ADVANCES IN STOCHASTICS OF HYDROCLIMATIC EXTREMES

PROGRESSI NELLA STOCASTICA DEGLI EVENTI IDROCLIMATICI ESTREMI

The 21st century has been marked by a substantial progress in hydroclimatic data collection and access to them, accompanied by regression in methodologies to study and interpret the behaviour of natural processes and in particular of extremes thereof. The developing culture of prophesising the future, guided by deterministic climate modelling approaches, has seriously affected hydrology. Therefore, aspired advances are related to abandoning the certainties of deterministic approaches and returning to stochastic descriptions, seeking in the latter theoretical consistency and optimal use of available data.

Keywords: Precipitation Extremes, Temperature Extremes, Stochastics, Knowable Moments.

Il XXI secolo è stato segnato da un sostanziale progresso nella raccolta e nell’accesso ai dati idroclimatici, accompagnato da una regressione nelle metodologie per studiare e interpretare il comportamento deiprocessi naturali e in particolare dei loro estremi. La cultura della previsione che avanza, guidata da approcci deterministici di modellazione del clima, ha seriamente influenzato l’idrologia. Perciò i progressi ai quali tende sono legati all’abbandono delle certezze degli approcci deterministici e al ritorno alle descrizioni stocastiche, mirando in queste alla ricerca della coerenza teorica e all’uso ottimale dei dati disponibili.


1. INTRODUCTION

One would assume that after Baldo Bacchi’s seminal contributions to the stochastics of hydrological extremes (Bacchi et al., 1992, 1994; Bacchi and Ranzi, 1996; Balistrocchi and Bacchi, 2011), there must have been several steps forward. Interestingly, however, the steps backward have prevailed. Specifically, there has been a cataclysm of hydrological studies prophesizing future extremes based on climate model projections, as if the latter were credible. However, the fact is that they do not pass reality tests (Koutsoyiannis et al. 2008, 2011; Anagnostopoulos et al., 2010; Tsaknias et al., 2016; Tyralis and Koutsoyiannis, 2017). Hence, their use in hydrological studies is not justified. A single example is depicted in Figure 1, reproduced from the study by Tsaknias et al. (2016), who tested the reproduction of extreme events by three climate models of the IPCC AR4 at 8 test sites in the Mediterranean with long time series of temperature and precipitation. It is evident that climate models are not able even to approach the natural behaviour in extreme events. In terms of the probabilistic distribution of extremes, the real rainfall intensities are five times (500%) higher that what is simulated by climate models (Figure 1, panels of the lower row), yet the modellers advise us to expect intensification of extremes of the order of 10% due to warming climate. In other words, while they err by 500%, they are able to discern a difference of 10%.

The second step backward is the promotion of the strange idea that “stationarity is dead” (Milly et al., 2008), an idea which proved to have a very high contagious rate (notably, not affecting Baldo Bacchi), triggering a quest of “trends everywhere” and resulting in a surge of related articles. Specifically, Iliopoulou and Koutsoyiannis (2020) have found that in 2018, among the scientific articles, registered by Google Scholar that contained the terms “precipitation”, “hydrology” and “extremes”, 89% also contained the word “trends”. While there has been a rising trend in the frequency of the word “trends” since 1960, near 2010 there has been an

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upward shift in that frequency (Figure 2). However, the entire popular exercise is based on a misinterpretation of the meaning of stationarity and nonstationarity and a confusion of the concepts of stationarity and dependence (Koutsoyiannis and Montanari, 2015; Koutsoyiannis, 2020a). Both stationarity and ergodicity are abstract mathematical concepts; hence they are immortal (Montanari and Koutsoyiannis, 2014).

2. REVERSING CURRENT TRENDS AND RETURNING TO A CONSISTENT STOCHASTIC FRAMEWORK

As discussed in the Introduction, the recent setbacks have their origin in the belief that the climate models can reliably represent the hydrological cycle and predict its future changes. However, a recent study on global scale by Koutsoyiannis (2020b) has shown that this is not the case. The standard hypotheses related to the presence of atmospheric water and the expectations, based on climate model results, of the intensification of the hydrological cycle and particularly the extreme events, are not confirmed by global-scale data from reanalyses and satellites.

If climate models were able to provide some useful information, it would be possible to incorporate it in a decent stochastic framework. Specifically, Tyralis and Koutsoyiannis (2017) have established a Bayesian framework to convert deterministic climate model predictions into formal stochastic ones. The underlining idea is that if the deterministic forecasts are good, then the Bayesian framework proposed takes them into account, otherwise they are automatically disregarded. The results of applying this framework to the average precipitation in a large area, namely the contiguous United States, which has the richest observation network, showed that conditioning on climate model outputs does not affect the results of a stationary stochastic model build on the basis of the real-world data. The reason is that the climate model results are irrelevant to reality and thus the framework...
effectively disregards them, as it should. Therefore, climate model predictions (or projections) can hardly justify a reason to be incorporated in studies of real-world processes.

The statistical counterpart of the futuristic endeavour, namely the fitting of trends based on data and projecting them to the future, was examined recently in a comprehensive study by Iliopoulou and Koutsoyiannis (2020), based on long precipitation series (> 150 years). The study compared four cases of projection to the future, namely (a) the mean estimated from the entire record, (b) a local time average estimated from the recent past, (c) a linear trend fitted to the entire record, and (d) a local linear trend estimated from the recent past. The process mean is a neutral predictor of the future (zero efficiency) but turns out to be better than predictions based on trends. In other words, the predictive skill of the trend models is poor, worse than using the mean. The model based on the local time average (case b) of previous years proves to be the best of the four. The reason is that in real-world processes there is temporal dependence. Hence, a local temporal average (of values of the recent past) can be a better predictor than the global (or the true) mean. On the other hand, the simplistic trend approach reflects a poor interpretation of a complex reality.

3. TIME DEPENDENCE

Time dependence in natural processes should normally be well understood as no one would expect the present to be fully independent from the past. However, the adherence to using classical statistics, in which everything is regarded independent from everything else, has affected geophysical sciences, including hydrology. Thus, patterns that are manifestations of dependence are commonly interpreted as nonstationarities and modelled as trends, in order to reinstate the illogical and ever failing hypothesis of time independence. However, by claiming nonstationary, one also rejects ergodicity (which cannot exist without stationarity) and, in sequel, one excludes the possibility of making inference from data, which actually is the desideratum (Koutsoyiannis and Montanari, 2015). Therefore, the entire endeavour is in vain and absurd.

Dependence manifests itself both on short and long time lags. It is also manifest on short and long time scales. It is rather easy to understand dependence on short time lags and scales; for example, after a day with high river flow one would expect that in the next day the flow would be high, too. However, at long time lags or scales, dependence manifests change, rather than “memory”. Long-term change (which should not be confused with nonstationarity), produced by different mechanisms acting on different time scales, is the rule in nature (Koutsoyiannis, 2013). The resulting picture in a time series is characterized by patterns and differs from random noise. It is exactly the presence of patterns which makes the autocorrelation (a measure of dependence) of the series different from zero. An illustration is given in the upper part of Figure 3. This is a plot of the longest available hydrological time series, the Nilometer series and, more specifically, the annual minimum water level of the Nile River for 849 years. For comparison, the lower part of Figure 3 depicts a purely random time series synthesized from an idealized (computer-simulated) roulette wheel. Clearly, the structured randomness of the upper plot is different from the pure randomness of the lower plot.

Figure 3 - (Upper) Structured randomness seen in the Nilometer data (annual minimum water level of the Nile River; 849 values). (Lower) A time series representing pure randomness: each value is the minimum of 36 roulette wheel outcomes. The value of 36 was chosen so that the standard deviation be equal to the Nilometer series. Adapted from Koutsoyiannis (2013).
The easiest way to quantify this difference is through the climacogram of the process, i.e. the variance of the time-averaged process at scale $k$ as a function of $k$. More formally, let $x(t)$ be a stationary stochastic process representing the instantaneous quantity of a certain hydrological or other physical process in continuous time $t$. Let $X(k) = \frac{1}{k} \int_0^k x(t)dt$ be the cumulative process over the time interval $[0,k]$ so that $X(k)/k$ be the averaged process at time scale $k$. Then the climacogram, $\gamma(k)$, is defined as:

$$\gamma(k) := \text{var} \left[ \frac{X(k)}{k} \right]$$  \hspace{1cm} (1)

The cumulative process enables representation of the process in discrete time $\tau$ (denoting the continuous-time interval $[(\tau - 1)D, \tau D]$, where $D$ is a time unit) by

$$X^{(\kappa)} := \frac{1}{D} \int_{(\tau-1)D}^{\tau D} x(u)du = \frac{X(\tau D) - X((\tau - 1)D)}{D}$$  \hspace{1cm} (2)

This can readily be expanded to define a discrete time process averaged at scale $k = \kappa D$, where $\kappa$ is an integer:

$$x^{(\kappa)} := \frac{X((\tau - 1)D) + \cdots + X(\tau D)}{\kappa}$$  \hspace{1cm} (3)

With reference to the Nilometer data, where we have a realization of the process $x$, of length 849, i.e. the time series, $x_1, x_2, ..., x_{849}$, we first calculate the sample estimate of variance $\gamma(1)$, for time scale 1 (year). Then we form a time series at time scale 2:

$$x_1^{(2)} := \frac{x_1 + x_2}{2}, \quad x_2^{(2)} := \frac{x_3 + x_4}{2}, \quad \cdots, \quad x_{424}^{(2)} := \frac{x_{847} + x_{848}}{2}$$

and calculate the sample estimate of the variance $\gamma(2)$. We repeat the same procedure and form a time series at time scale 3, 4, … (years), up to scale 84 (1/10 of the record length), and calculate the variances $\gamma(3), \gamma(4), \ldots, \gamma(84)$. The climacogram, the variance $\gamma(k)$ as a function of scale $k$, is visualized as a double logarithmic plot in Figure 4.

If the time series $x$, represented a purely random process, the climacogram would be a straight line with slope $-1$. In real world processes, the slope is different from $-1$, designated as $2H - 2$, where $H$ is the so-called Hurst parameter ($0 < H < 1$). The resulting model is expressed by:

$$\gamma(k) = \frac{\gamma(1)}{k^{2-2H}}$$  \hspace{1cm} (4)

where this scaling law defines the Hurst-Kolmogorov (HK) process. High values of $H$ (> 0.5) indicate enhanced change at large scales, else known as long-range dependence or long-term persistence. It indicates strong clustering (grouping) of similar values. For the Nilometer series, $H = 0.87$. This high value indicates that (a) long-term changes are more frequent and intense than commonly perceived, and (b) future states are much more uncertain and unpredictable on long time horizons than implied by pure randomness.

**Figure 4 - Climacogram of the Nilometer time series.** The empirical climacogram is estimated from the data, while all other are theoretical. A purely random process ($H = 0.5$) has a constant slope -1. A Markov process has a curved climacogram for small scales, which becomes a straight line with slope -1 at large scales. The best fitted model is the Hurst-Kolmogorov with $H = 0.87$, whose slope is constant. If we take into account the bias in the empirical estimation (which is high, shown as the difference in the two red lines in the plot), the resulting adapted theoretical climacogram fits well the empirical climacogram. Adapted from Koutsoyiannis (2013).
4. EXAMPLES FROM NORTHERN ITALY

Does the behaviour of modern hydroclimatic time series resemble that of the Nilometer or that of random noise? To answer this question, we can investigate time series from Northern Italy, where we can find some of the longest records of the world.

Figure 5 shows the daily precipitation in Bologna for 206 years (1813-2018) as well as the hourly precipitation for 23 years (1990-2013 with 2008 missing). In addition to the daily values, the 10-year average and maximum values are also plotted. It is clearly seen that the 10-year climatic values have varied irregularly by a factor of 2 for the average daily precipitation and by a factor > 3 for the maximum daily precipitation. This is consistent with the Hurst-Kolmogorov behaviour and not with the trendy climate-change culture of monotonic trends attributed to global warming. Similar upward and downward fluctuations are seen for hourly rainfall, even though the shorter length of record does not allow to view them on a 10-year scale; rather, a 2-year scale has been used in the figure.

Figure 6 shows the daily minimum and maximum temperature in Milano for 246 years (1763-2008, almost uninterrupted), along with the maxima and minima of a 30-year long sliding window.

Again, we have upward and downward fluctuations; the upward ones prevail. A clearer illustration of temperature extremes in Milano is provided in Figure 7, where for each month the second highest value of maximum temperature and the second lowest of minimum temperature are plotted in a sliding window of a 30-year length (containing $\approx 30 \times 30 = 900$ values). Again, upward and downward fluctuations are seen with the upward ones prevailing, particularly in the low extremes. The climatic range, measured as the difference of the high and low extremes, was 47 °C in 1800, increased to 53 °C in 1860 (worst period), deceased to 45 °C in 2000 (best period), and increased to 48 °C in 2008.

Figure 5 - Rainfall in Bologna, Italy (44.50°N, 11.35°E, +53.0 m). (Upper) Daily data for 206 years; for the period 1813-2007 (195 years, uninterrupted) the data are from the Global Historical Climatology Network (GHCN); for the period 2008-2018, the data are provided by the repository Dext3r of ARPA Emilia Romagna. (Lower) Hourly rainfall data for the period 1990-2013 (23 years full coverage, while the entire 2008 is missing), provided by the Dext3r repository. Source for daily data: Koutsoyiannis (2020a); for hourly data: Lombardo et al. (2019).
A typical interpretation of the prevalence of the upward segments would be to attribute them to global warming, which in turn would be attributed to human CO$_2$ emissions, etc. However, other factors, more local than global, may have been more important and it would be prudent to investigate them. One of them is urbanization, a depiction of which is given in Figure 8.

By comparing the temporal evolution of maxima in Milano and two nearby stations (Monte Cimone and Paganella; Figure 9), in which there is no increasing trend, it appears that urbanization is the principal factor causing temperature increase, rather than global effects.

A necessary subsequent step would be to investigate the stochastic properties of the time series and try to build a consistent stochastic representation (such as that in Koutsoyiannis et al. 2007), rather than relying on computationally complex but conceptually simplistic deterministic approaches. As a first observation, the climacograms of daily maximum and minimum temperature in Milano, shown in Figure 10, suggest a pronounced Hurst-Kolmogorov behaviour (evident for time scales $> 1$ year) with a large value of Hurst parameter, approaching 1. This suggests a very large climatic uncertainty about the future.
While the climacogram is a powerful tool in characterizing the dependence in a stochastic process, it is relevant to remark that the climacogram is based on second-order characteristics of the process. On the other hand, it is well known that the behaviour of extremes cannot be captured by second-order moments but is related to higher-order ones. One may think of introducing climacograms of high-order moments to serve the purpose of studying extremes. However, as shown by Koutsoyiannis (2019), classical moments of order higher than 2-3 are unknowable, in the sense that their values cannot be reliably estimated from typical samples, even when in theory their estimators are unbiased.

To overcome this problem Koutsoyiannis (2019) introduced the knowable moments (or K-moments), a category of which, most useful for the study of extremes, is the noncentral knowable moment of order \( (p, 1) \) defined as:
\( K'_p = pE \left[ \left( F(x) \right)^{p-1} x \right], \quad p \geq 1 \) \hfill (5)

The relevance of K-moments for extremes results from the following relationship, rendering them expected values of maxima:

\( K'_p = E[x_{(p)}] = E[\max(x_1, x_2, \ldots, x_p)] \) \hfill (6)

Their knowability stems from the fact that we can construct estimators with good properties such as unbiasedness, small variance and fast convergence to the true value. The unbiased estimator of \( K'_p \) for any positive order \( p \) (usually, but not necessarily, integer), up to the sample size \( n \), is (Koutsoyiannis, 2020a):

\[
\hat{K}_p = \sum_{i=1}^{n} b_{\text{inp}} x_{(i:n)}, \quad b_{\text{inp}} = \begin{cases} 0, & i < p \\ \frac{\Gamma(n-p+1)}{n} \frac{\Gamma(i)}{\Gamma(i-p+1)}, & i \geq p \geq 0 \end{cases}
\] \hfill (7)

where \( x_{(i:n)} \) is the \( i \)th order statistic (representing the \( i \)th smallest value) in a sample of size \( n \) and \( \Gamma(\cdot) \) is the gamma function.

A K-moment is a characteristic of the marginal distribution of a process and therefore it is not affected by the dependence structure. However, its estimator is affected. Indeed, temporal dependence induces bias to estimators of K-moments. Thus, the unbiasedness ceases to hold in stochastic processes. For a Markov process the effect of autocorrelation is negligible, unless \( n \) is low and \( r \) high (e.g. > 0.90). However, for an HK process this effect can be substantial. In that case, what we estimate by equation (7) is not an estimate of the \( K_p \) for order \( p \), but one for a lower order \( p' \), where (Koutsoyiannis, 2020a):

\[ p' \approx 2\theta + (1 - 2\theta)p(1+\theta)^2, \quad \theta(n,H) = \frac{K_{p'} - K_p}{K_p} \approx \frac{2H(1-H)}{n-1} - \frac{1}{2(n-1)^2 - 2H} (< 0) \] \hfill (8)

Thus, the K-moment framework allows to take into account the dependence structure of the process. An exceptional characteristic of the K-moments is the fact that to each estimated value of \( K_p \) we can easily assign a return period, which makes the framework ideal for studying extremes. Recalling that for maxima, the return period of a value \( x \) is given as \( T(x) = D(1 - F(x)) \), where \( D \) is a reference time unit (e.g. 1 year) and \( F \) is the distribution function of \( x \), the assignment of return periods to \( K'_p \) values is made through the coefficients \( \Lambda_p \), defined as:

\[
\Lambda_p = \frac{T(K'_{p1})}{D p} = \frac{1}{p \left(1 - F(K'_{p1}) \right)}
\] \hfill (9)

It happens that \( \Lambda_p \) varies only slightly with \( p \). Any symmetric distribution will give exactly \( \Lambda_1 = 2 \) because \( K'_1 \) is the mean, which equals the median and thus has a return period of 2\( D \). Hence, a rough approximation is the rule of thumb, \( \Lambda_1 = 2 \). Generally, the exact value \( \Lambda_1 \) is easy to determine, as it is the return period of the mean, while the exact value of \( \Lambda_\infty \) (for order \( p \to \infty \)) depends only on the tail index \( \xi \) of the distribution (Koutsoyiannis, 2020a):

\[
\Lambda_1 = \frac{1}{1 - F(\mu)} = \frac{T(\mu)}{D}, \quad \Lambda_\infty = \begin{cases} \frac{1}{e^\gamma}, & \xi \neq 0 \\ \frac{1}{1 - F(\mu)} = \frac{T(\mu)}{D}, & \xi = 0 \end{cases}
\] \hfill (10)

where \( \gamma = 0.577 \) is the Euler’s constant. These enable simple approximation of \( \Lambda_p \) and hence of the return period by:

\[
\Lambda_p \approx \Lambda_\infty + (\Lambda_1 - \Lambda_\infty) \frac{1}{p}, \quad \frac{T(K'_p)}{D} = p\Lambda_p \approx \Lambda_\infty p + (\Lambda_1 - \Lambda_\infty) \] \hfill (11)

A better approximation of \( \Lambda_p \), almost perfect for any distribution function, is (Koutsoyiannis, 2020a):

\[
\Lambda_p \approx \Lambda_\infty + \left( \Lambda_1 - \Lambda_\infty - B \ln \left( 1 + \frac{\beta}{2\beta - 1} \right) \right) \frac{1}{p} + B \ln \left( 1 + \frac{\beta}{(p+1)^\beta - 1} \right)
\] \hfill (12)
This involves two constants $\beta$ and $B$, which depend on the distribution function. The effectiveness of this approximation for a number of distributions is depicted in Figure 11. A detailed presentation of the K-moment framework and its application in several cases of assessment and modelling of extremes can be found in Koutsoyiannis (2020a). Even the most demanding applications, such as the construction of reliable ombrian curves (else known as intensity-duration-frequency curves) for time scales ranging from minutes to decades, has been well served by this framework.

6. CONCLUDING REMARKS
Following the current trend in hydrology may mean taking steps back. An advice to avoid them would be: better classic than trendy (cf. Iliopoulou and Koutsoyiannis, 2020). A non-trendy stochastic framework, equipped with the central concepts of stationarity, ergodicity and time dependence (particularly long-range one), offers a good representation of the hydrological processes and their extremes. The newly introduced knowable moments (K-moments) are powerful tools that unify other statistical moments (classical, L-, probability weighted) and order statistics, offering several advantages, including their theoretical consistency and optimal use of available data.

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Data sources
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