National Technical University of Athens

School of Civil Engineering

Estimating the risk of large investments using Hurst-Kolmogorov dynamics in interest rates

Supplementary material

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Supplementary-Introduction

Infrastructures are vital for our survival and our prosperity. Society uses economics and money to organize itself [1,2] and achieve economies of scale with the help of infrastructure [3,4]. Investing in infrastructures presents certain risks. These may originate in the macroeconomic environment, disrupting the organization of society [5,6]. Interest rates are among the most important elements in the calculation of the unit cost of infrastructure. However, engineers commissioned to estimate the cost of infrastructure must depend on very broad assumptions of fixed interest rates. In this presentation we use stochastic modeling to realistically simulate the macroeconomic environment.

The calculation of unit cost is ultimately the most important factor in the decision-making process. The aim is to present a range of possible unit costs to decision makers. Under no circumstances should we aim to provided a single prediction which would likely not be precise given the unpredictability of real life.

Supplementary-Choosing the data

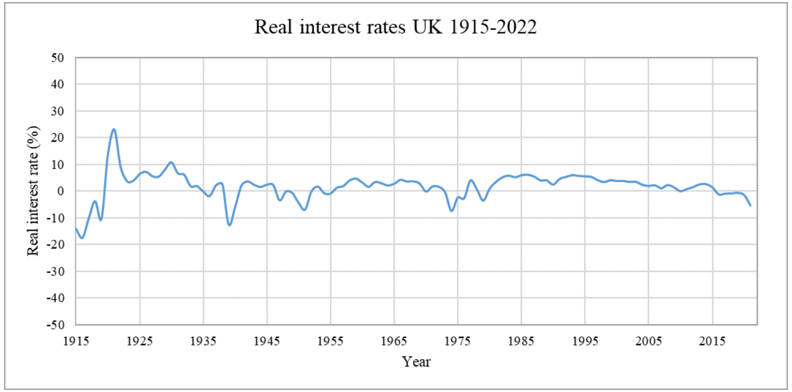
To properly adjust the methodology, we chose monthly historical interest rates from the United kingdom for the period of 1915-2022 [7].

The timeseries was chosen for its reliability as well as for its excellent representation of the impact of a variety of historical events to the interest rate. During the period in question [8], the following events took place:

- World War 1
- World War 2
- The Spanish Influenza Pandemic
- The COVID-19 Pandemic
- The Great Depression
- The Great Recession

We insist that in such models the data series includes pre-WW2 data, since the recent period of economic stability has been historically unprecedented.

Real interest rates are derived from the Fisher equation [9] in which the inflation rate is subtracted from the nominal interest rate of government treasury bonds.



Supplementary-SMA method

The SMA scheme [10,11] is appropriate to preserve any type of dependence.

With the use of the SMA method (Symmetric Moving Average) we can reproduce the HK behavior.

Based on the climacogram we chose the GHK (Generalized Hurst-Kolmogorov) process, which is a particular case of the HHK in which:

$$\gamma(k) = \frac{\lambda}{(1 + k/q)^{2-2H}}$$

Where $\gamma(k)$ is variance, k is the time scale, λ is the variance at this timescale k, H is the hurst parameter and q is the characteristic time parameter.

The previous methodology [12] uses the Marcovian framework, in which we produce an Autoregressive Model with Lag 1, therefore AR(1).

The AR(1) model was used in a case study of the water supply system of Western Mani [13-19].

Supplementary-Hurst Kolmogorov behavior

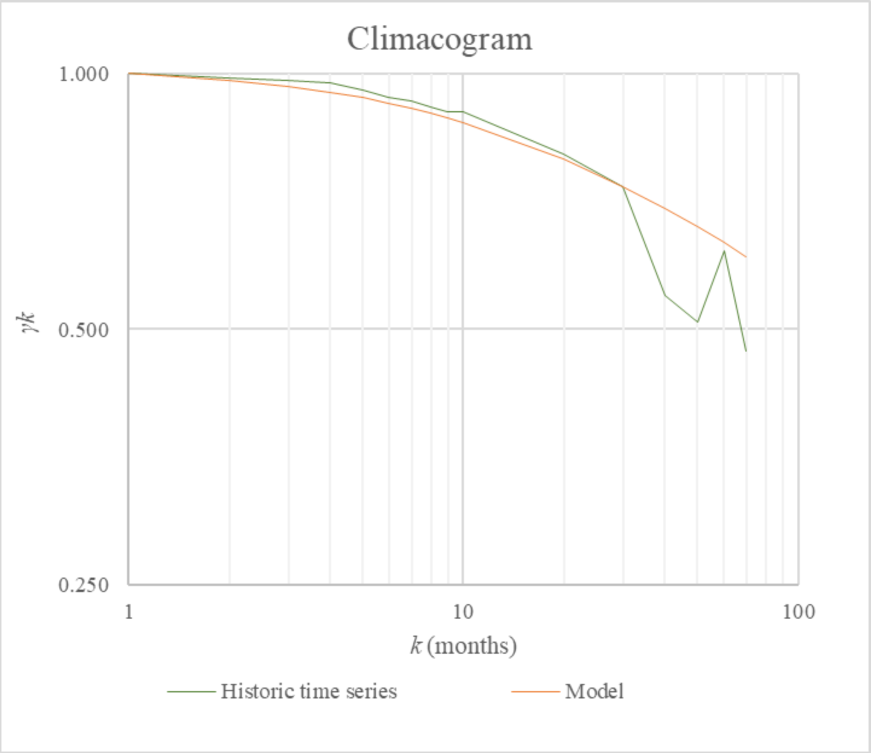
The data clearly present Hurst Kolmogorov behavior.

We note that even longer timeseries would be required to produce better models.

We use the GHK model and estimate the parameters:

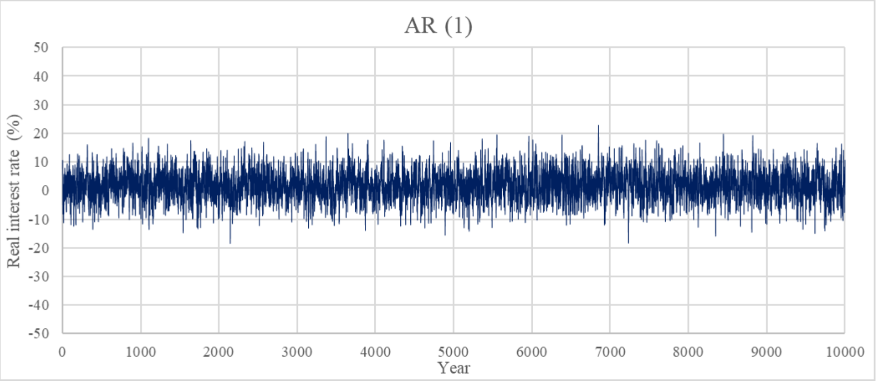
- H=0.85
- q=15.36

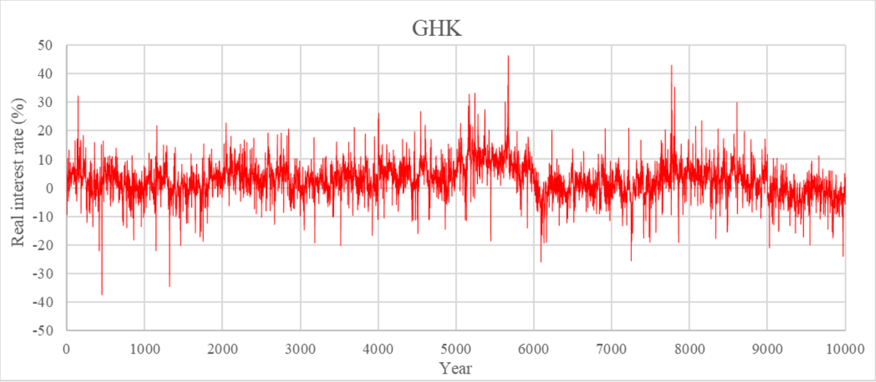
The monthly data do not present seasonality, which gives us the opportunity to use their averages in annual timeseries.



Supplementary-Synthetic timeseries

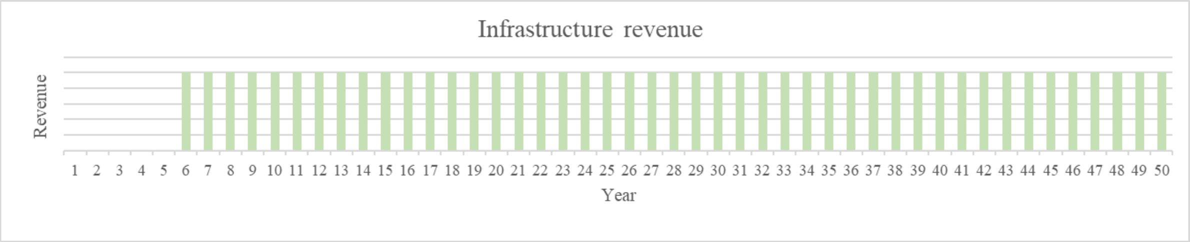
- With the use of the parameters, we produce synthetic timeseries [20].
- The AR(1) and GHK models produce different results. In particular, the AR(1) model seems unable to reproduce macroeconomic events such as periods of extraordinary prosperity or the economic aftermath of major wars.
- In general, we note the existence of prolonged periods of persistently high or persistently low interest rates, a fact that is not reflected in the AR(1) model but is approximated in the GHK model.

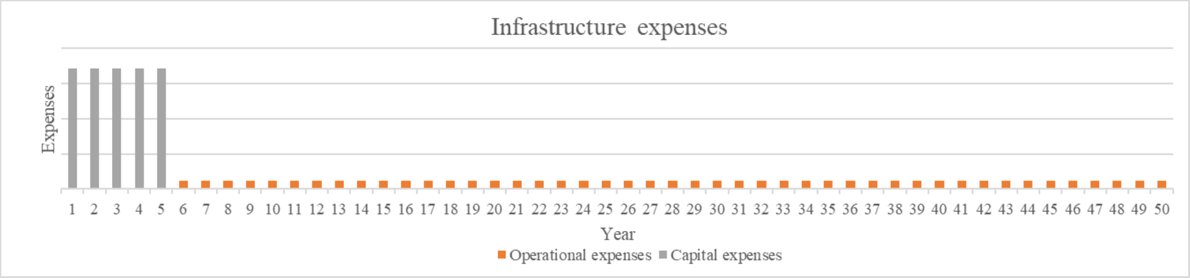




Supplementary-Toy Model

We used a simple toy model to simulate the construction and operation of a large infrastructure project with the lifetime of 50 years.
The cost analysis uses the real interest rate as the discount rate to determine the present value of all income and expenses [21] and thus determine the cost.
We produced 200 simulations with 50-year long timeseries for each model and calculated the unit cost.





Supplementary-Unit cost calculation

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(a) (b) (c)

Figure 2. (**a**) Magic potion in the bath. The capital investment; (**b**) Withdrawals from the bath (incomes) (**c**) The capital investment (Magic potion) is expanded with positive IR.

This somewhat childish analogy is nevertheless the basis to form an understanding of the problem.

An IR can be viewed as the percentage of an amount of money that is paid for its use over a period [53]. In essence, the IR reflects the fact that the cost of the money varies [54]. Usually, when studying the cost of infrastructure, one would use the methodology in which a single initial capital investment takes place, and over the years a stable amount is paid annually to cover the debt at a stable IR [55].

The real IR (for a single year) is defined as the yield of the one-year treasury bond of a country minus the inflation rate for that year (Fisher equation) [56]; however, this should not be confused with the cost of capital that expresses the rate available for borrowers by banking institutions [57].

In our calculations, we use the real IR, i_t, $t = 1, \ldots, T$, where all i_t, are assumed as stochastic variables. In order to estimate the total investment, we use the following equations.

The total expenditure during a given year is provided by the Equation (1):

$$E_T = C_T + O_T \quad (1)$$

where: E_T: Total expenditures during year T; C_T: Amount of that was spent in the year T for buying or building fixed assets; O_T: Amount that was spent to maintain operations in the year T (note: because the assumption is that all infrastructure will operate at maximum capacity during the entire lifespan of the project, the operational cost is assumed constant and not a function of units produced or sold). The revenue during a given year is provided by Equation (2):

$$R_T = UN \quad (2)$$

where: R_T: revenue during year T; U: cost of unit; N: number of units produced, (note: during construction revenue is 0). A stochastic estimation of the present value of the expenditures of a given year T is provided by Equation (3), and the present value of all future expenditures is provided by Equation (4):

$$P_T = E_T(1+i_1)^{-1}(1+i_2)^{-1}\ldots(1+i_{T-1})^{-1}(1+i_T)^{-1} \quad (3)$$

$$Q_n = P_1 + P_2 + \ldots + P_T \quad (4)$$

where: Q_n : present value of all future expenditures for a period T; P_T: Present value of a future expense in the year T; i_T: IR for year T.

The present value of all future expenditures, assuming that the IR is fixed, is described by Equation (5):

$$P_T = E_T(1+i)^{-T} \quad (5)$$

In the same manner, we replace expenses with revenue. Therefore, a stochastic estimation of the present value of the revenue of a given year T is provided by Equation (6) and the present value of all future expenditures is provided by Equation (7):

$$A_T = R_T(1+i_1)^{-1}(1+i_2)^{-1}\ldots(1+i_{T-1})^{-1}(1+i_T)^{-1} \quad (6)$$

$$A_n = A_1 + A_2 + \ldots + A_T \quad (7)$$

For calculation purposes and during the years with no revenues, we consider revenues to be 0.

The present value of all future revenues, assuming that the IR is fixed, is described by Equation (8):

$$A_T = \mathrm{R}_T(1+i)^{-T} \quad (8)$$

Using the above equations, we calculate the expenditures for a given year (Equation (1)). We then calculate the present value of expenditures for each given year (Equation (3)). Afterwards, we calculate the present value of expenses during the lifespan of the project (Equation (4)). Finally, we assume the present value of all expenses to be equal to the present value of all future revenue, repeating the process backwards (Equations (2), (6) and (7), and enabling us to calculate the unit cost in each case.

It is arguably unusual for the calculation of the cost of water to engage in such complicated modeling. Various common methodologies assume the IR to be fixed [58], and a lot of research has been put into deciding the 'correct' stable IR [59]. The fact that the IR is assumed as fixed, is not a simplification of calculations but rather a reflection of the way public infrastructures are financed, and comprised of stable financial vehicles such as bond issuance [60]. Nevertheless, it is important, in our view, to study the IR as a variable function, since interest payments ultimately are not stable. The above can take many forms, including a change in expectations of investment returns or, in the case of governments, a change in economic conditions, which in turn may cause changes in the government's budget [61].

Multiple financial tools exist to take advantage of, or shield, an investment from that fact, such as issuing bonds of different durations. Furthermore, if the infrastructure project is partially or fully funded by private entities, then these funds would be considered as opportunity cost, which is simply the ability to invest in other more promising available investments [62], when making investment decisions.

4. Modeling of Investments' Risks

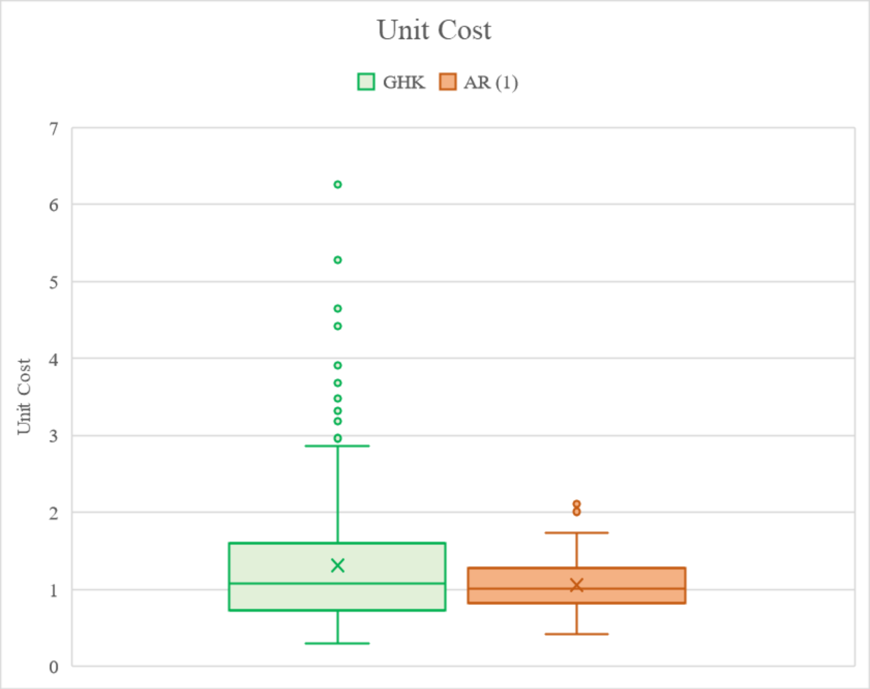
From the above, one may get the wrong impression that this work attempts to predict or forecast future IR values. Forecasting, in our view, is a fundamentally different process than stochastic modeling., and the main difference is ultimately a philosophical one. Modern-day forecasting was born out of Keynesian thought, and it ultimately attempted to predict the future behavior of the economy [63]. This was necessary in order to conduct macroeconomic public policy; however, the first such attempt [64] failed to predict the Great Recession. In subsequent years, despite ever-increasing computing power, economists have very poorly predicted in advance any major macroeconomic events. On the other hand, weather forecasting has become better at predicting tornadoes [65].

The tradition of forecasting usually aims at producing a single prediction, but this impacts the operational principles of forecasting. Specifically, general principles on forecasting [66] suggest: 'Do not forecast cycles; Adjust for events expected in the future; Be conservative in situations of high uncertainty or instability'.

These previous principles stand against the tradition of hydrological engineering. The seasonality and unpredictability of hydrological phenomena, in no case, allows us to implement such methodologies, especially when these are what we seek to 'design around'. Furthermore, we under no circumstances aim to make predictions. Our aim is to produce a range of outcomes along with the associated distribution function (i.e.,

Supplementary-Results

The unit cost presents significant larger variation in the GHK model.
The existence of periods of consistently high interest rates produce instances in which the unit cost is many times higher than the median.
The distribution of the unit costs has a particularly "fat tail" in which unlikely events may take place and radically increase the unit cost.



Supplementary-Discussion and conclusions

- The unit cost is extremely sensitive to interest rates.
- Furthermore, interest rates present very high Long-Term Persistence (LTP).
- Therefore, this characteristic needs to be included in long term economic calculations of infrastructure investments to properly simulate the financial risk.
- Commitment towards an infrastructure project is an extremely risky act, in a financial sense. As such, infrastructure investments require both long-term thinking and confidence.
- Confidence from decision makers relies in public participation, rationality and competence. Properly designed built and operated infrastructure can survive extraordinary financial events.

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