ᾽ ὅσον ἡ τοῦ πράγματος φύσις ἐπιδέχεται***
… look for precision in each class of things just so far as the nature of the subject admits
(Aristotle, Nicomachean Ethics 1094b)

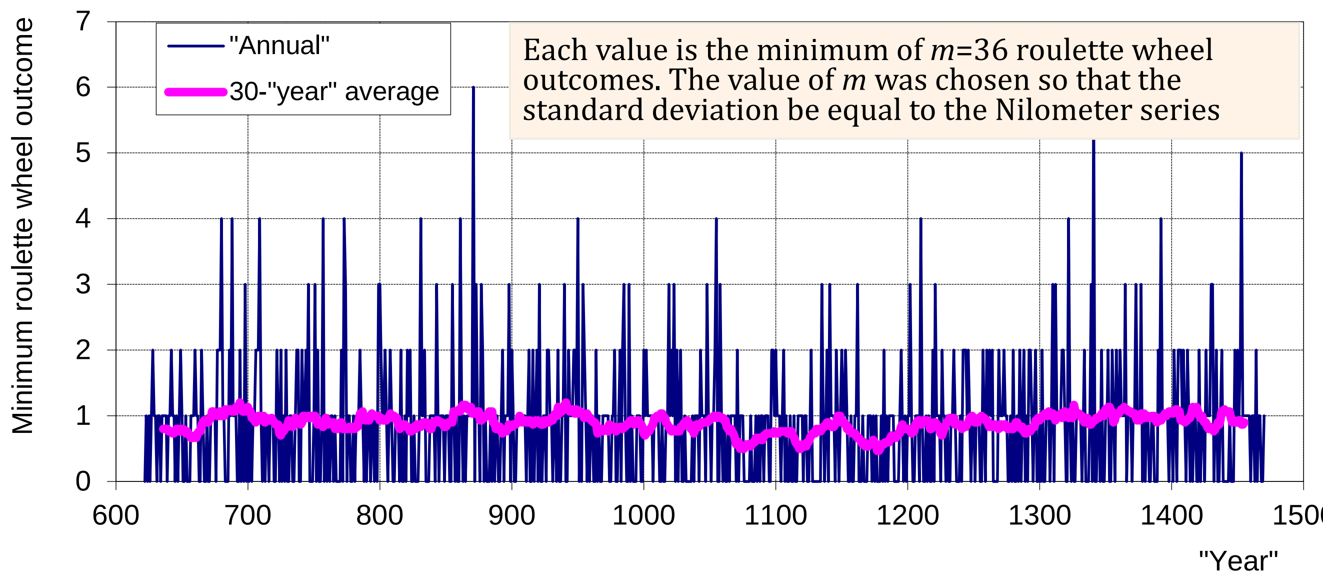
# Hurst-Kolmogorov dynamics—Or: Earth's perpetual change

Structured random




Nilometer data: Koutsoyiannis (2013a)

Purely random



# The climacogram: A simple statistical tool to quantify change across time scales

- Take the Nilometer time series, $x_1, x_2, \ldots, x_{849}$, and calculate the sample estimate of variance $\gamma^{(1)}$, where the superscript (1) indicates time scale (1 year)
- Form a time series at time scale 2 (years):

$$x_1^{(2)} := \frac{x_1 + x_2}{2}, x_2^{(2)} := \frac{x_3 + x_4}{2}, \ldots, x_{424}^{(2)} := \frac{x_{847} + x_{848}}{2} \quad (1)$$

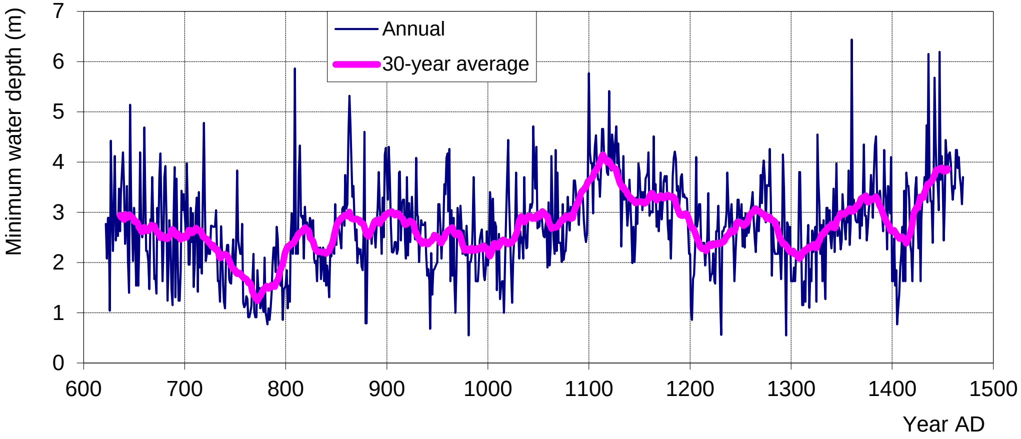
  and calculate the sample estimate of the variance $\gamma^{(2)}$.
- Repeat the same procedure and form a time series at time scale 3, 4, … (years), up to scale 84 (1/10 of the record length) and calculate the variances $\gamma^{(3)}, \gamma^{(4)}, \ldots \gamma^{(84)}$.
- The **climacogram** is the variance $\gamma^{(\kappa)}$ as a function of scale $\kappa$; it is visualized as a double logarithmic plot of $\gamma^{(\kappa)}$ vs. $\kappa$ (or alternatively of the standard deviation $\sigma^{(\kappa)}$).
- If the time series $x_\tau$ represented a pure random process, the climacogram would be a straight line with slope –1 (the proof is very easy).
- In real world processes, the slope is different from –1, designated as $2H - 2$, where $H$ is the so-called Hurst coefficient ($0 < H < 1$).
- The scaling law $\gamma^{(\kappa)} = \gamma^{(1)} / \kappa^{2-2H}$ defines the **Hurst-Kolmogorov (HK) process**.
- High values of $H$ (> 0.5) indicate **enhanced change** at large scales, else known as **long-term persistence**, or strong **clustering** (grouping) of similar values.

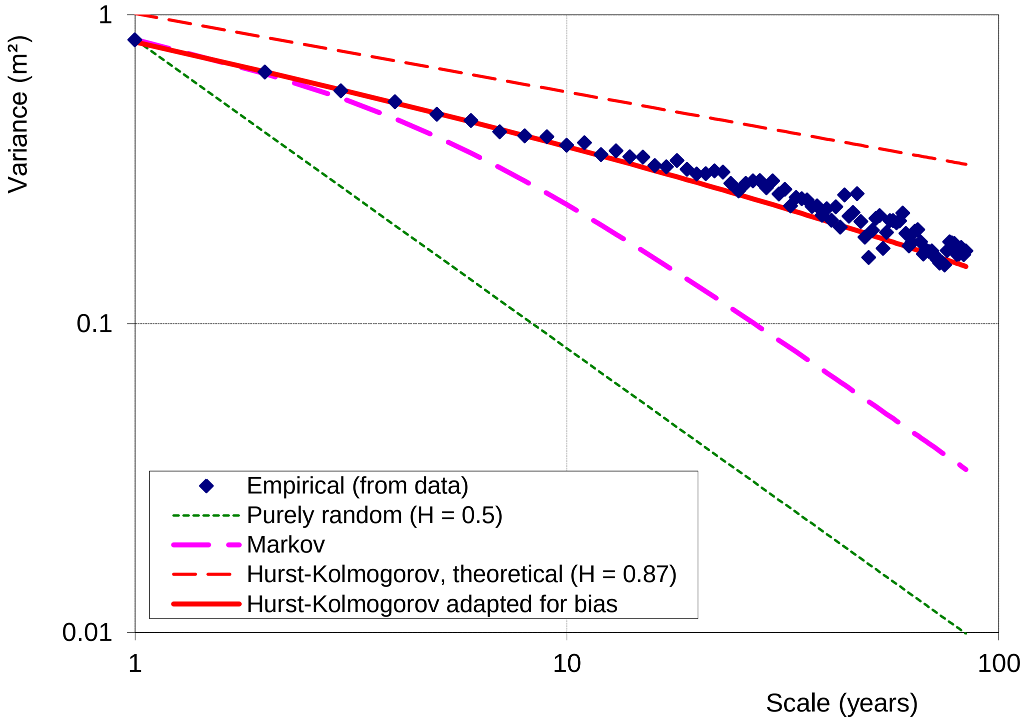
# The climacogram of the Nilometer time series

- The Hurst-Kolmogorov process seems consistent with reality.
- The Hurst coefficient is $H = 0.87$ (Similar $H$ values are estimated from the simultaneous record of maximum water levels and from the modern, 131-year, flow record of the Nile flows at Aswan).
- The Hurst-Kolmogorov behaviour, seen in the climacogram, indicates that:

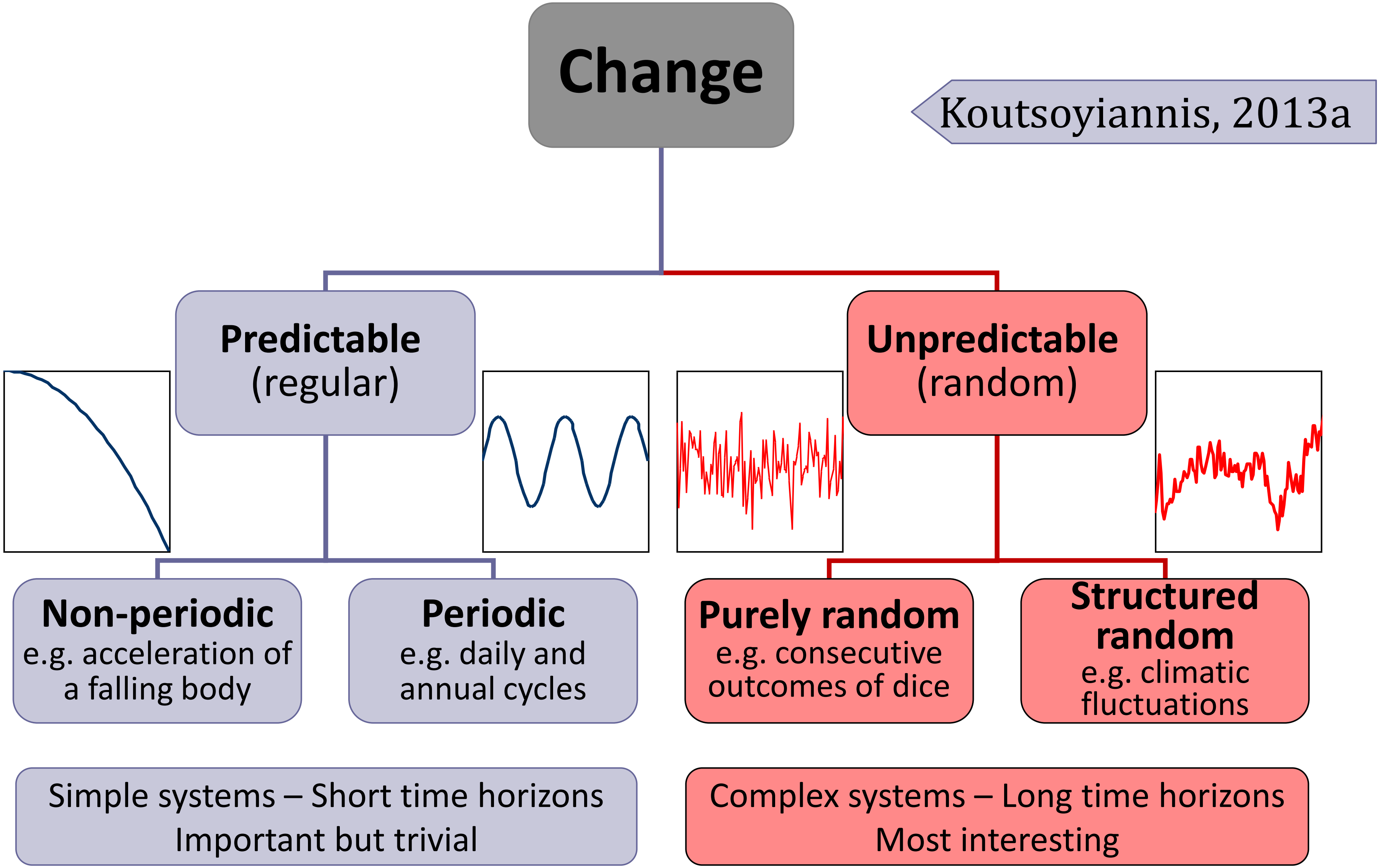
  (a) long-term changes are more frequent and intense than commonly perceived, and

  (b) future states are much more uncertain and unpredictable on long time horizons than implied by pure randomness.



# Change and predictability



# The cause of change: © Peter Atkins

ENTROPY

THE SPRING OF CHANGE

THE GREAT IDEA

*All change is the consequence of the purposeless collapse of energy and matter into disorder*

*Not knowing the Second Law of thermodynamics is like never having read a work of Shakespeare*[1]

C. P. SNOW

Atkins, 2004

timents. The second law is of central importance in the whole of science, and hence in our rational understanding of the universe, because it provides a foundation for understanding why *any* change occurs. Thus, not only is it a basis for understanding why engines run and chemical reactions occur, but it is also a foundation for understanding those most exquisite consequences of chemical reactions, the acts of literary, artistic, and musical creativity that enhance our culture.

Atkins, 2007

# Entropy ≡ Uncertainty quantified

- Historically entropy was introduced in thermodynamics but later it was given a rigorous definition within probability theory (owing to Boltzmann, Gibbs and Shannon).
- Thermodynamic and probabilistic entropy are essentially the same thing (Koutsoyiannis, 2010, 2013b, 2014; but others have different opinion).
- Entropy acquires its importance from the **principle of maximum entropy** (Jaynes, 1957), which postulates that the entropy of a random variable should be at maximum, under the conditions (constraints) which incorporate the available information about this variable.
- The tendency of entropy to become maximal explains a spectrum of phenomena from the random outcomes of dice to the Second Law of thermodynamics as the driving force of natural change.
- Entropy is a dimensionless measure of uncertainty:

| **Discrete random variable $\underline{z}$** | **Continuous random variable $\underline{z}$** |
|---|---|
| $\Phi[\underline{z}] := \mathrm{E}[-\ln P(\underline{z})] = \sum_{j=1}^{w} P_j \ln P_j$ <br> where $P_j := P\{\underline{z} = z_j\}$ (probability) | $\Phi[\underline{z}] := \mathrm{E}\left[-\ln \frac{f(\underline{z})}{h(\underline{z})}\right] = -\int_{-\infty}^{\infty} \ln \frac{f(z)}{h(z)} f(z) \mathrm{d}z$ <br> where $f(z)$ is probability density and $h(z)$ is the density of a background measure |

# Memorable moments in the history of stochastics

**Ludwig Boltzmann**

(1844 –1906, Universities of Graz and Vienna, Austria, and Munich, Germany)

**1877** Explanation of the concept of **entropy** in probability theoretic context.

**1884/85** Introduction of the notion of **ergodic*** systems which however he called "isodic"

* The term is etymologized from Greek words but which ones exactly is uncertain (options: (a) έργον + οδός; (b) έργον + είδος; (c) εργώδης; see Mathieu, 1988).

**George D. Birkhoff**

(1884 – 1944; Princeton, Harvard, USA)

**1931** Discovery of the **ergodic (Birkhoff–Khinchin) theorem**

**Aleksandr Khinchin**

(1894 – 1959; Moscow State University, Russia)

**1933** Purely measure-theoretic proof of the **ergodic (Birkhoff–Khinchin) theorem**

**1934** Definition of **stationary stochastic processes** and probabilistic setting of the **Wiener-Khinchin theorem** relating autocovariance and power spectrum

**Andrey N. Kolmogorov**

(1903 – 1987; Moscow State University, Russia)

**1931** Introduction of the terms **process** to describe change of a certain system and **stationary** to describe a probability density function that is unchanged in time

**1933** Definition of the concepts of **probability** & **random variable**

**1937-1938** Probabilistic exposition of the **ergodic (Birkhoff–Khinchin) theorem** and **stationarity**

**1947** Definition of **wide sense stationarity**

# Stationarity and nonstationarity

Central to the notion of a stochastic process are the concepts of **stationarity** and **nonstationarity**, two widely misunderstood and broadly misused concepts (Montanari and Koutsoyiannis, 2014; Koutsoyiannis and Montanari, 2015); their **definitions apply only to stochastic processes** (e.g., time series cannot be stationary, nor nonstationary).

**Reminder of definitions**

Following Kolmogorov (1931, 1938) and Khinchin (1934), a process is **(strict-sense) stationary** if its **statistical properties are invariant to a shift of time origin**, i.e. the processes $\underline{x}(t)$ and $\underline{x}(t')$ have the same statistics for any $t$ and $t'$*.

Following Kolmogorov (1947), a stochastic process is **wide-sense stationary** if its mean is constant and its autocovariance depends on time difference only, i.e.:

$$\mathrm{E}[\underline{x}(t)] = \mu = \text{constant}, \quad \mathrm{E}[(\underline{x}(t) - \mu)\,(\underline{x}(t+\tau) - \mu)] = c(\tau) \tag{2}$$

Conversely, a process is **nonstationary** if some of its statistics are changing through time and **their change is described as a deterministic† function of time**.

---

*See further details in Papoulis (1991); see also further explanations in Koutsoyiannis (2006a, 2011) and Koutsoyiannis and Montanari (2015).

† See Koutsoyiannis(2000, 2011) and Koutsoyiannis and Montanari (2015) about clarification of "deterministic".

# Ergodicity

- Stationarity is also related to **ergodicity**, which in turn is a prerequisite to make inference from data, that is, induction.
- By definition (e.g., Mackey, 1992, p. 48; Lasota and Mackey, 1994, p. 59), a **transformation of a dynamical system is ergodic if all its invariant sets are trivial (have zero probability)**. In other words, in an ergodic transformation starting from any point, a trajectory will visit all other points, without being trapped to a certain subset. (In contrast, in non-ergodic transformations there are invariant subsets, such that a trajectory starting from within a subset will never depart from it).
- The **ergodic theorem** (Birkhoff, 1931; Khinchin, 1933; see also Mackey, 1992, p. 54) allows **redefining ergodicity within the stochastic processes domain** (Papoulis 1991 p. 427; Koutsoyiannis 2010) in the following manner: A stochastic process $\underline{x}(t)$ is ergodic if the **time average** of any (integrable) function $g(\underline{x}(t))$, **as time tends to infinity, equals the true** (ensemble) **expectation** $\mathrm{E}[g(\underline{x}(t))]$, i.e., $\lim_{T\to\infty} \frac{1}{T}\int_0^T g\left(\underline{x}(t)\right) dt = \mathrm{E}[g(\underline{x}(t))]$.
- If the system that is modelled in a stochastic framework has **deterministic dynamics** (meaning that a system input will give a single system response, as happens for example in most hydrological models) then a theorem applies (Mackey 1992, p. 52), according to which a dynamical system has a stationary probability density *if and only if* it is ergodic. Therefore, **a stationary system is also ergodic and vice versa**, and **a nonstationary system is also non-ergodic and vice versa**.
- If the system **dynamics is stochastic** (a single input could result in multiple outputs), then **ergodicity and stationarity do not necessarily coincide**. However, recalling that a stochastic process is a model and not part of the real world, **we can always conveniently device a stochastic process that is ergodic** (see example in Koutsoyiannis and Montanari, 2015).

# Both stationarity and ergodicity are immortal!*

From a practical point of view ergodicity can always be assumed when there is stationarity. Without stationarity and ergodicity inference from data would not be possible. Ironically, several studies use time series data to estimate statistical properties, as if the process were ergodic, while at the same time what they (cursorily) estimate may falsify the ergodicity hypothesis.

**Misuse example 1**: By analysing the time series $x_\tau$ (where $\tau$ denotes time), I concluded that it is nonstationary and I identified an increasing trend with slope $b$.

**Corrected example 1**: I analysed the time series $x_\tau$ based on the hypothesis that the stochastic process $\underline{x}_\tau - b\tau$ is stationary and ergodic, which enabled the estimation of the slope $b$.

**Misuse example 2**: From the time series $x_\tau$, I calculated the power spectrum and found that its slope for low frequencies is steeper than −1, which means that the process is nonstationary.

**Possible correction (a) of example 2**: I cursorily interpreted a slope steeper than −1 in the power spectrum as if indicated nonstationary, while a simple explanation would be that the frequencies on which my data enable calculation of the power spectrum values are too high.

**Possible correction (b) of example 2**: I cursorily applied the concept of the power spectrum of a stationary stochastic process, forgetting that the empirical power spectrum of a stationary stochastic process is a (nonstationary) stochastic process per se. The high variability of the latter (or the inconsistent numerical algorithm I used) resulted in a slope for low frequencies steeper than −1, which is absurd. Such a slope would suggest a non-ergodic process while my calculations were based on the hypothesis of a stationary and ergodic process.

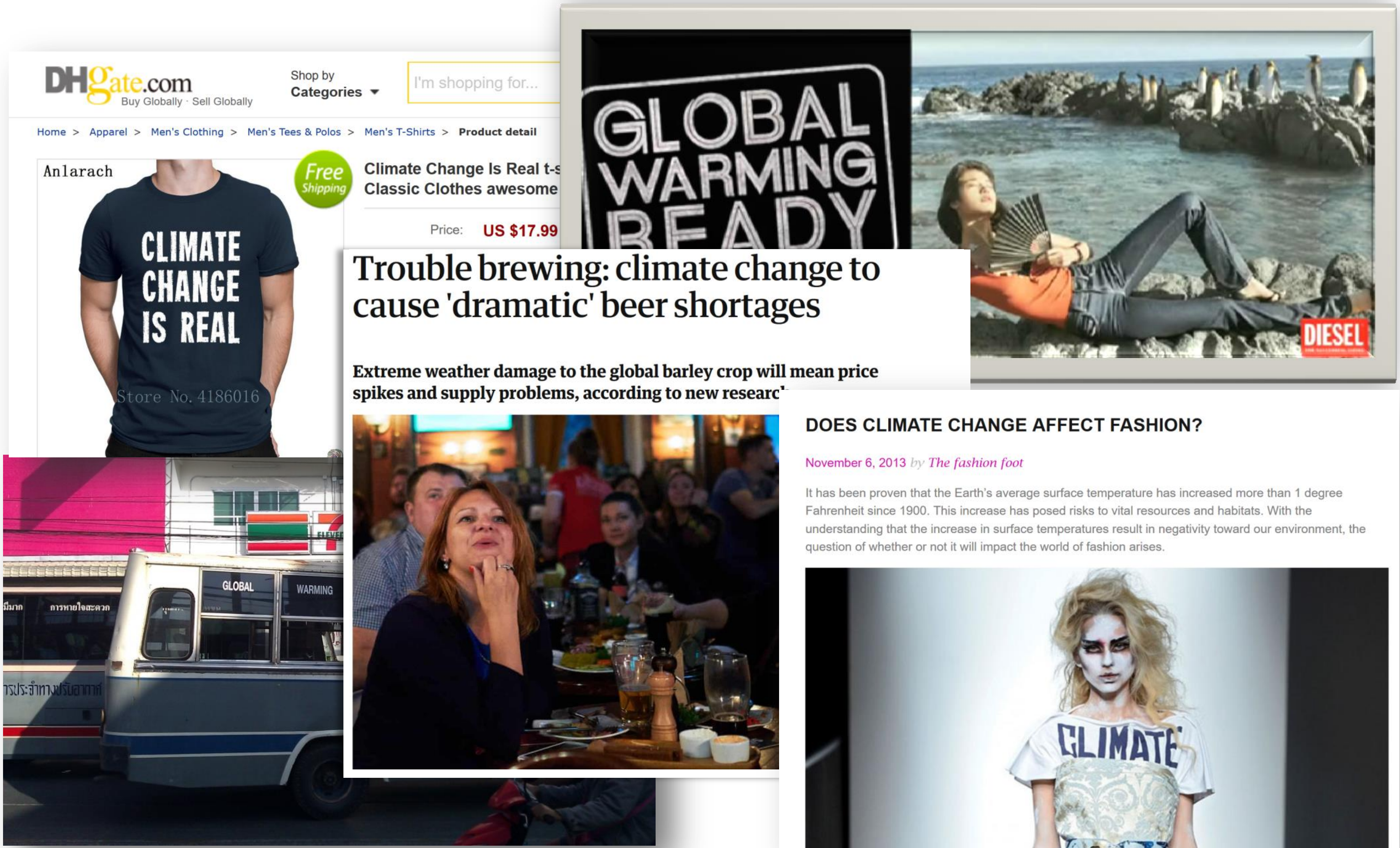
**Possible correction (c) of example 2**: I cursorily applied the concept of the power spectrum of a stationary stochastic process using a time series which is realization of a nonstationary stochastic process and I found an inconsistent result; therefore, I will repeat the calculations recognizing that the power spectrum of a nonstationary stochastic process is a function of two variables, frequency and "absolute" time.

*Montanari and Koutsoyiannis (2014)

# Part B
# Change in rainfall
# in the era of climate change concerns

**(or how to avoid one step back)**

# Climate change concerns are real and affect people



# Climate change concerns affect hydrology: The surge of studies of nonstationary extremes

AGU PUBLICATIONS
Water Resources Research
RESEARCH ARTICLE
10.1002/2016WR019676
Predicting nonstationary flood frequencies: Evidence supports an updated stationarity thesis in the United States

ERDC/CHL CHETN-X-2
March 2016
US Army Corps of Engineers
Bayesian Inference of Nonstationary Precipitation Intensity-Duration-Frequency Curves for Infrastructure Design

atmosphere
MDPI
Article
An Uncertainty Investigation of RCM Downscaling Ratios in Nonstationary Extreme Rainfall IDF Curves

Water Resour Manage (2015) 29:5533–5550
DOI 10.1007/s11269-015-1133-5
CrossMark
Nonstationary Flood Frequency Analysis for Annual Flood Peak Series, Adopting Climate Indices and Check Dam Index as Covariates

Journal of Hydrology 519 (2014) 2040–2048
Contents lists available at ScienceDirect
ELSEVIER
Journal
journal homepage: ww
A nonstationary index-flood technique
for annual maximum streamflow

Nat. Hazards Earth Syst. Sci. Discuss., https://doi.org/10.5194/nhess-2018-290
Manuscript under review for journal Nat. Hazards Earth Syst. Sci.
Discussion started: 16 October 2018
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Natural Hazards and Earth System Sciences
Discussions
EGU
Projected intensification of sub-daily and daily rainfall extremes in convection-permitting climate model simulations over North America: Implications for future Intensity-Duration-Frequency curves

in Water Resources 118 (2018) 83–94
ts lists available at ScienceDirect
es in Water Resources
ge: www.elsevier.com/locate/advwatres
ing for nonstationary hydrology

SCIENTIFIC REPORTS
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SUBJECT AREAS:
CLIMATE SCIENCES
HYDROLOGY
Received
19 June 2014
Nonstationary Precipitation Intensity-Duration-Frequency Curves for Infrastructure Design in a Changing Climate

HYDROLOGICAL SCIENCES JOURNAL, 2018
VOL. 63, NO. 3, 325–352
https://doi.org/10.1080/02626667.2018.1426858
IAHS AISH
Taylor & Francis
Check for updates
Techniques for assessing water infrastructure for nonstationary extreme events: a review

# Is “stationarity dead” and is there “rainfall intensification”?


POLICYFORUM

CLIMATE CHANGE

**Stationarity Is Dead: Whither Water Management?**

P. C. D. Milly,[1*] Julio Betancourt,[2] Malin Falkenmark,[3] Robert M. Hirsch,[4] Zbignie Kundzewicz,[5] Dennis P. Lettenmaier,[6] Ronald J. Stouffer[7]


LETTERS

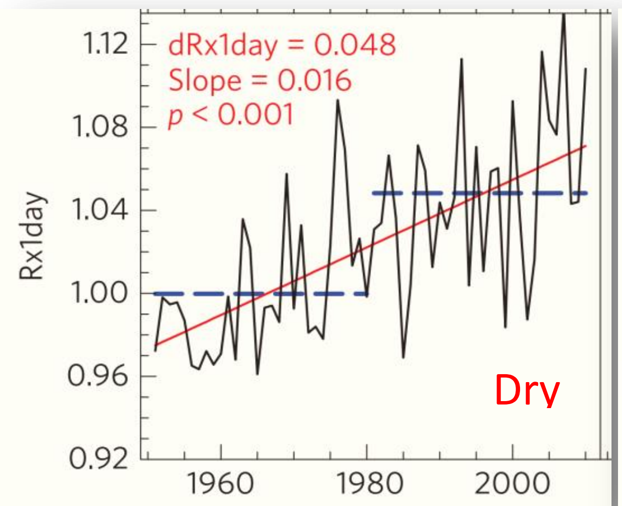
PUBLISHED ONLINE: 7 MARCH 2016 | DOI: 10.1038/NCLIMATE2941

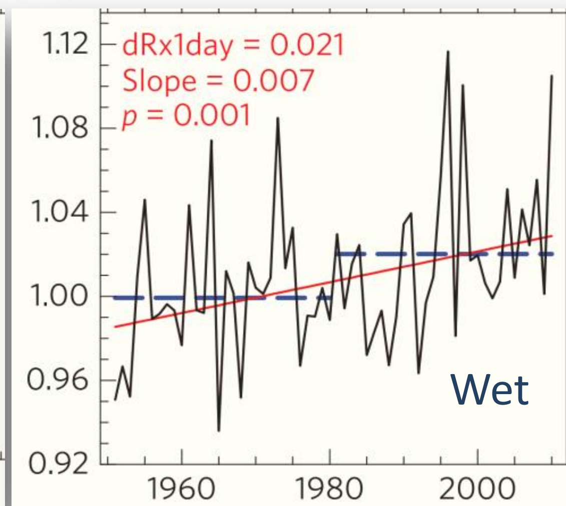
nature climate change

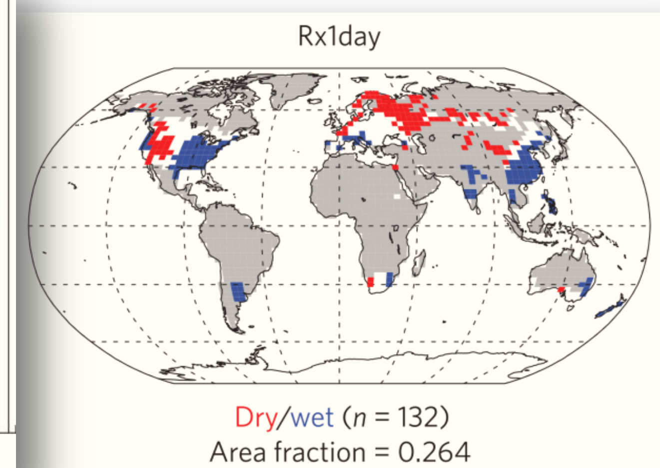
**More extreme precipitation in the world’s dry and wet regions**

Markus G. Donat[1*], Andrew L. Lowry[1], Lisa V. Alexander[1], Paul A. O’Gorman[2] and Nicola Maher[1]

The climatic value of annual maximum daily rainfall of the 30-year period 1980 – 2010, compared to that of 1960-80, is greater by 5% for dry areas and by 2% for wet areas Donat et al. (2016).

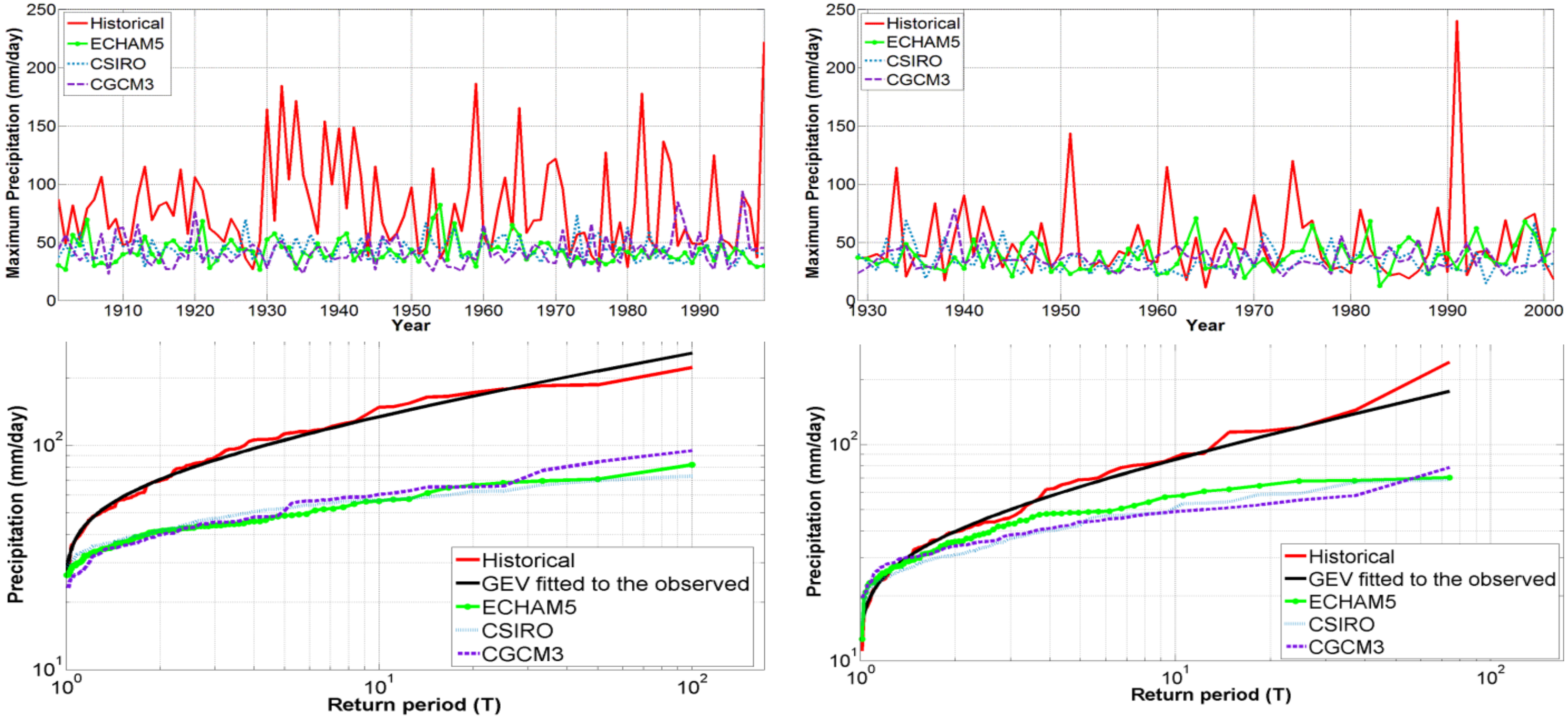


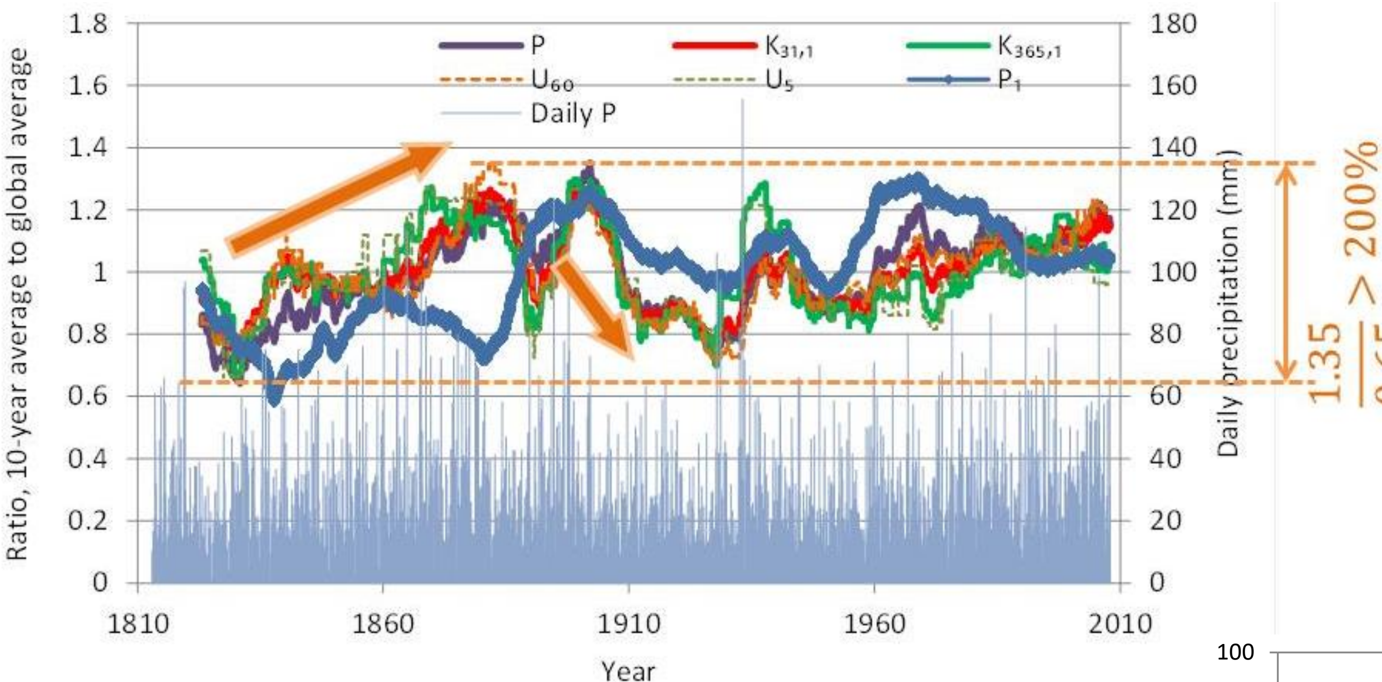
# Do climate models allow a nonstationary approach on rainfall extremes or do they simulate a process other than rainfall?

- Tsaknias et al. (2016—**multirejected paper**) tested the reproduction of extreme events by three climate models of the IPCC AR4 at 8 test sites in the Mediterranean which had long time series of temperature and precipitation.
- They concluded that model results are irrelevant to reality as they seriously underestimate the size of extreme events.




Upper row: Daily annual maximum precipitation at Perpignan and Torrevieja;
Lower row: empirical distribution functions of the data in upper row (Tsaknias et al., 2016)

# Decadal change as seen in a long daily precipitation record

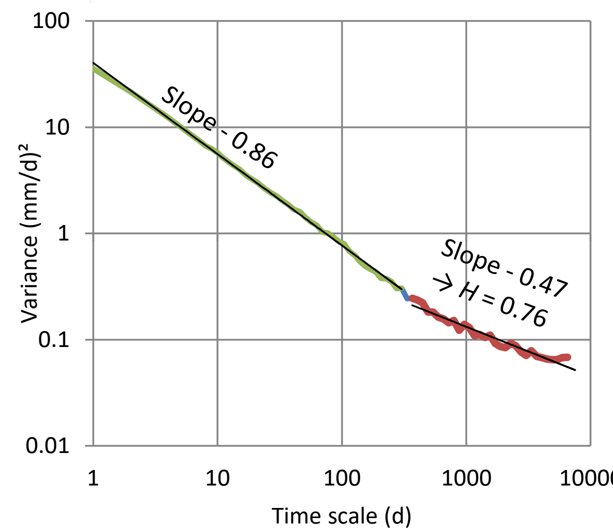



**Dataset details**
Station: BOLOGNA, Italy, 44.50°N, 11.35°E, +53.0 m a.m.s.l.,
period: 1813-2007 (195 years);
https://climexp.knmi.nl/gdcnprcp.cgi?WMO=ITE00100550

The plots show moving averages of ratios for a time window of 10-year length.

P: precipitation; $K_{351,1}$ and $U_5$ represent the maximum daily rainfall intensity for a return period of 2 years. Same are $K_{31,1}$ and $U_{60}$ but for a return period of 2 months (see definitions below); $P_1$: probability wet.

The Hurst-Kolmogorov behaviour is evident.



# Part C
# The non-scientific concept of upper limits and the Probable Maximum Precipitation

**(or how to avoid a second step back)**

# Deterministic thinking and its impact on rainfall modelling

- Deterministic thinking in science is strong enough and has been dominant even in extreme rainfall, despite strong resistance of the rainfall process to comply with deterministic descriptions and despite spectacular failures in adequate modelling of rainfall based on first principles.
- Perhaps the oldest of the attempts, yet very popular even today, aims to determine physical upper bounds to precipitation that could be used to design risk-free constructions or practices.
- The resulting concept of probable maximum precipitation (PMP), that is, an upper bound of precipitation that is physically feasible (WMO, 1986, 2009), is perhaps one of the biggest failures in hydrology but it is still in wide use as a research topic*. In addition, the method is still quite popular in engineering studies.
- Even the terminology is self-contradictory, and thus not scientific. Namely, the word "*probable*" contradicts the existence of a deterministic upper limit.
- Note that the "probable maximum" concept began as "maximum possible" and was later renamed in an attempt to salvage the failed concept (Benson, 1973).
- Rational thinking and fundamental philosophical and scientific principles can help dispel such fallacies. In particular, the Aristotelian notions of *potentia* (potentiality; Greek 'δύναμις') and of potential infinity (Greek 'άπειρον'; Aristotle, Physics, 3.7, 206b16) that "*exists in no other way, but ... potentially or by reduction*" (and is different from mathematical complete infinity) would help us to avoid the PMP concept.

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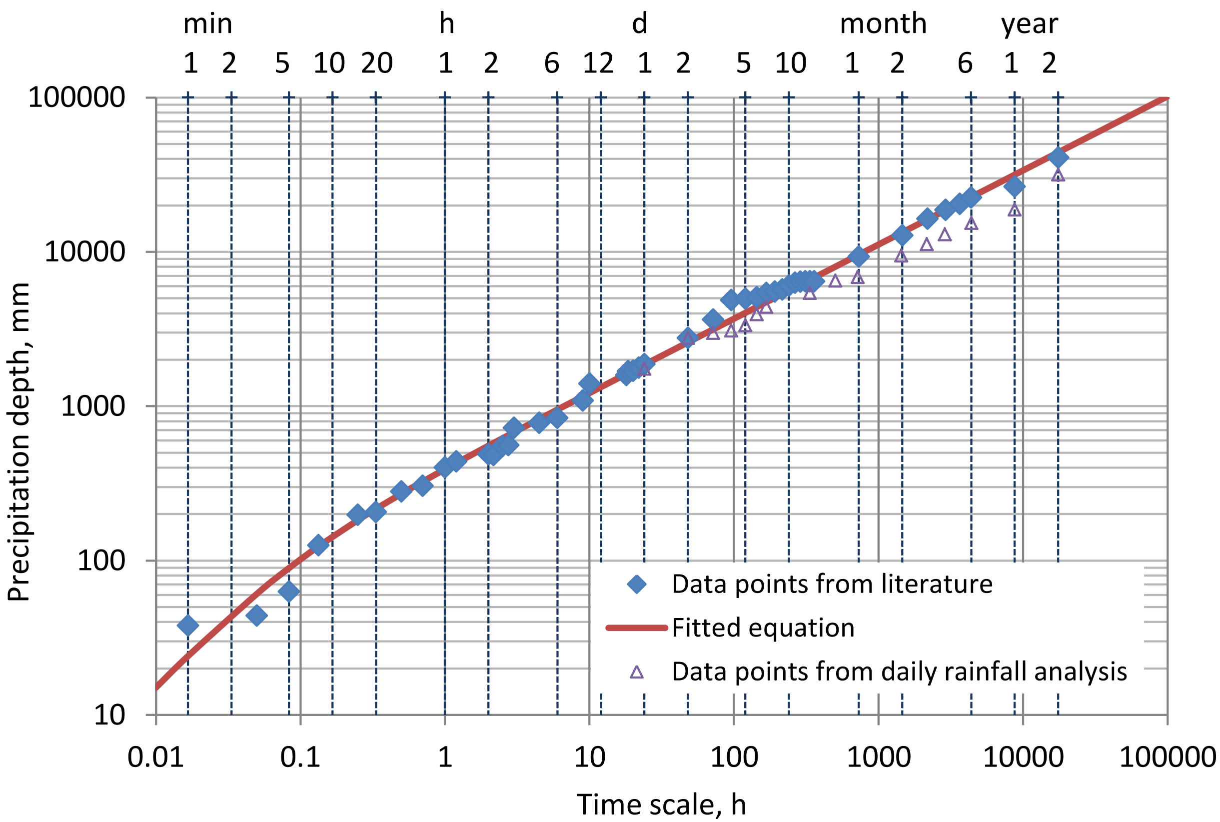
*A Google Scholar search reveals that about 1000 recent publications (since 2014) include the term "probable maximum precipitation". More than 100 of them contain the term (or the acronym PMP) in their title, and half of them also refer to climate change.

# World records of rainfall depth: are they upper limits?




Depiction of the record rainfall values over the globe from literature and from daily rainfall analysis in Koutsoyiannis and Papalexiou (2017).

The equation fitted is

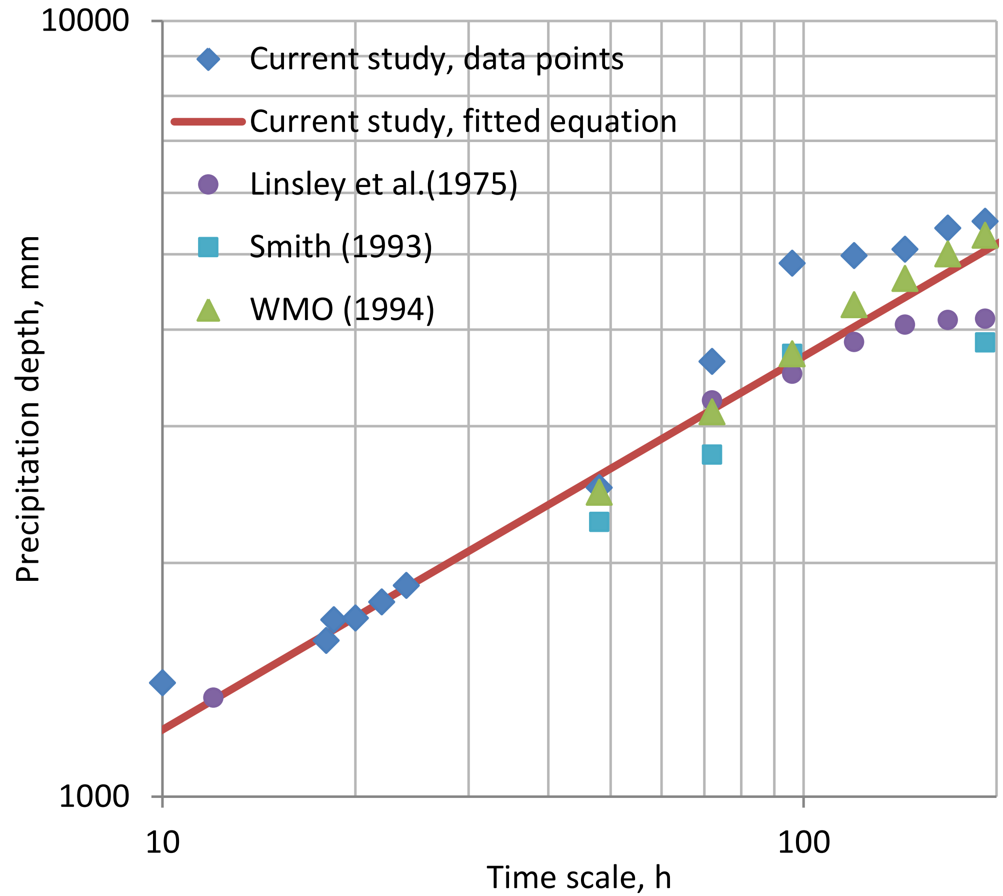
$$i = \frac{a}{(1 + d/\theta)^{\eta}}, \qquad h = \frac{ad}{(1 + d/\theta)^{\eta}} \tag{3}$$

where $i$ is rainfall intensity averaged over time scale $d$ and $h$ is the corresponding rainfall depth, whereas the parameter values are $a$ = 1615 mm/h, $\theta$ = 0.07 h and $\eta$ = 0.52.

# Records are not upper limits; sooner or later they will be broken

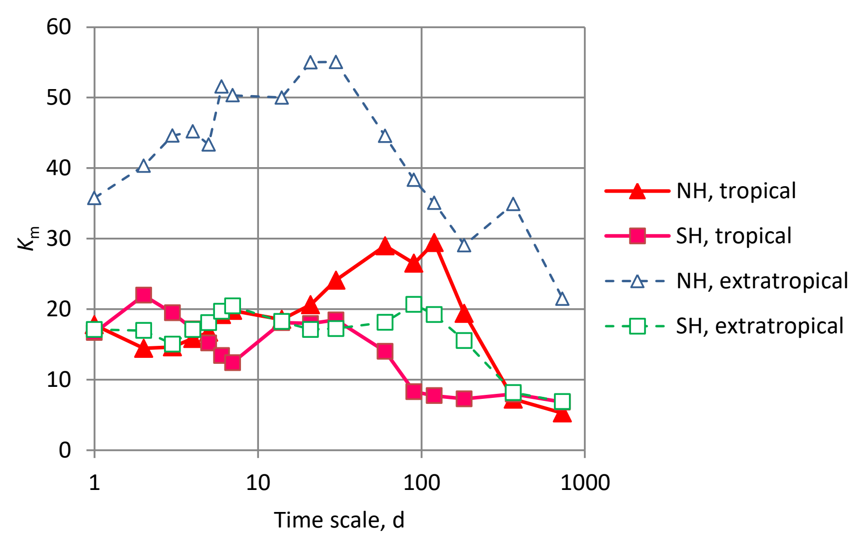
- Apparently, the values shown the figure of world rainfall records do not represent any physical upper bound of precipitation rate. They just represent what was observed as record rainfall.
- Certainly, higher rates have occurred in places where no raingages exist or in longer periods of history.
- Furthermore, values registered in older publications as record values no longer represent record values. Obsolete record values from older publications, which now have been exceeded, are shown in the figure on the right.
- Logically, we can be confident that the records presented here will surely be broken in the future.

The graph from Koutsoyiannis and Papalexiou (2017) shows some points that had been registered as world rainfall records in the three indicated publications and are broken now.



# On Hershfield's statistical approach to PMP

- Several methods to determine PMP exist in literature and are described by WMO (1986, 2009). All suffer logically from the fallacious concept of an upper limit.
- Thorough examination of each of the specific methods separately will reveal that each one is affected by additional logical inconsistencies. While they all assume the existence of a deterministic upper limit, they determine this limit statistically (inference from data) rather than deducing it in a causative deterministic manner using physical principles.
- The statistical character is clear in the so-called "*statistical approach*" by Hershfield (1961, 1965), who used about 95 000 station-years of annual maximum daily rainfall belonging to 2645 stations, standardized each record and found the maximum over the 95 000 standardized values, which was $K_{\mathrm{m}} = 15$.
- Naturally, one of the 95 000 standardized values would be the greatest of all others, but this is not a deterministic limit to call PMP. In fact Koutsoyiannis (1999) provided a consistent statistical analysis of the data set and showed that the values are consistent with a Pareto distribution without an upper bound.
- Thus the logical problem here is the incorrect interpretation that an observed maximum in precipitation is a physical upper limit.
- Koutsoyiannis and Papalexiou (2017) examined a bigger data set (17 490 stations, 1 394 593 station-years) and found much larger values of $K_{\mathrm{m}}$ as shown in the graph on the right.



# On the hydrometeorological approach to PMP

- "Physically based" or "hydrometeorological" methods are only seemingly so and may be even more problematic than Hershfield's statistical approach.
- Among hydrometeorological methods, most representative and most popular is the so-called moisture maximization approach, which is based on the simple formula $h_\mathrm{m} = (W_\mathrm{m}/W)\, h$, where $h_\mathrm{m}$ is the maximized rainfall depth, $h$ is the observed precipitation, $W$ is the precipitable water in the atmosphere during the day of rain, estimated by the corresponding dew point $T_\mathrm{d}$, and $W_\mathrm{m}$ is the maximized precipitable water.
- The latter is estimated from the maximum dew point for the corresponding month, which is either the maximum recorded value from a sample of at least 50 years length, or the value corresponding to a 100-year return period, for samples smaller than 50 years (WMO, 1986).
- The method suffers twice by the incorrect interpretation that an observed maximum is a physical upper limit (Papalexiou and Koutsoyiannis, 2006):
  - It uses a record of observed dew point temperatures to determine an upper limit, which is the maximum observed value.
  - Then it uses this "limit" for the so called "maximization" of an observed sample of storms, and asserts the largest value among them as PMP.
- Even as a pure statistical approach, this is a questionable, particularly because it is based only on one observed value (known in statistics as the largest order statistic), rather than on the whole sample, and thus it is enormously sensitive to one particular observation of the entire sample (Papalexiou and Koutsoyiannis, 2006; Koutsoyiannis, 2007).
- Furthermore, the logic of moisture maximization at a particular location is unsupported given that a large storm at this location depends on the convergence of atmospheric moisture from much greater areas.

# Towards a probabilistic approach to extreme rainfall

- According to Popper (1982) the extension of the Aristotelian idea of *potentia* in modern terms is the notion of probability.
- Probability provides a different way to perceive the intense rainfall and flood, and by assigning to each value a certain probability of exceedence it avoids:
  - the delusion of an upper bound of precipitation;
  - the fooling of decision makers that we can build risk-free constructions;
  - the logical, technical, philosophical and ethical issues that are associated with the PMP concept (Benson, 1973)
- Naturally, the probabilistic approach is inductive and relies on local rainfall observations as only observations can provide a sound basis for quantification of extreme rainfall. Currently, the analysis of global rainfall behaviours (Koutsoyiannis, 2004a,b; Papalexiou and Koutsoyiannis, 2013) assist in formulating the probability distribution function.
- The typical arguments against the use of probabilistic approaches and in favor of PMP are naïve indicating ignorance of probability theory.[*]
- In general, stochastic approaches in modelling the rainfall process —in particular its extremes— are more logical and efficient than deterministic ones and thus the latter ought to be abandoned.

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[*] One example is the statement by Horton (1931; from Klemes, 2000): "*It is, however, important to recognize the nature of the physical processes involved and their limitations in connection with the use of statistical methods. … Rock Creek cannot produce a Mississippi River flood any more than a barnyard fowl can lay an ostrich egg.*" A discussion of this naïve argument can be found in Koutsoyiannis and Papalexiou (201?).

# Part D
# Elements of a consistent stochastic methodology

**(or how to make one step forward)**

# Introductory notes on statistical moments

- As mentioned in the introduction, statistical inference (induction) is based on expectations of functions or random variables, and in particular, moments, which are estimated from samples by virtue of stationarity and ergodicity.
- The ergodic theorem enables, in theory, estimation of moments from data as $n \to \infty$, but what happens for finite $n$?
- It is recalled that the classical definitions of raw and central moments of order $p$ are:

$$\mu'_p \coloneqq \mathrm{E}\left[\underline{x}^p\right], \qquad \mu_p \coloneqq \mathrm{E}\left[\left(\underline{x} - \mu\right)^p\right] \tag{4}$$

respectively, where $\mu \coloneqq \mu'_1 = \mathrm{E}\left[\underline{x}\right]$ is the mean of the random variable $\underline{x}$. Their standard estimators from a sample $\underline{x}_i$, $i = 1, \ldots, n$, are

$$\underline{\hat{\mu}}'_p = \frac{1}{n}\sum_{i=1}^{n} \underline{x}_i^p, \qquad \underline{\hat{\mu}}_p = \frac{b(n,p)}{n}\sum_{i=1}^{n}\left(\underline{x}_i - \hat{\mu}\right)^p \tag{5}$$

where $a(n, p)$ is a bias correction factor (e.g. for the variance $\mu_2 =: \sigma^2$, $b(n, 2) = n/(n-1)$).
- The estimators of the raw moments $\underline{\hat{\mu}}'_p$ are in theory unbiased*, but it is practically impossible to use them in estimation for $p > 2$:

  cf. Lombardo et al. (2014), "**Just two moments**".
- When dealing with maxima, two moments may not be enough, as the behaviour of maxima is strongly related to high-order moments.

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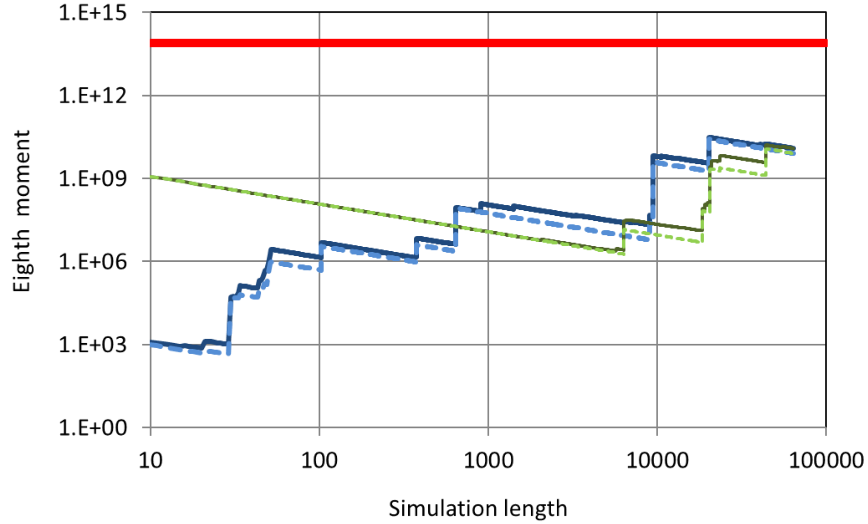
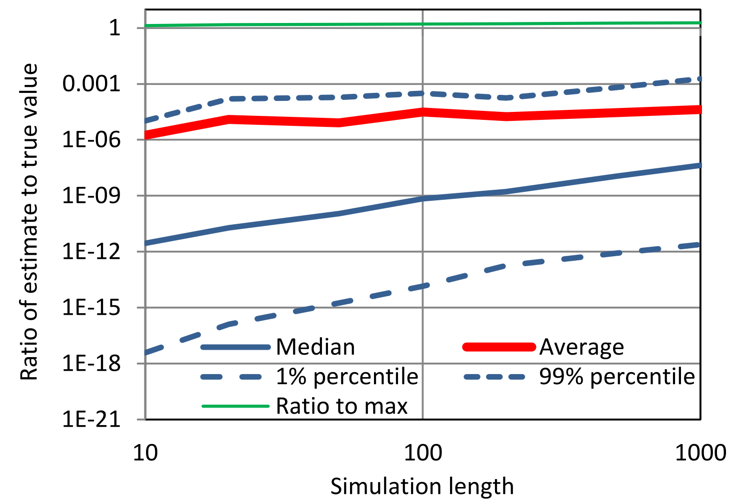
* Central moment estimators $\underline{\hat{\mu}}_p$ are also unbiased in the (uncommon) case that $\mu$ is a priori known, in which case it replaces $\hat{\mu}$ in the rightmost equation in (5), while $b(n, p)$=1.

# Illustration of slow convergence of moment estimates




Convergence of the sample estimate of the eighth non-central moment to its true value (thick horizontal line) corresponding to a lognormal distribution LN(0,1) where the process is an exponentiated Hurst-Kolmogorov process with Hurst parameter $H = 0.9$. The sample moments ($\sum_{i=1}^{n} x_i^p / n$ with $p = 8$; continuous lines), are estimated from a single simulation of length 64 000, subset to sample size $n$ from 10 to 64 000, with the subsetting being done either from the beginning to the end or from the end to the beginning. Dashed lines represent maximum values $(\max_{1\le i\le n}(x_i))^p / n$.

As in the example on the left but for 200 simulated series of length 1000 each. The sampling distribution of the eighth moment estimator $\sum_{i=1}^{n} x_i^8 / n$ is visualized by the percentiles, the median and the average, plotted as ratios to the true value. Theoretically, the ratio should be 1, but it is smaller by many orders of magnitude, and the convergence to 1 is very slow. (The convergence of the average could also be achieved if we used millions of simulated series instead of 200). In contrast, the ratio to $(\max_{1\le i\le n}(x_i))^8 / n$ is ≈1.

# The reason of slow convergence

- What is the result of **raising to a power and adding**, i.e. $\sum_{i=1}^{n} x_i^p$ – like in estimating moments?

| Linear, $p$ = 1 | Pythagorean, $p$ = 2 | Cubic, $p$ = 3 | High order, $p$ = 8 |
|---|---|---|---|
| 3 + 4 = 7 | $3^2 + 4^2 = 5^2$ | $3^3 + 4^3 = 4.5^3$ | $3^8 + 4^8 \approx 4^8$ |
| 3 + 4 +12 = 19 | $3^2 + 4^2 + 12^2 = 13^2$ | $3^3 + 4^3 + 12^3 = 12.2^3$ | $3^8 + 4^8 + 12^8 \approx 12^8$ |

- Symbolically, for relatively large $p$ the estimate of $\mu'_p$ is*:

$$\hat{\mu}'_p = \frac{1}{n}\sum_{i=1}^{n} x_i^p \approx \frac{1}{n}\left(\max_{1\leq i\leq n}(x_i)\right)^p \tag{6}$$

- Thus, for an unbounded variable $\underline{x}$ and for large $p$, we can conclude that $\underline{\hat{\mu}}'_p$ is **more an estimator of an extreme quantity**, i.e., the $n$th order statistic (the largest) raised to power $p$, **than an estimator of $\boldsymbol{\mu'_p}$**.
- Thus, unless $p$ is very small, $\boldsymbol{\mu'_p}$ **is not a *knowable* quantity**: we cannot infer its value from a sample. **This is the case even if *n* is very large**!
- Also, the various $\underline{\hat{\mu}}'_p$ are not independent to each other as they only differ on the power to which the maximum value is raised.

* This is precise if $x_i$ are positive; see also p. 39. Note that for large $p$ the term $(1/n)$ in the rightmost part of the equation could be omitted with a negligible error.

# Definition of K-moments

- To derive **knowable moments** for high orders $p$, in the expectation defining the $p$th moment we raise $(\underline{x} - \mu)$ to a **lower power** $q < p$ and for the remaining $(p - q)$ terms we **replace** $(\underline{x} - \mu)$ with $(2F(\underline{x}) - 1)$, where $F(x)$ is the distribution function. This leads to the following definition of the central *K-moment* of order $(p, q)$ (Koutsoyiannis, 2018):

$$K_{pq} \coloneqq (p - q + 1)\mathrm{E}\left[\left(2F\left(\underline{x}\right) - 1\right)^{p-q}(\underline{x} - \mu)^q\right] \tag{7}$$

- Likewise, we define the non-central K-moment of order $(p, q)$ as:

$$K'_{pq} \coloneqq (p - q + 1)\mathrm{E}\left[\left(F\left(\underline{x}\right)\right)^{p-q} \underline{x}^q\right] \tag{8}$$

- The quantity $\left(2F\left(\underline{x}\right) - 1\right)^{p-q}$ is estimated from a sample without using powers of $\underline{x}$. Specifically, for the $i$th element of a sample $x_{(i)}$ of size $n$, sorted in ascending order, $F(x_{(i)})$ and $(2F(x_{(i)}) - 1)$ are estimated as,

$$\hat{F}\left(x_{(i)}\right) = \frac{i-1}{n-1}, \quad 2\hat{F}\left(x_{(i)}\right) - 1 = \frac{2i-n-1}{n-1} \tag{9}$$

taking values in [0, 1] and [−1, 1], respectively, irrespective of the values $x_{(i)}$. Hence, the estimators are:

$$\underline{\hat{K}}'_{pq} = \frac{p-q+1}{n}\sum_{i=1}^{n}\left(\frac{i-1}{n-1}\right)^{p-q}\underline{x}_{(i)}^{q}, \quad \underline{\hat{K}}_{pq} = \frac{p-q+1}{n}\sum_{i=1}^{n}\left(\frac{2i-n-1}{n-1}\right)^{p-q}\left(\underline{x}_{(i)} - \hat{\mu}\right)^{q} \tag{10}$$

# Rationale of the definition

1. Assuming that the distribution mean is close to the median, so that $F(\mu) \approx 1/2$ (this is precisely true for a symmetric distribution), the quantity whose expectation is taken in (7) is $A(\underline{x}) \coloneqq \left(2F(\underline{x}) - 1\right)^{p-q} (\underline{x} - \mu)^q$ and its Taylor expansion is

$$A(\underline{x}) = \left(2f(\mu)\right)^{p-q} (\underline{x} - \mu)^p + (p - q)\left(2f(\mu)\right)^{p-q-1} f'(\mu)(\underline{x} - \mu)^{p+1} + O((\underline{x} - \mu)^{p+2}) \quad (11)$$

where $f(x)$ is the probability density function of $\underline{x}$. Clearly then, $K_{pq}$ depends on $\mu_p$ as well as classical moments of $\underline{x}$ of order higher than $p$. The **independence of $\boldsymbol{K_{pq}}$ from classical moments of order < $\boldsymbol{p}$ makes it a good knowable surrogate of the unknowable $\boldsymbol{\mu_p}$**.

2. As $p$ becomes large, by virtue of the multiplicative term $(p - q + 1)$ in definition (7), $K_{pq}$ shares similar asymptotic properties with $\hat{\mu}_p^{q/p}$ (the estimate, not the true $\mu_p^{q/p}$). To illustrate this for $q$ = 1 and for independent variables $\underline{x}_i$, we consider the variable $\underline{z}_p \coloneqq \max_{1 \le i \le p} \underline{x}_i$ and denote $f(\ )$ and $h(\ )$ the probability densities of $\underline{x}_i$ and $\underline{z}$, respectively. Then (Papoulis, 1990, p. 209):

$$h(z) = p f(z)\left(F(z)\right)^{p-1} \quad (12)$$

and thus, by virtue of (8),

$$\mathrm{E}\left[\underline{z}_p\right] = p\mathrm{E}\left[\left(F(\underline{x})\right)^{p-1} \underline{x}\right] = K'_{p1} \quad (13)$$

On the other hand, for positive $\underline{x}$ and large $p \to n$,

$$\left(\mathrm{E}\left[\underline{\hat{\mu}}'_p\right]\right)^{1/p} = \left(\mathrm{E}\left[\left(\frac{1}{n}\sum_{i=1}^{n} \underline{x}_i^p\right)\right]\right)^{1/p} \approx \left(\mathrm{E}\left[\frac{1}{n}\max_{1\le\, i \le n}\left(\underline{x}_i^p\right)\right]\right)^{1/p} \approx n^{-1/p}\mathrm{E}\left[\max_{1\le\, i \le n} \underline{x}_i\right] \approx \mathrm{E}\left[\underline{z}_n\right] \quad (14)$$

Note also that the multiplicative term $(p - q + 1)$ in definition (7) and (8) makes K-moments generally increasing functions of $p$.

# Asymptotic properties of moment estimates

- Generally, as $p$ becomes large approaching $n^*$, estimates of both classical and K moments, central or non-central, become estimates of expressions involving extremes such as $\left(\max_{1\le i\le p} x_i\right)^q$ or $\max_{1\le i\le p}(x_i-\mu)^q$. For negatively skewed distributions these quantities can also involve minimum, instead of maximum quantities.
- For the K-moments this is consistent with their theoretical definition. For the classical moments this is an inconsistency.
- A common property of both classical and K moments is that symmetrical distributions have all their odd moments equal to zero.
- For unbounded variables both classical and K moments are non-decreasing functions of $p$, separately for odd and even $p$.
- **In geophysical processes we can justifiably assume that the variance** $\mu_2 \equiv \gamma_1 \equiv \sigma^2 \equiv K_{22}$ **is finite** (an infinite variance would presuppose infinite energy to materialize, which is absurd). Hence, high order K-moments $K_{p2}$ will be finite too, even if classical moments $\mu_p$ diverge to infinity beyond a certain $p$ (i.e., in heavy tailed distributions).

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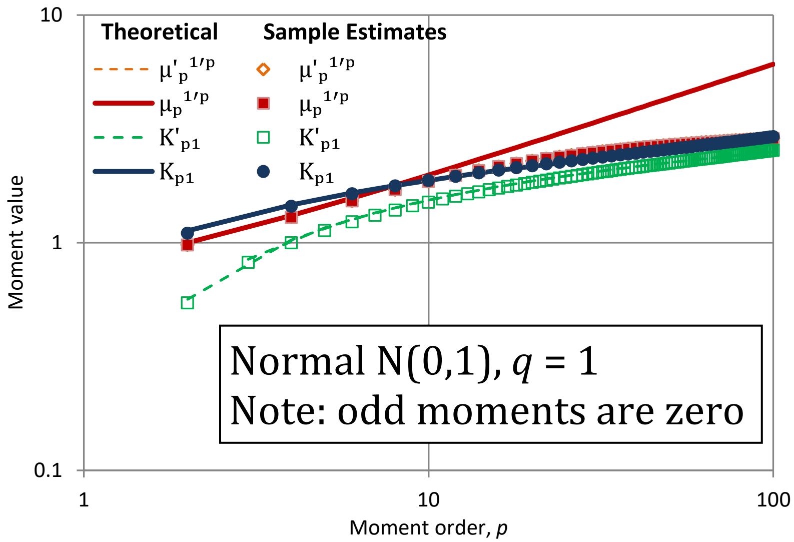
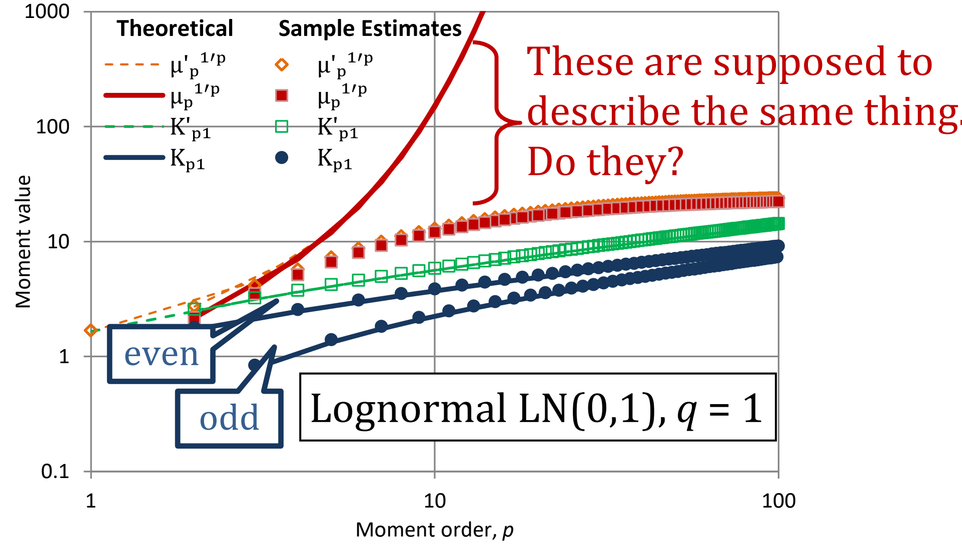
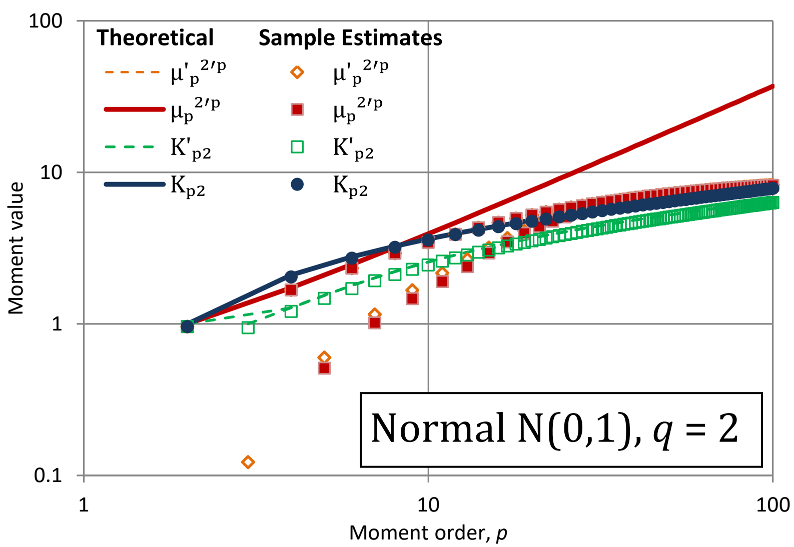
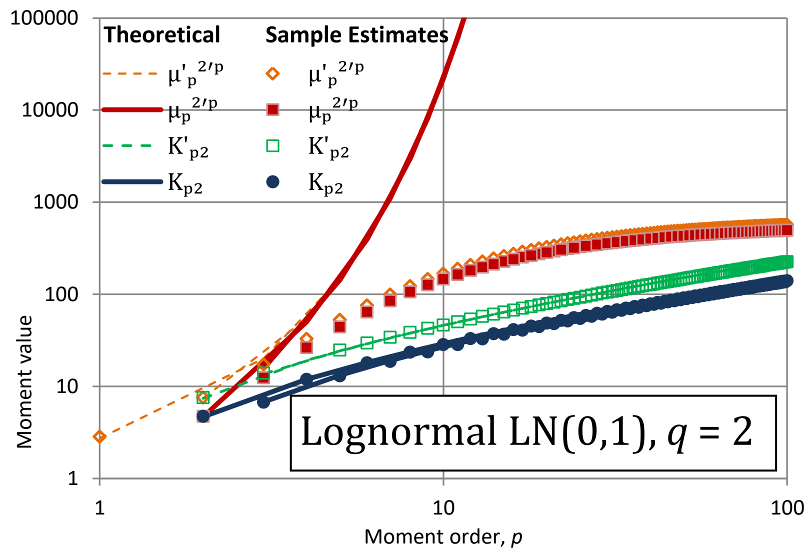
* It is possible to take $p > n$ and get a $K_{pq}$ value that is extrapolation beyond the maximum contained in the sample. In contrast, with the classical moments we cannot get any value beyond the sample maximum.

# Justification of the notion of *unknowable* vs. *knowable*




Note: Sample sizes are ten times higher than the maximum $p$ shown in graphs, i.e., 1000.

# Relationship among different moment types

- The **classical moments** can be recovered as a **special case of K-moments**: $M_p \equiv K_{pp}$. In particular, in **uniform distribution, classical and K-moments are proportional** to each other:

$$K'_{pq} \coloneqq (p-q+1)\mu'_p, \;\; K_{pq} \coloneqq (p-q+1)\mu_p \tag{15}$$

- The **probability weighted moments** (PWM) can also be recovered from the K- moments. The typical PWM form $\beta_p \coloneqq \mathrm{E}\left[\underline{x}\left(F(\underline{x})\right)^p\right]$ is a special case of K- moments corresponding to $q = 1$:

$$K'_{p1} = p\beta_{p-1} \tag{16}$$

- The **L-moments** are defined as $\lambda_p \coloneqq \frac{1}{p}\sum_{k=0}^{p-1}(-1)^k \binom{p-1}{k} \mathrm{E}\left[\underline{x}_{(p-k):p}\right]$, where $\underline{x}_{k:p}$ is the $k$th order statistic in a sample of size $p$. **L-moments are also related to PWM and through them to K-moments**. The relationships for the different types of moments for the first four orders are:

$$\begin{aligned}
K'_{11} &= \mu = \beta_0, & K_{11} &= 0 \\
K'_{21} &= 2\beta_1, & K_{21} &= 2(K'_{21}-\mu) = 4\beta_1 - 2\beta_0 = 2\lambda_2 \\
K'_{31} &= 3\beta_2, & K_{31} &= 4(K'_{31}-\mu) - 6(K'_{21}-\mu) = 12\beta_2 - 12\beta_1 + 2\beta_0 = 2\lambda_3 \\
K'_{41} &= 4\beta_3, & K_{41} &= 8(K'_{41}-\mu) - 16(K'_{31}-\mu) + 12(K'_{21}-\mu) \\
&& &= 32\beta_3 - 48\beta_2 + 24\beta_1 - 4\beta_0 = \frac{8}{5}\lambda_4 + \frac{12}{5}\lambda_2
\end{aligned} \tag{17}$$

- Both **PWM and L-moments** are better estimated from samples than classical moments but they are all of first order in terms of the random variable of interest. **PWM and L-moments** are good to characterize independent series or to infer the marginal distribution of stochastic processes, but they **cannot characterize** even **second order dependence** of processes; **K-moments can**.

# Characteristics a marginal distribution using K-moments

- Within the framework of K-moments, while respecting the rule of thumb "Just two moments" in terms of the power of $\underline{x}$, i.e. $q$ = 1 or 2, we can obtain knowable statistical characteristics for much (even enormously) higher orders $p$.
- In this manner, for $p$ > 1 we have two alternative options to define statistical characteristics related to moments of the distribution, as in the table below. (Which of the two is preferable depends on the statistical behaviour, and in particular, the mean, mode and variance, of the estimator.)

| Characteristic | Order $p$ | Option 1 | Option 2 | Option 3* |
|---|---|---|---|---|
| Location | 1 | $K'_{11} = \mu$ (the classical mean) | | |
| Variability | 2 | $K_{21} = 2(K'_{21} - \mu) = 2\lambda_2$ | $K_{22} = \mu_2 = \sigma^2$ (the classical variance) | |
| Skewness (dimensionless) | 3 | $\dfrac{K_{31}}{K_{21}} = \dfrac{\lambda_3}{\lambda_2}$ | $\dfrac{K_{32}}{K_{22}}$ | $\dfrac{K_{33}}{K_{22}^{3/2}} = \dfrac{\mu_3}{\sigma^3}$ |
| Kurtosis (dimensionless) | 4 | $\dfrac{K_{41}}{K_{21}} = \dfrac{4}{5}\dfrac{\lambda_4}{\lambda_2} + \dfrac{6}{5}$ | $\dfrac{K_{42}}{K_{22}}$ | $\dfrac{K_{44}}{K_{22}^2} = \dfrac{\mu_4}{\sigma^4}$ |

* Option 3 is based on the classical moments and is not recommended for distribution fitting.

# Statistical behaviour of variability, skewness and kurtosis indices

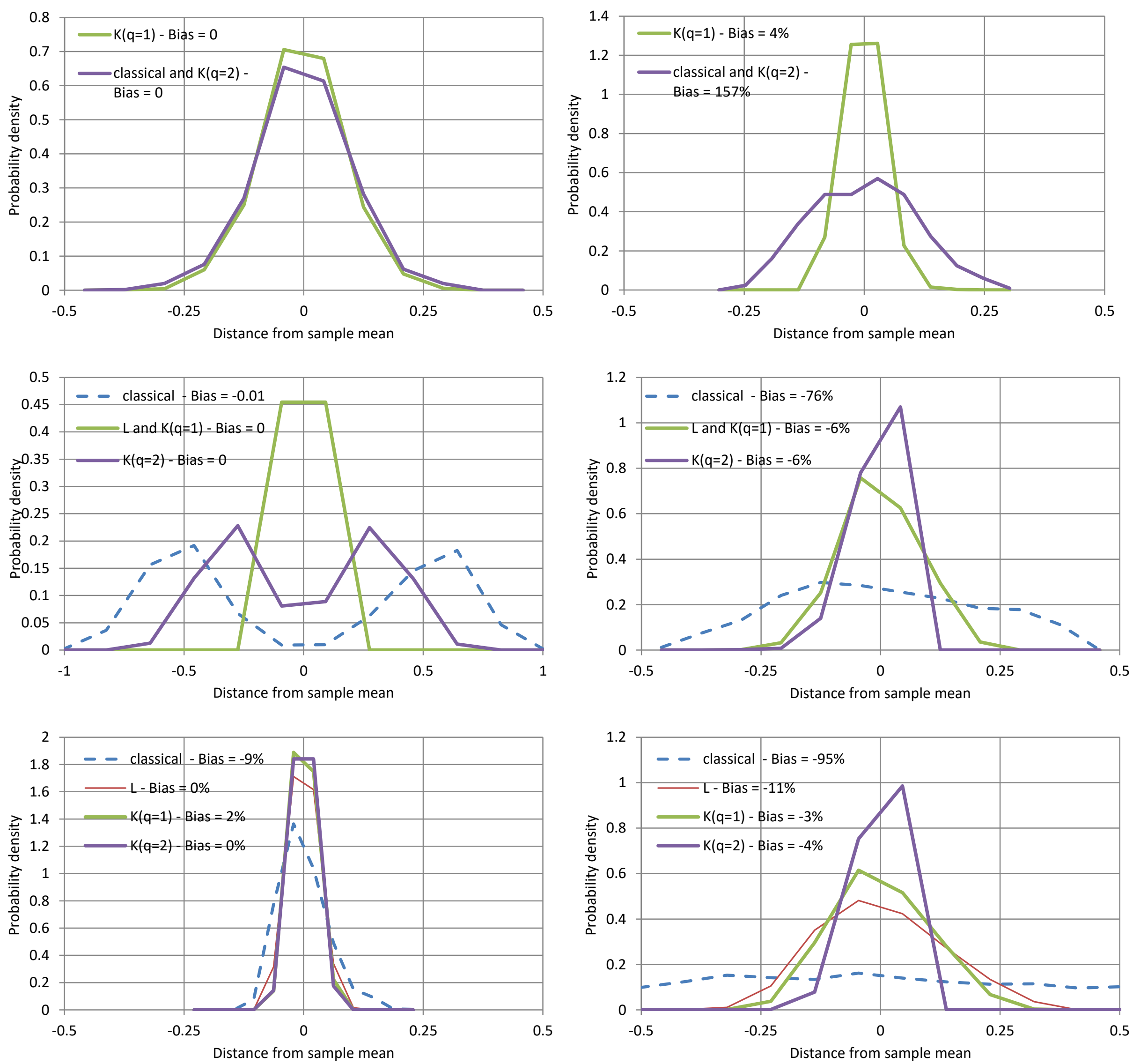



Illustration of the probability density function of:

**(upper) variability index** ($K_{11}/K_{21}$, $\mu/\sigma \equiv K_{11}/\sqrt{K_{22}}$; note that the latter is inverse of the common coefficient of variation);

**(middle) skewness index** ($\mu_3^{1/3}/\sigma$, $K_{31}/K_{21}$, $\operatorname{sign}(K_{32})\sqrt{|K_{32}|/K_{22}}$);

**(lower) kurtosis index** ($\mu_4^{1/4}/\sigma$, $\lambda_4/\lambda_2$, $K_{41}/K_{21}$, $\sqrt{K_{42}/K_{22}}$).

The panels of the left column correspond to the normal distribution N(0,1) and those of the right column to the lognormal distribution LN(0, 2).

# High order moments for stochastic processes: the K-climacogram and the K-climacospectrum

- The full description of the third-order, fourth-order, etc., properties of a stochastic process requires functions of 2, 3, …, variables.
- For example, the **third order properties are expressed in terms of the two-variable function**:

$$c_3(h_1, h_2) := \mathrm{E}[(\underline{x}(t) - \mu)\ (\underline{x}(t + h_1) - \mu)\ (\underline{x}(t + h_2) - \mu)] \quad (18)$$

- Such a description is **not parsimonious and its accuracy holds only in theory**, because sample estimates are not reliable.
- This problem is remedied if we **introduce single-variable descriptions for any order** $p$, expanding the idea of the climacogram and climacospectrum based on K-moments.

K-climacogram: $$\gamma_{pq}(k) = (p - q + 1)\mathrm{E}\left[\left(2F\left(\underline{X}(k)/k\right) - 1\right)^{p-q}\left(\underline{X}(k)/k - \mu\right)^q\right] \quad (19)$$

K-climacospectrum: $$\zeta_{pq}(k) = \frac{k\left(\gamma_{pq}(k) - \gamma_{pq}(2k)\right)}{\ln 2} \quad (20)$$

where $\gamma_{22}(k) \equiv \gamma(k)$ and $\zeta_{22}(k) \equiv \zeta(k)$.

- While the standard climacogram $\gamma_{22}(k) \equiv \gamma(k)$ provides a description precisely equivalent to the classical, this is not the case for $q > 2$. In this case, the single-variable K-climacogram description is obviously not equivalent to the multivariate high-order one. However, it suffices to define the marginal distribution at any scale $k$.

# Part E
# The ombrian curves

# The concept of ombrian curves

- An **ombrian relationship** or **ombrian curve** (from the Greek 'όμβρος', rainfall) is a mathematical relationship estimating the average rainfall intensity $i$ over a given time scale $d$ for a given return period $T$.
- Ombrian curves are a major tool in hydrologic design.
- They are more widely known by the misnomer rainfall **intensity-duration-frequency (IDF)** curves (the "duration" is not duration but time scale and the "frequency" is not frequency but return period, or inverse frequency).
- Several forms of ombrian relationships are found in the literature, most of which have been empirically derived and validated by the long use in hydrologic practice. However it has been shown (Koutsoyiannis et al. 1998) that the empirical considerations usually involved in the construction of ombrian curves are not at all necessary, and create difficulties and confusion.
- Attempts to give them a theoretical basis (e.g. in multifractal literature) have often used inappropriate assumptions and resulted in oversimplified and inaccurate relationships.
- In fact, as shown in Koutsoyiannis et al. (1998), an ombrian relationship is none other than a family of distribution functions of rainfall intensity for multiple time scales. This is because, the return period is tied to the distribution function, i.e., $T = \mathit{\Delta}/(1 - F(x))$, where $\mathit{\Delta}$ is the mean interarrival time of an event that is represented by the variable $\underline{x}$. When annual maxima are examined, $\mathit{\Delta}$ = 1 year, while when the complete time series (at time step $d$) is analysed, $\mathit{\Delta} = d$.
- Thus, the distribution function is at the same time an ombrian relationship, once generalized for a multitude of time scales.
- The marginal distribution and the dependence structure of the rainfall process, once known, determine the form of the ombrian curves. However their mathematical expression may be difficult to derive from the stochastic structure of the process.

# Basic assumptions for expressing ombrian curves

1. **Separability** of the influences of return period and time scale (Koutsoyiannis et al., 1998), i.e.,

$$i(d,T) = \frac{a(T)}{b(d)} \tag{21}$$

   where $a(T)$ and $b(d)$ are mathematical expressions to be determined.

2. **Pareto distribution** for the rainfall intensity over some threshold at any time scale (it corresponds to EV2 distribution of maxima). This readily provides a simple expression for $a(T)$ of the form $\lambda((T/\Delta)^{\kappa} - \psi)$, where $\lambda$ and $\psi$ are scale and location parameters of the Pareto distribution, while $\kappa$ is the shape parameter (Koutsoyiannis et al., 1998, Koutsoyiannis, 2004a,b).

3. **General power law expression** of $b(d)$, i.e., $b(d) \sim (1 + d/\theta)^{\eta}$, where $\theta > 0$ and $\eta > 0$ are parameters.

Based on assumptions 1-3, we deduce that the final form of the ombrian relationship is

$$i(d,T) = \lambda \frac{(T/\Delta)^{\kappa} - \psi}{(1 + d/\theta)^{\eta}} \tag{22}$$

Equation (22) is dimensionally consistent, provided that $\theta$ and $\Delta$ have units of time, $\lambda'$ has units of intensity, and $\kappa$ and $\psi$ are dimensionless.

# Justification of the assumptions

- We first note that the assumptions are **not valid for the entire ranges of scales and distribution quantiles** as a fully consistent multiscale distribution function may be too complicated. They provide satisfactory approximations for the ranges of scales and quantiles which are **of interest in engineering studies**. Specifically they are good for relatively small time scales, up to several days, and for the distribution tails.
- The **Pareto** distribution is satisfactory for the **distribution tail** and for relatively small scales. This has theoretical support based on maximum entropy considerations (Koutsoyiannis, 2005) as well as extended empirical backing (Koutsoyiannis, 2004b; Papalexiou and Koutsoyiannis, 2006; Koutsoyiannis and Papalexiou, 2017).
- Once the Pareto distribution is assumed, the shape parameter $\kappa$ should be the same for all time scales $d$. This can be understood considering that the moments of the distribution of order > $1/\kappa$ are infinite and if they are infinite for one time scale $d_1$ they **should be infinite for any other time scale** $d$. This is consistent with the assumption of separability.
- The general power law expression $b(d) \sim (1 + d/\theta)^\eta$ is more consistent in comparison to the frequently used simple power law expression $b(d) \sim d^\eta$. The latter entails infinite intensity of the instantaneous process ($d$ = 0), which would require infinite energy to materialize, and hence it is absurd. Further justification of the general power law expression, based on maximum entropy considerations, can be found in Koutsoyiannis (2006b).

# Notes on parameter estimation methods

- Ombrian curve fitting is equivalent to probability distribution fitting. However, because **ombrian curves focus on a specific part of the distribution** (the tail and not the body) typical fitting methods (moments, L-moments, maximum likelihood) are not suitable.
- Consistent parameter estimation techniques for ombrian relationships have been discussed in Koutsoyiannis et al. (1998). Two methods have been proposed, the **one-step least squares method** and a **two-step procedure**, where in the first the parameters of the **denominator** of (22) are estimated so that, the quantities $i(d, T)(1 + d/\theta)^{\eta}$ have the same distribution for all time scales $d$ and in the second step the parameters of the **numerator** of (22) are estimated.
- These methods make use of **order statistics** of the statistical sample. By definition the order statistic $\underline{x}_{(i)}$ is equal to the $i$th-smallest value of the sample of size $n$, while the (unbiased) estimate of its distribution function is $\hat{F}\left(x_{(i)}\right) = \ i/(n+1)$.
- Likewise, the estimate of the non-exceedence probability of the $j$th-largest value, $\underline{U}_j \coloneqq \underline{x}_{(n+1-j)}$ will be $1 - \hat{F}\left(x_{(n+1-j)}\right) = 1 - \hat{F}(\underline{U}_j) = \ j/(n+1)$.
- As an alternative to order statistics, below we develop **a methodology based on the K-climacogram**.
- The statistical sample used to estimate parameters can be of two types:
  (a) **complete series of rainfall intensities at a time step** $D$ for a large period of $N$ years; the total number of data values is $n = N\Delta/D$, where $\Delta = 1$ year, but a portion of them (the largest $N$ or so) are used in the fitting;
  (b) time block (typically **annual**) **maxima**, e.g. one value per year forming a sample of $N$ values.
- Eventually, the ombrian curves should correspond to the actual rainfall process, naturally represented in the sample of type (a). Therefore, if a sample of type (b) is used, an adaptation of the distribution function is required (see Koutsoyiannis et al., 1998, for details); in particular, the distribution function to be fitted should be EV2 instead of Pareto.

# Data requirements for parameter estimation

1. The parameter $\theta$, which is typically smaller than 1 h, needs sub-hourly data to be estimated. These can be provided by observations from autographic rain recorders with high temporal resolution, or from digital sensors with sub-hourly time step. Without such data the estimated $\theta$ tends to approach zero.
2. For the estimation of the parameter $\eta$, data for hourly or multi-hour time step can also be quite useful.
3. The parameters of the numerator of equation (22) are better deduced from daily raingage data rather than from autographic rain recorder data, because the latter are generally available for shorter periods and are more susceptible to measurement errors.
4. For the parameters $\psi$ and $\lambda$ of the numerator (which are, respectively, the location and scale parameters of the Pareto/EV2 distributions), daily raingage data of an adequate length (of several decades) usually suffice for a reliable estimation.
5. Finally the parameter $\kappa$ (which is the shape parameter of the Pareto/EV2 distributions), unless the period of observations is very large, should be estimated based on multi-station data of the area, or be assumed independently of data.
   In particular, the literature supports a typical value of $\kappa = 0.15$ (Koutsoyiannis, 2004b), if a method based on least squares on order statistics is used, while the L-moments method results in lower estimates ($\kappa = 0.10$ in Koutsoyiannis, 2004b or $\kappa = 0.114$ in Papalexiou and Koutsoyiannis, 2013).
   An absolute minimum value is $\kappa = 0$, for which the Pareto/EV2 distribution becomes Exponential/EV1 and equation (22) becomes

$$i(d,T) = \lambda \frac{\ln(T/\Delta) - \psi}{(1 + d/\theta)^{\eta}} \tag{23}$$

# Ombrian curves as K-climacograms

- In consistency with (8) and (19), we can define the non-central K-climacogram $\gamma'_{p1}(k)$ as:

$$\gamma'_{p1}(k) = p\mathrm{E}\left[\left(F\left(\underline{X}(k)/k\right)\right)^{p-1} (\underline{X}(k)/k)^q\right] \tag{24}$$

- By definition, it represents the expected value of the maximum of $p$ realizations of a process averaged on time-scale $k$. Therefore, for appropriate $p$ determined in connection to the return period $T$, and for time scale $k = d$, $\gamma'_{p1}(d)$ represents the ombrian curve $i(d, T)$.
- To determine the theoretical return period $T(\gamma'_{p1}(d))$ we introduce the ratio $\Lambda_p$ which happens to vary very slightly with $p$:

$$T(\gamma'_{p1}(d)) = \frac{d}{1-F\left(\gamma'_{p1}(d)\right)}, \quad \Lambda_p := \frac{T(\gamma'_{p1}(d))}{d\ p} = \frac{1}{p\left(1-F\left(\gamma'_{p1}(d)\right)\right)} \tag{25}$$

- The slight variation of $\Lambda_p$ with $p$ can be very well approximated if we first accurately determine from (24) the specific values $\Lambda_1$ and $\Lambda_\infty$. For the approximation of $\Lambda_p$ we use any of the following relationships:

$$\Lambda_p \approx \Lambda_\infty \left(\frac{\Lambda_1}{\Lambda_\infty}\right)^{\frac{1}{p^c}}, \quad \Lambda_p \approx \Lambda_\infty + (\Lambda_1 - \Lambda_\infty)\frac{1}{p^c} \tag{26}$$

where $c$ is a constant depending on the distribution function with default value $c = 1$.

# Exact and approximate relationships between $p$ and $T$

- For given $p$ and distribution function $F\big(\underline{X}(k)/k\big)$, $\gamma'_{p1}(k)$ is theoretically determined from (24); then $T(\gamma'_{p1}(d))$ is determined from (25) (left part). In absence of analytical solutions, we can establish an exact relationship between $p$ and $T$ by doing numerical calculations for several $p$.
- Alternatively, we can make exact calculations only for $\Lambda_1$ and $\Lambda_\infty$ (the latter as a limit for large $p$) and use the approximate equations (26) for any $p$. The table below provides such results.

*Table for $\Lambda_1$ and $\Lambda_\infty$ for customary distributions (along with some exact relationships for $\Lambda_p$).*

| Distribution | Distribution | $\Lambda_1$ | $\Lambda_\infty$ | Exact relationship |
|---|---|---|---|---|
| Normal | $f(x) = \frac{\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}{\sqrt{2\pi}\,\sigma}$ | $\Lambda_1 = 2$ | $\Lambda_\infty = \mathrm{e}^{1/2} = 1.649^{\S}$ | |
| Exponential | $f(x) = \mathrm{e}^{-x/\beta}/\beta$ | $\Lambda_1 = \mathrm{e} = 2.718$ | $\Lambda_\infty = \mathrm{e}^{\gamma} = 1.781$ | $\Lambda_p = \mathrm{e}^{H_p}/p$ |
| Gamma | $f(x) = \frac{x^{\alpha-1}\mathrm{e}^{-x/\beta}}{\beta^{\alpha}\,\Gamma(\alpha)}$ | $\Lambda_1 = \frac{\Gamma(\alpha)}{\Gamma_\alpha(\alpha)}$ | $\Lambda_\infty = \mathrm{e}^{\gamma} = 1.781^{\S}$ | |
| Pareto | $F(x) = 1 - \left(1 + \kappa\left(\frac{x}{\beta} - \psi\right)\right)^{-\frac{1}{\kappa}}$ | $\Lambda_1 = \left(\frac{1}{1-\kappa}\right)^{\frac{1}{\kappa}}$ | $\Lambda_\infty = \left(\frac{\pi}{\sin(\kappa\,\pi)\Gamma(\kappa)}\right)^{\frac{1}{\kappa}}$ | $\Lambda_p = \left(\frac{\pi}{\sin(\kappa\,\pi)\mathrm{B}(\kappa, p+1-\kappa)}\right)^{\frac{1}{\kappa}}\frac{1}{p}$ |
| Lognormal* | $f(x) = \frac{\exp\left(-\frac{(\ln(x/\beta))^2}{2\sigma^2}\right)}{\sqrt{2\pi}\,\sigma\,x}$ | $\Lambda_1 = \frac{2}{\mathrm{erfc}(\sigma/2^{3/2})}$ | $\Lambda_\infty \approx 1.31\Lambda_1^{0.4}$ | |

*For the lognormal distribution, the value $c = \Lambda_\infty/\Lambda_1$ gives a better approximation than the default value $c = 1$.
§To be cross-checked.

Explanations of symbols: $\gamma = 0.5772$ the Euler constant; $\Gamma(\ )$ the gamma function; $\Gamma_\alpha(\ )$ the incomplete gamma function; erfc( ) the complementary error function; $H_p \coloneqq \sum_{i=1}^{p} 1/i$ the harmonic number.

# Practical considerations for the relationships between *p* and *T*

- Depending on the required degree of accuracy, we can determine the moment order $p$ for a specified return period $T$ by either of the following relationships (with the first being very rough and the last being exact).

$$p^{(A)} = \frac{T}{2d}, \quad p^{(B)} = \frac{T}{\Lambda_\infty d} - \frac{\Lambda_1}{\Lambda_\infty} + 1, \quad p^{(C)} = p(T/d, \boldsymbol{\alpha}) \tag{27}$$

  where $\boldsymbol{\alpha}$ is a vector of the shaper parameters of the distribution function (the location and scale parameters should not affect the relationship between $p$ and $T$).

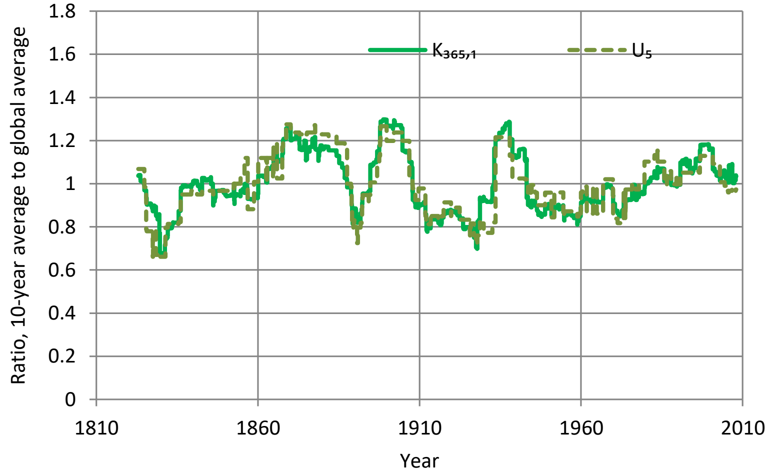
- The rationale of $p^{(A)}$ is that the table in the previous page supports a rough approximation (for preliminary estimates) of $\Lambda_p \approx \Lambda_1 \approx \Lambda_\infty \approx 2$. As an example, any symmetric distribution for $p$ = 1 will give exactly $\Lambda_1$= 2 because $\gamma'_{11}(k)$ is the mean, which because of symmetry is equal to the median and thus has a return period of $2d$.
- The justification for $p^{(B)}$ is that it is readily derived combining (25) (right part) and (26) (right part with the default $c$ = 1).

*Example with comparison of the three options for Pareto distribution with shape parameter κ = 0.15.*

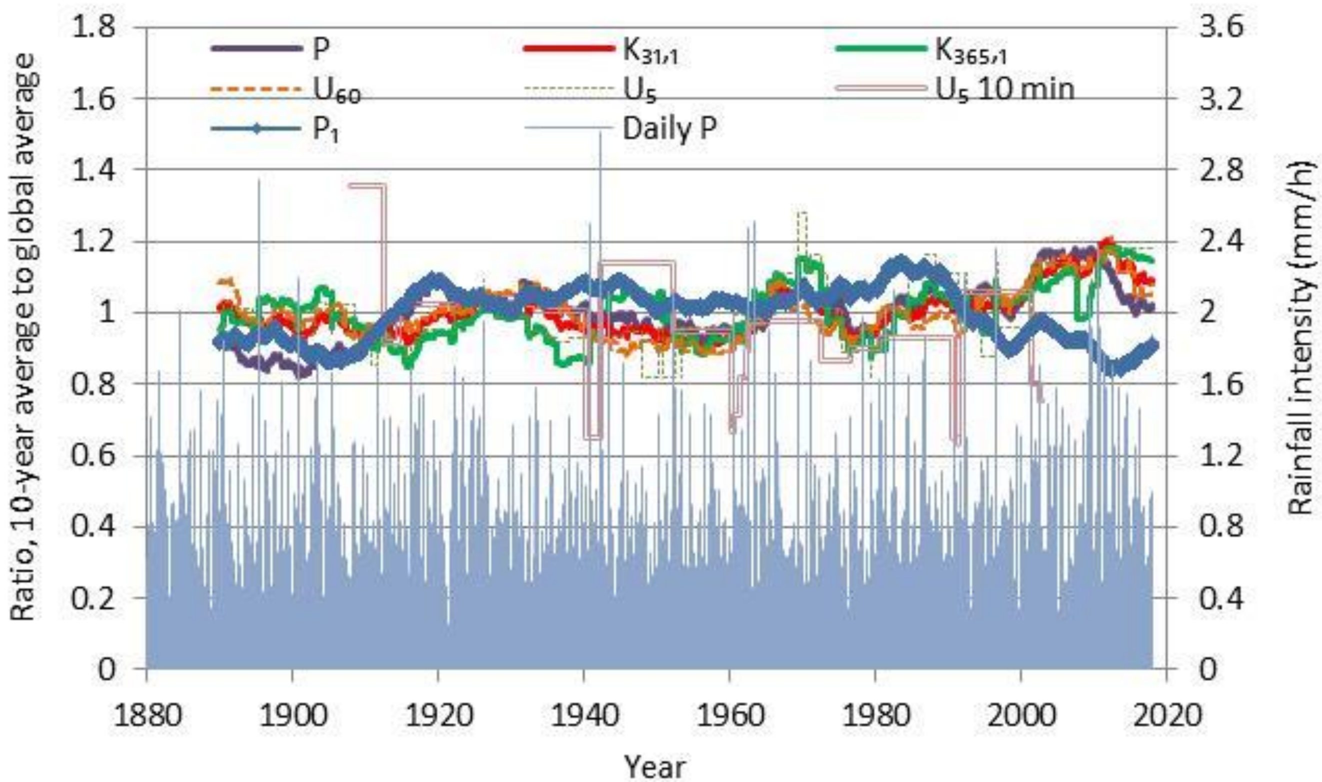
| | $d$ = 10 min | | | $d$ = 1 h | | | $d$ = 1 d | | |
|---|---|---|---|---|---|---|---|---|---|
| | $p^{(A)}$ | $p^{(B)}$ | $p^{(C)}$ | $p^{(A)}$ | $p^{(B)}$ | $p^{(C)}$ | $p^{(A)}$ | $p^{(B)}$ | $p^{(C)}$ |
| $T$ = 2 months | 4 383 | 4 307 | 4307 | 731 | 717 | 717 | 30 | 29 | 29 |
| $T$ = 1 year | 26 298 | 25 842 | 25 842 | 4 383 | 4 307 | 4 307 | 183 | 179 | 179 |
| $T$ = 2 years | 52 596 | 51 684 | 51 684 | 8 766 | 8 614 | 8 614 | 365 | 358 | 358 |
| $T$ = 100 years | 2 629 800 | 2 584 212 | 2 584 212 | 438 300 | 430 702 | 430 702 | 18 263 | 17 945 | 17 945 |

# Advantages of using K-climacogram in ombrian curve fitting

- Generally, the **K-moments** contain **similar information with order statistics**. However, the **order statistics** can be evaluated only at a **limited number of points** as $\hat{F}(U_j) = j/(n+1)$, $j$ = 1, …, $n$. For example, assuming a sample covering a period of 100 years, the largest order statistic corresponds to a return period of 101/1 = 101 years and the next one to a return period of 101/2 = 50.5 years, while no information can be extracted from the sample for return periods between 50.5 and 101 years . In contrast, there is **no such limitation in using K-moments**, where, referring to this example, we can make estimates of maxima for any return period, by appropriately specifying the order $p$ as detailed above.
- Even for smaller return periods, the K-moments admit a more detailed representation of the behaviour of maxima, in comparison to order statistics, whose variation appears in the form of steps (see figure below, which contains those of the curves appearing in the figure in p. 25 which correspond to return period of 2 years; see also graph on p. 55).
- In contrast to **classical moments, whose estimates are reliable only for very low orders** (usually 1 and 2, and, for very large sample sizes, 3 to 4), the **K-moments can give reliable estimates** for orders of **several millions** (see graph on p. 56), such as those appearing in the table of the previous page.
- Similar to classical moments and unlike order statistics whose estimation depends on one data value only, **K-moments represent the entire data set** (with bigger weights on larger values); this is consistent with the "Save the observations" principle (Volpi et al. 2018).



# Application to the Uccle climatological station, Belgium



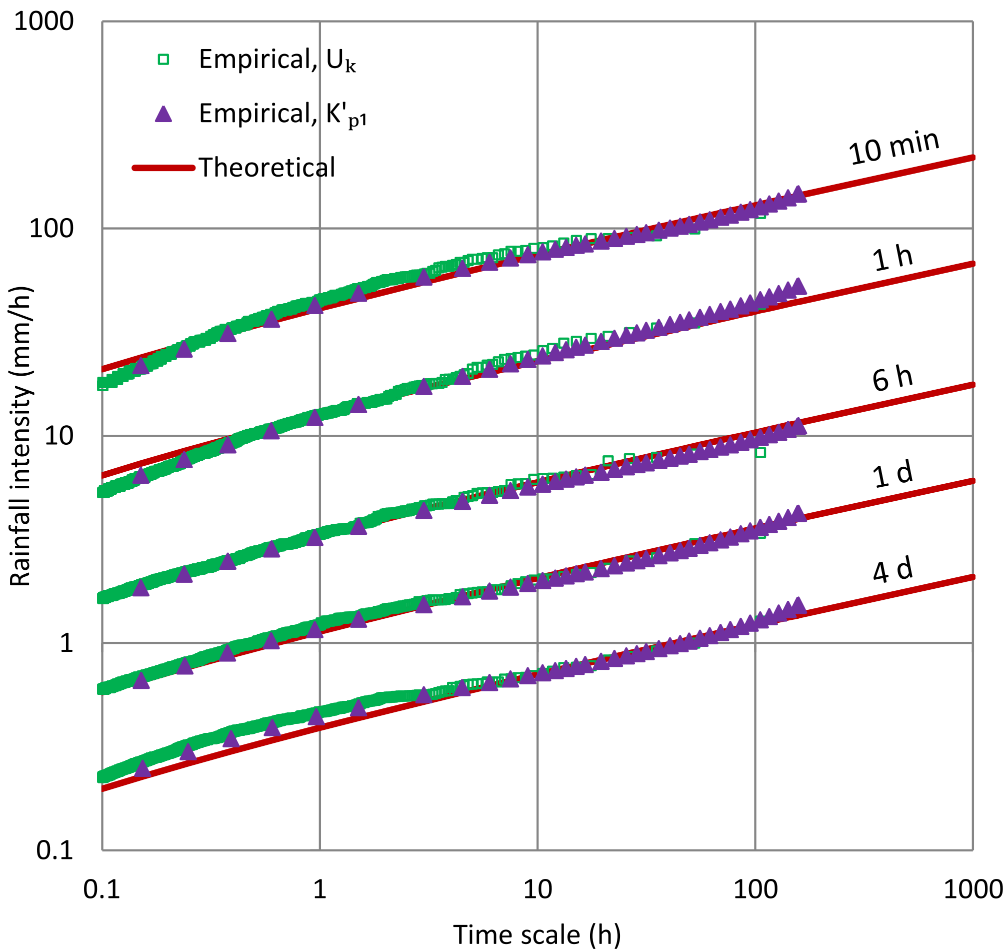

**Data set**: The long-term 10-min rainfall series at the climatological station of the Royal Meteorological Institute of Belgium at Uccle (1898-2002) (Demarée, 2003, De Jongh et al., 2006, Ntegeka and Willems, 2008).

The data are homogeneous (same location, same measuring instrument and measuring accuracy, identical quality of processing since 1898) and without gaps.

The daily data of the station plotted above (available at http://climexp.knmi.nl/data/bpeca17.dat) is longer, covering the period 1880-2018. The daily data are fully consistent with the 10-min data.

The graph, based on the daily rainfall record, which is similar to that of Bologna in p. 25 (with an additional plot of the order statistics for 10 min rainfall), shows the typical fluctuations at the 10-year scale, but indicates smaller variation than in Bologna (i.e., smaller Hurst parameter).

# Ombrian curves at Uccle




Ombrian curve expression:

$$i(d, T) = (137.2 \text{ mm/h}) \, \frac{(T/(1 \text{ year}))^{0.217} - 0.2}{(1 + d/(0.065 \text{ h}))^{0.773}}$$

The ombrian curves have been fitted based on the K-climacogram $\gamma'_{p1}(d)$, evaluated at several return periods up to ~150 years (extrapolating beyond the observation period of 105 years). The entire data set was used in a single optimization step, with the objective function being the overall square error of the fitting to be minimized.

The graph shows the obtained ombrian curves in comparison to empirical estimates of $K'_{p1}$ and order statistics $U_k$. The two series of points are in good agreement to each other and to theoretical curves.

**Note**: It is clarified that the analyses and the final result refer to specified time scales $d$ regularly formed, without considering the case of a moving time window and “optimizing” its location so as to obtain the maximum possible $U_k$ or $K_{p1}$. This problem is not covered here.

# Concluding remarks

- Awareness of stochastics is important in analysing and modelling geophysical processes.
- Nature is perpetually changing, yet stochastics describe it efficiently in terms of stationary stochastic processes.
- So-called "nonstationary" analyses of rainfall and flood extremes are not actually nonstationary, as they do not describe change in causative deterministic terms; their use is dangerous as they underestimate the actual uncertainty that is related to long-term variations.
- Statistical description of extremes is related to high-order classical moments but in fact these are unknowable for typical geophysical samples.
- The recently introduced knowable moments (K-moments and K-climacograms), which constitute alternatives to both classical moments and order statistics, can support the consistent stochastic modelling of extremes and the fitting of distributions of extremes or distribution tails.
- A simple and consistent formula based on justified assumptions provides a possibly universal frame for ombrian (intensity – time scale – return period) curves.
- The case study using a 105-year long uninterrupted rainfall record at a 10-min time step shows an impressive fitting of the general ombrian curve and illustrates the usefulness of the K-climacogram in its construction and visualisation.

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