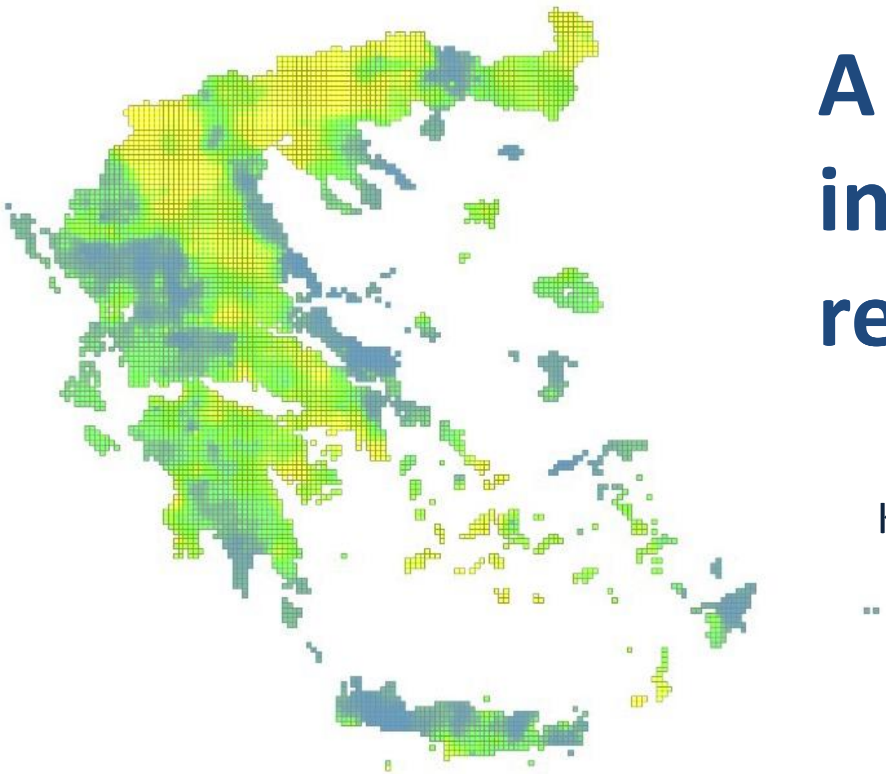
EGU General Assembly 2024

HS3.8: Advances in stochastic analysis, modelling, simulation and prediction for hydrological and water-related processes



A stochastic framework for rainfall intensity-timescale-return period relationships regionalized over Greece

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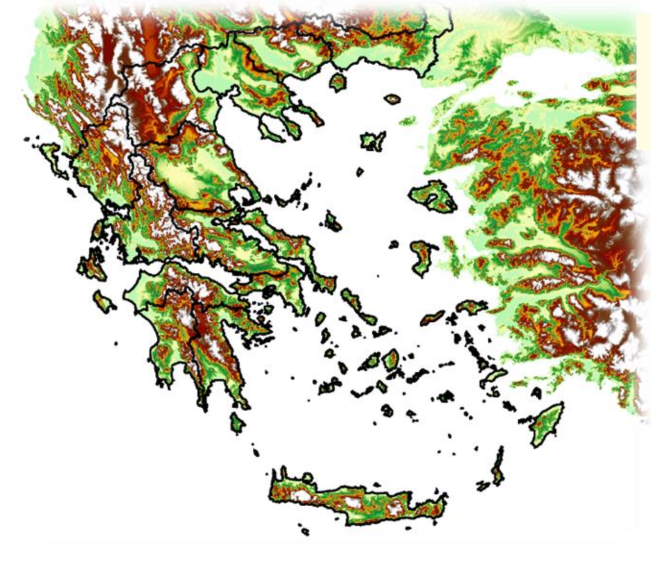
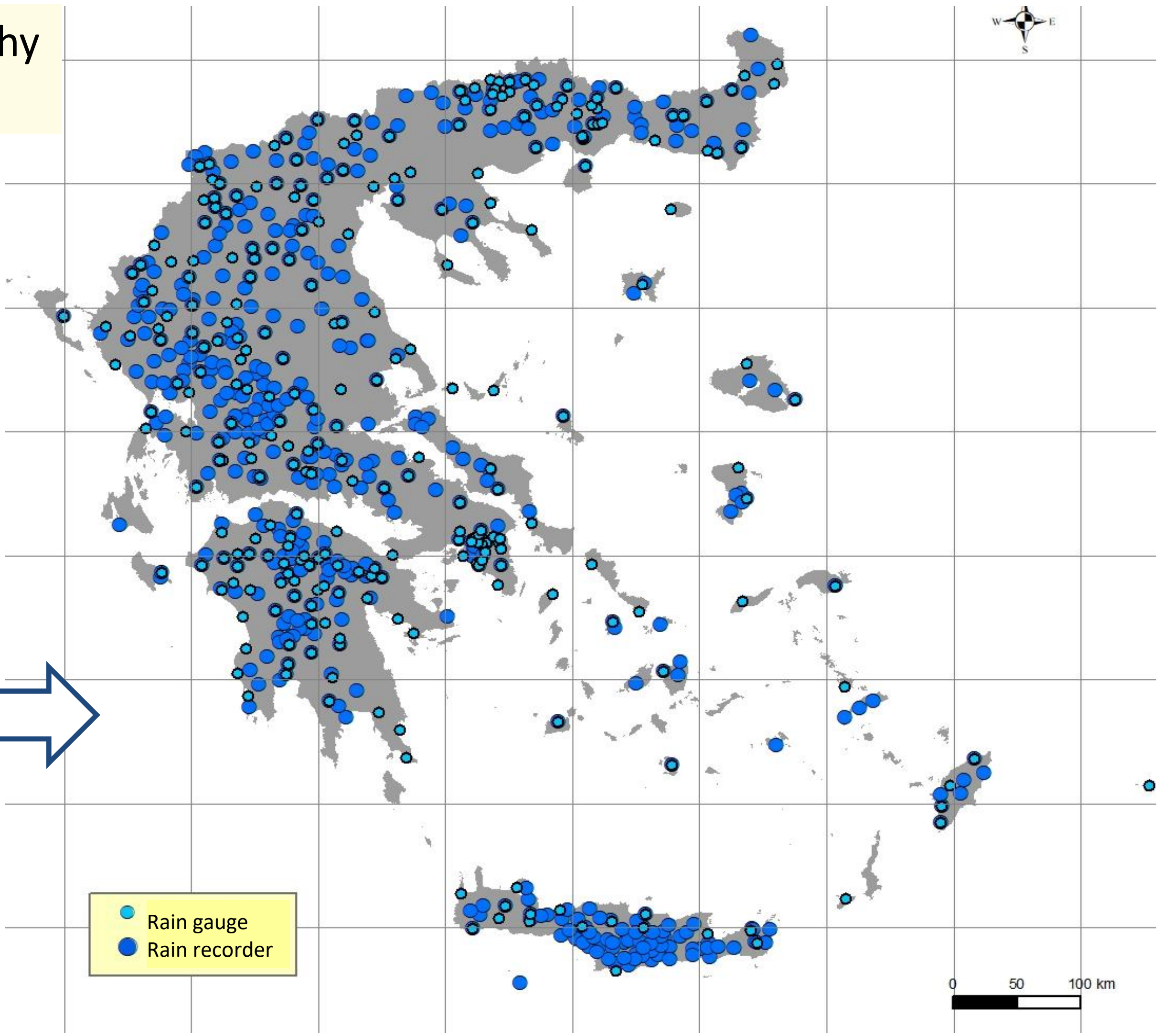
Compiling a rainfall maxima dataset for Greece



- Highly variable topography
- Diverse rainfall regimes

After extensive data collection efforts and quality control, we compile a dataset of **783 stations**, comprising:

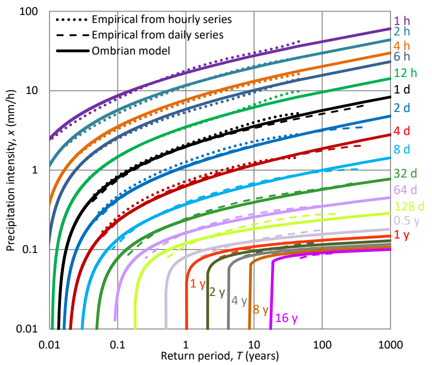
- 503 **daily** rain gauges,
- 280 rain gauges with **sub-daily** resolution (rain recorders).



Modelling approach

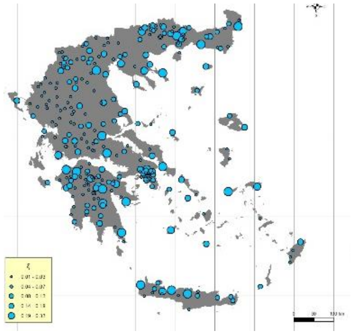
1 At-site independent estimation of parameters

- using Koutsoyiannis' (2022) new framework for **rainfall intensity-timescale-return period** relationships (else known as idf/ombrian curves)

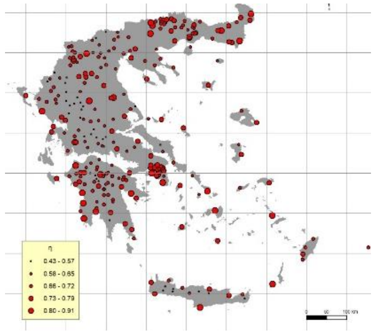


2 Evaluation of spatial variability of parameters:

- mostly **random patterns**

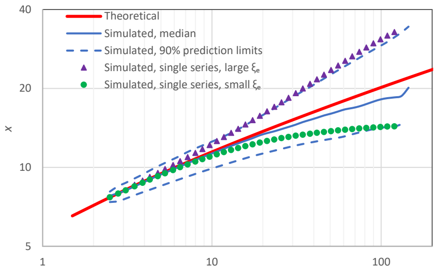


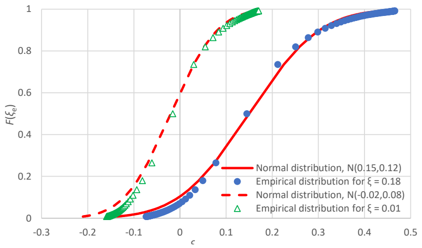
- mostly **systematic patterns**



3 Identification of **common** parameter values

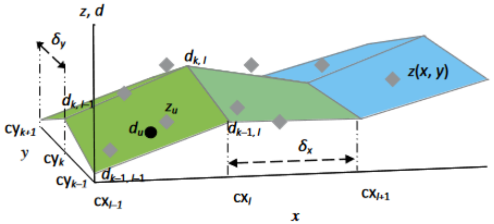
- using simultaneous optimization
- and stochastic simulations

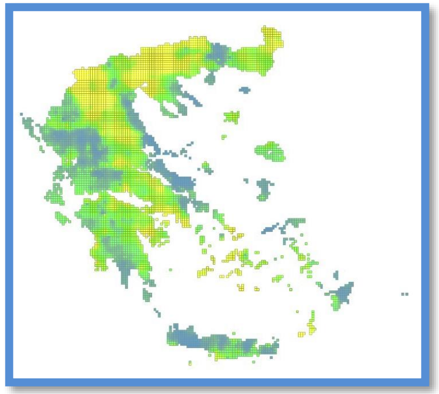




4 Regionalization using **spatial models**

- Inverse Distance Weighted (IDW)
- Bilinear surface smoothing (BSS)
- Ordinary Kriging (OK)





Final product

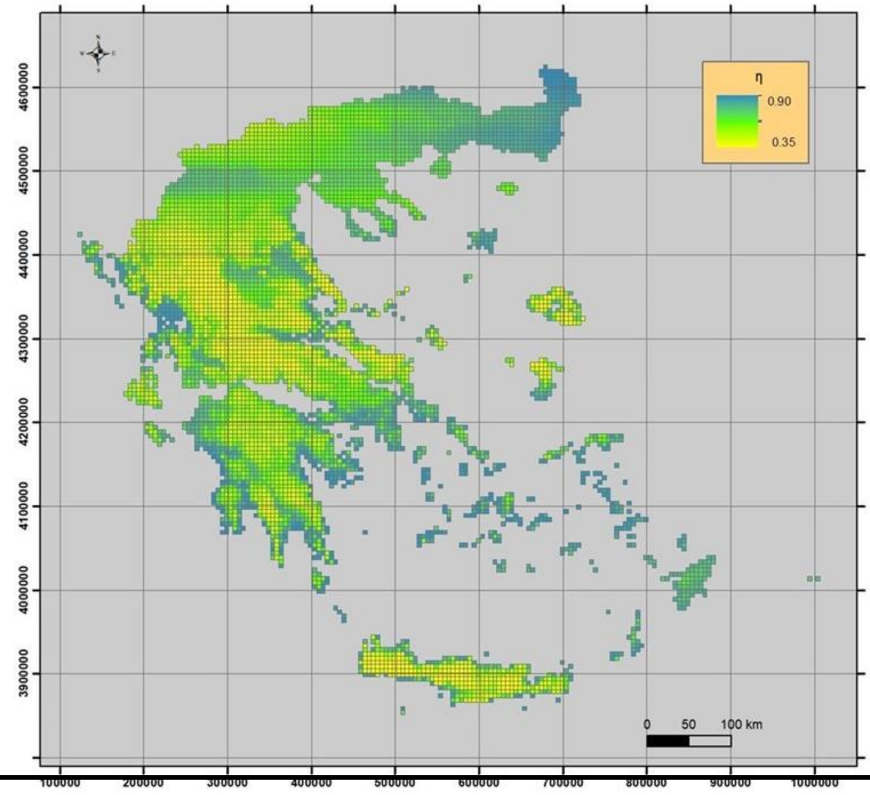
The following generalized form of ombrian curves is derived for rainfall intensity *x* (mm/h), return period *T* (years) and temporal scale *k* (h):

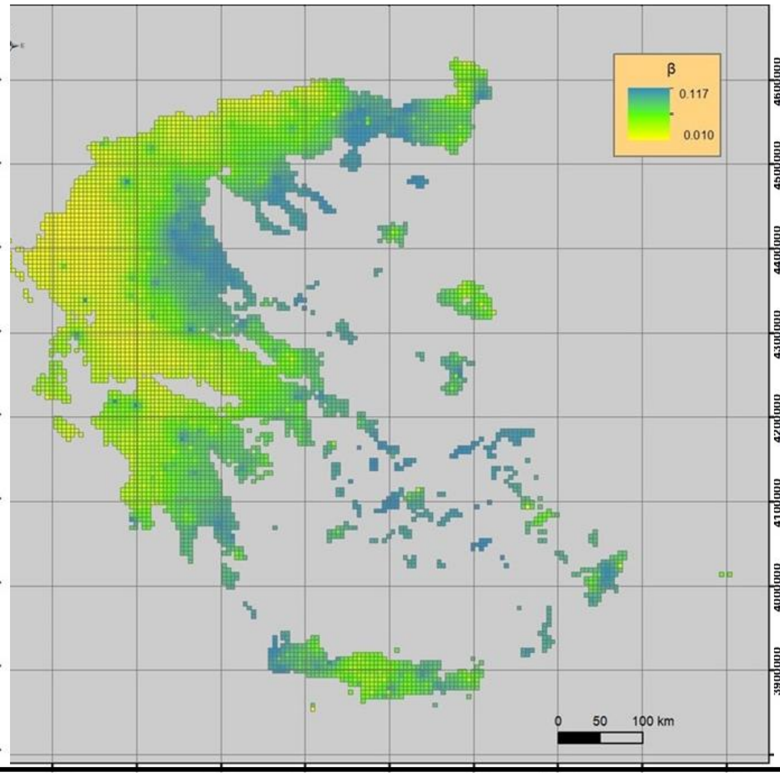
$$x = \lambda_* \frac{(T/\beta_*)^{\xi} - 1}{(1 + k/\alpha)^{\eta_*}}$$

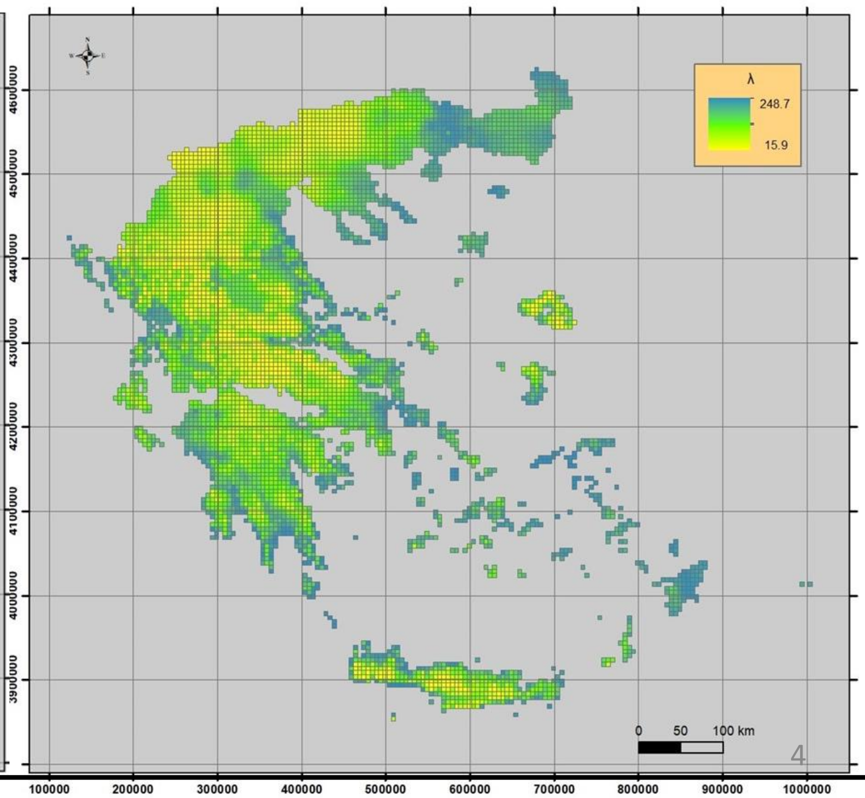
with the following **five** parameters:

- characteristic timescale α = 0.18 h
- tail index ξ = 0.18,
- three spatially varying parameters $\eta_*[-]$, β_* (years) and λ_* (mm/h):

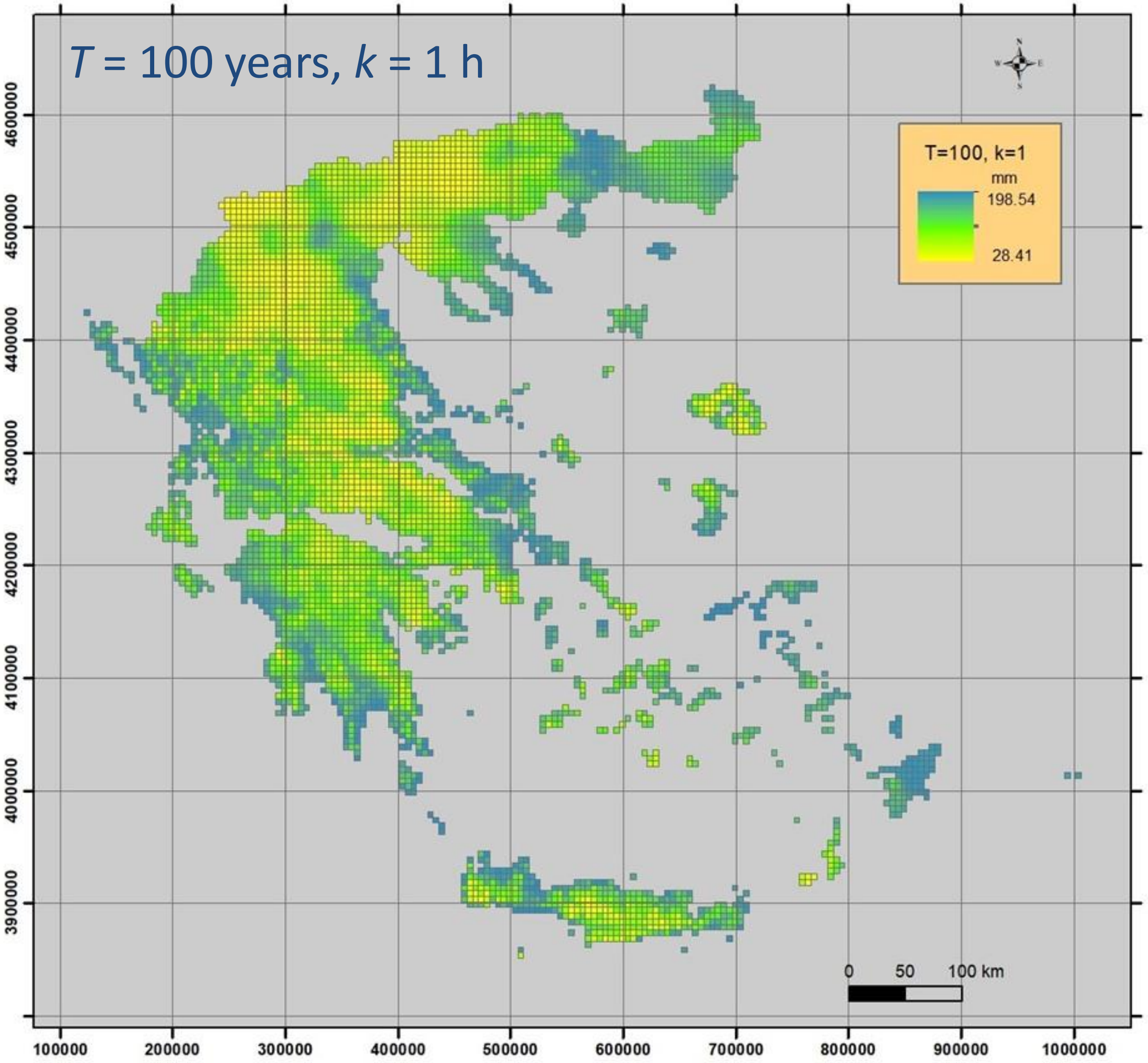
For the first time, we have produced a geographically distributed design rainfall model for the entire Greek territory.

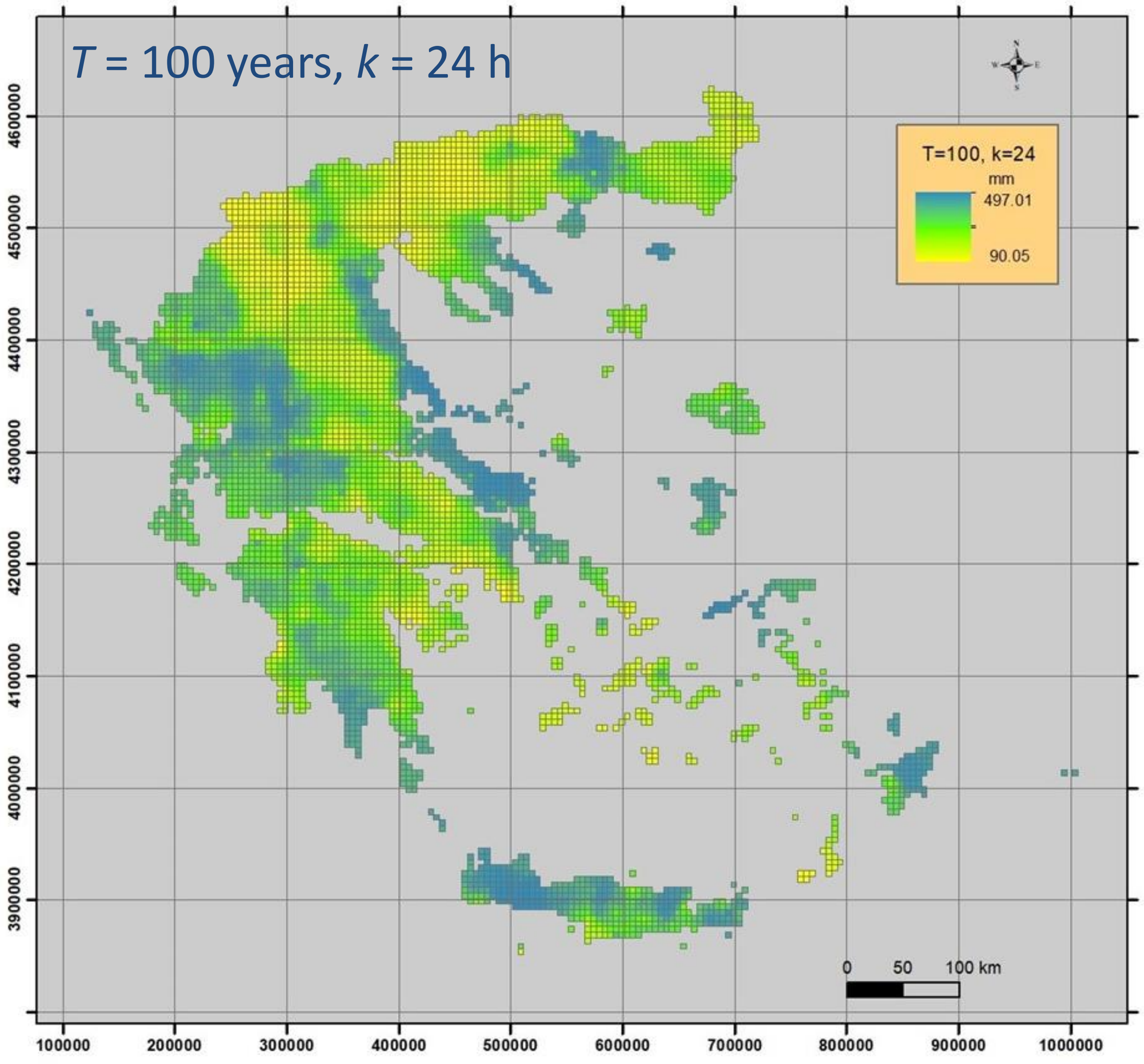






Design rainfall maps

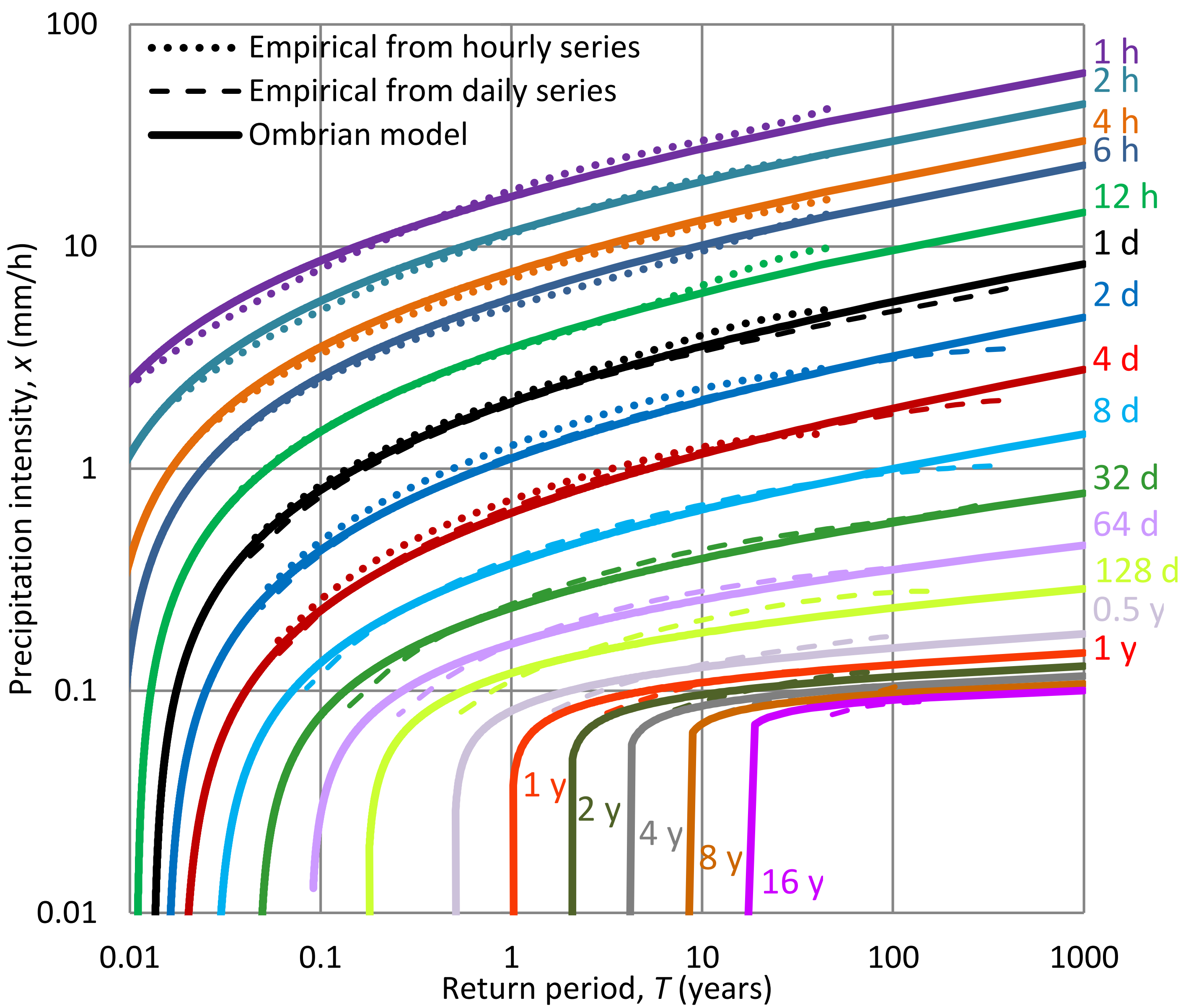




Overview of theoretical framework (I)

- Koutsoyiannis (2022) developed **a new framework for ombrian modelling that can be applied at any timescale**, however large or small.
- The example shown is for Bologna, Italy (a station with 206 years of data), for timescales from 1 h to 16 years.
- For large timescales the mathematics are somewhat involved.

➢ See also PythOm toolbox, by Iliopoulou and Koutsoyiannis (2021)



Overview of theoretical framework (II)

- Under **some simplifying assumptions** the rainfall intensity x for **small timescales** k (of the order of minutes to a few days) and return period T is given by the following relationships, resulting from the full-scale ombrian model can be derived as:
  - for return period estimated from a full series or of rainfall exceedances over threshold:

$$x = \frac{b(T)}{a(k)} = \lambda \frac{(T/\beta)^{\xi} - 1}{(1 + k/\alpha)^{\eta}}, \qquad \xi > 0$$

  - from series of annual maxima (where Δ = 1 year):

$$x = \lambda \frac{(-(\beta/\Delta)\ln(1 - \Delta/T))^{-\xi} - 1}{(1 + k/\alpha)^{\eta}}, \qquad \xi > 0$$

Theoretically equivalent for all T and for the same parameter values; giving virtually same values for T > 10 years

- The simplified model parameters are:

- λ a characteristic rainfall intensity (scale parameter) in units of x (e.g., mm/h);
- β a time parameter, related to the mean distance of wet periods, in units of the return period (e.g., years);
- α a timescale parameter in units of timescale (e.g., h) with $\alpha > 0$;
- η a dimensionless parameter, expressing persistence, with $0 < \eta < 1$;
- $\xi > 0$ the tail index of the process distribution.

5 parameters with physical meaning

Two-step fitting procedure

An attractive feature of this simplification, related to the separable function $x = b(T)/a(k)$, is that it allows the parameters to be estimated **by a convenient, two-step procedure**.

- **First step:**

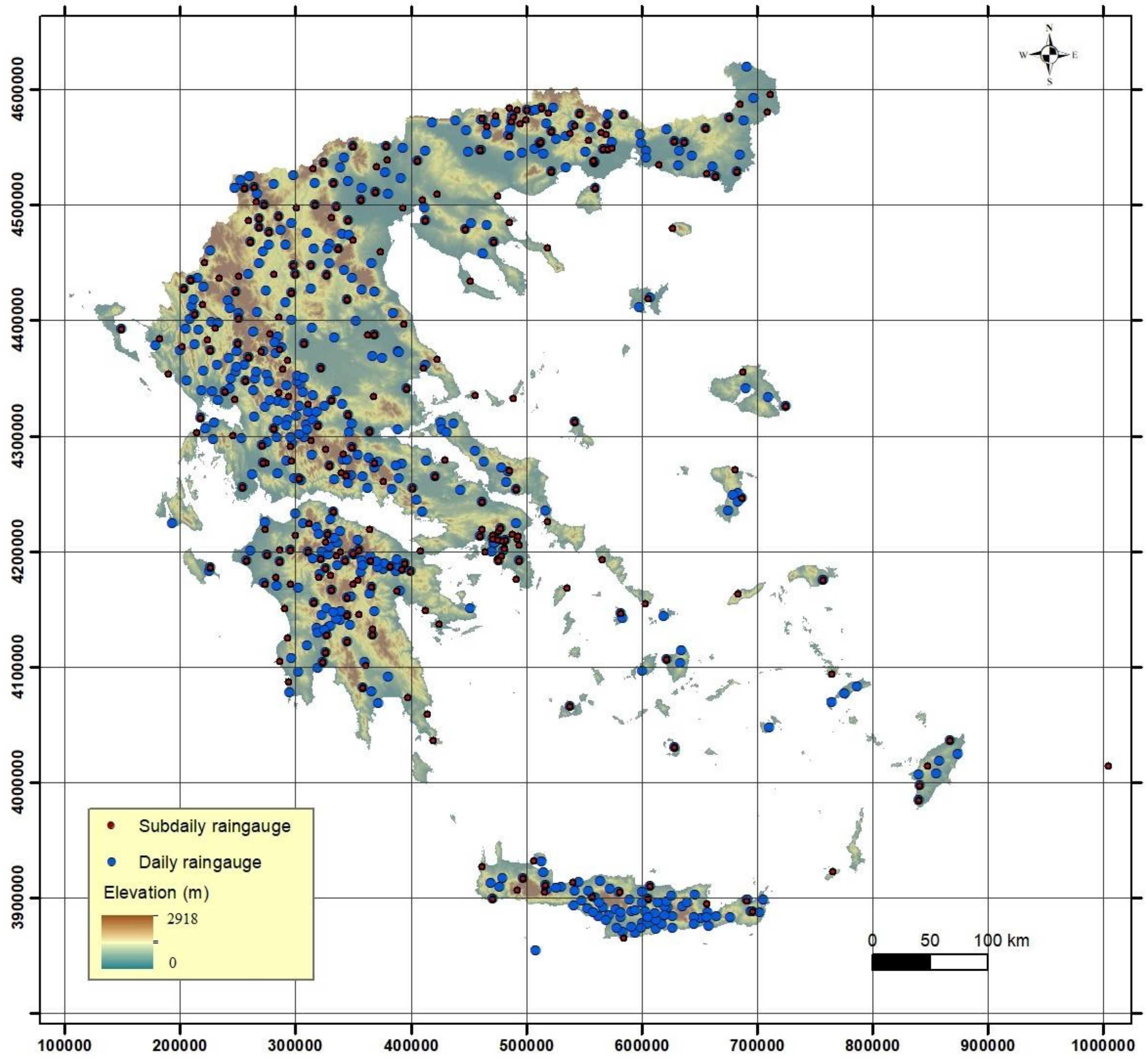
The timescale parameters (of the expression $a(k)$) obtained by Koutsoyiannis et al. (1998) optimization procedure.

- **Second step:**

The distribution parameters (of the expression $b(T)$) are obtained by **the newly introduced method of K-moments** (Koutsoyiannis, 2020), which has the following important properties:

- ✓ Intuitive formulation, as the K-moment of order *p* equals the expected value of the maximum of *p*; independent stochastic variables identical to $\underline{x}$, i.e., $K'_p = \mathrm{E}\left[\max\left(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_p\right)\right]$;
- ✓ Can be readily assigned an empirical return period;
- ✓ Knowable even for very large orders and with determinable bias in the case of (spatial and temporal) dependence

Greece's rainfall network



- From the initial set of 940 stations, and after meticulous quality control processing, we compiled a final dataset of 783 stations, comprising:
  - 503 daily rain gauges, 130 of which at locations where there is also a rain recorder;
  - 280 rain gauges (rain recorders) with sub-daily resolution.
- The stations are distributed over 651 geographical locations.
- The longest available record (in Athens) covers the period from 1860 to 2022.

Spatial variability characterization & Regionalization

Independent at-site procedure

- First, we perform a spatially-independent fitting of the ombrian curves for each location.
- Then, we assess the resulting patterns of variability, and we identify the parameters exhibiting random spatial variation and the ones robust spatial patterns.

Regionalization procedure

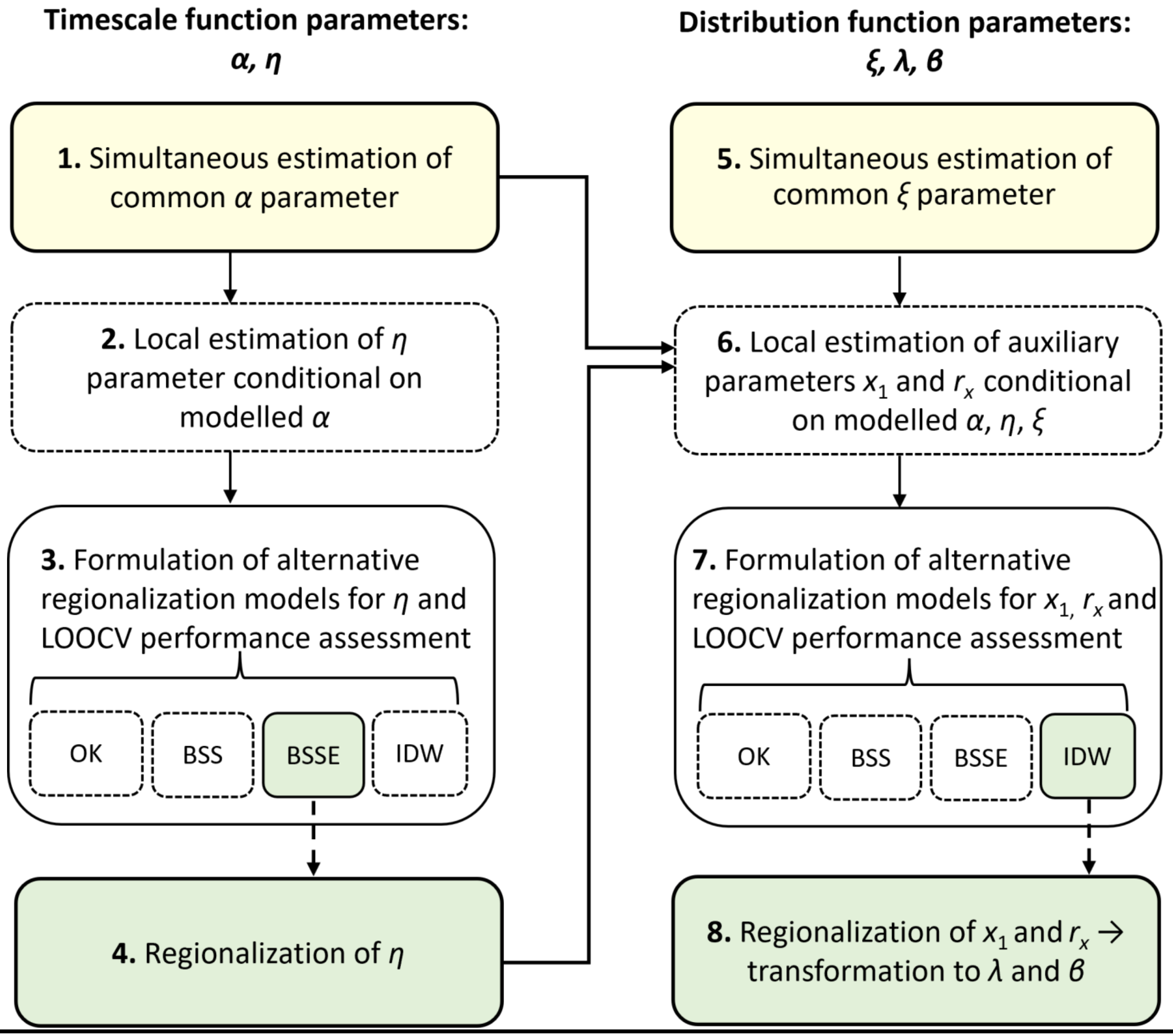
1. We perform a combined (simultaneous) estimation of the parameters exhibiting random variation in space using the most reliable and relevant data for each case, e.g. we exploit the longest sub-hourly records for the estimation of the α parameter and the longest daily records for the estimation of the tail-index parameter ξ.

2. With the common parameters now fixed for all stations, we re-estimate the other parameters and assess their geographical variation.

3. In case that systematic patterns are identified, we model their geographic variation using both spatial smoothing and interpolation models, and evaluate their performance based on the accuracy of the fit and cross-validation metrics.

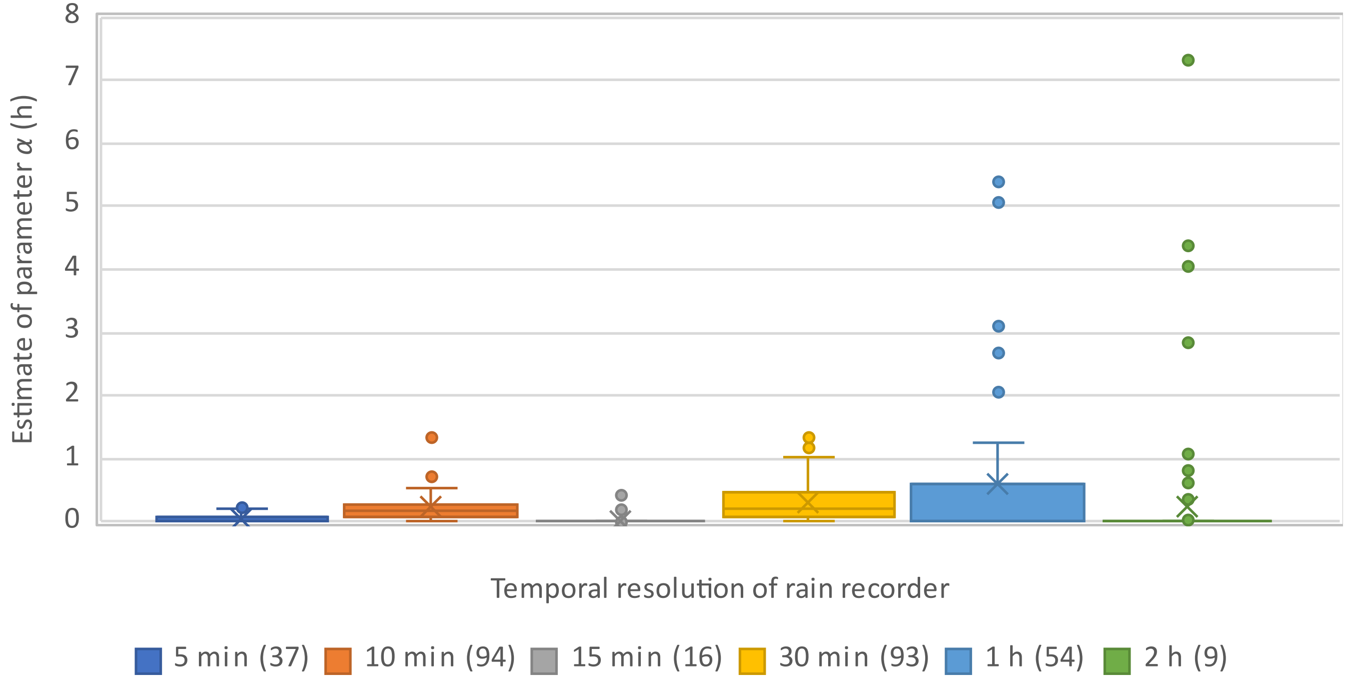
4. The best spatial model per parameter is chosen and a map with 5 km resolution is produced with the spatially varying parameters over Greece.

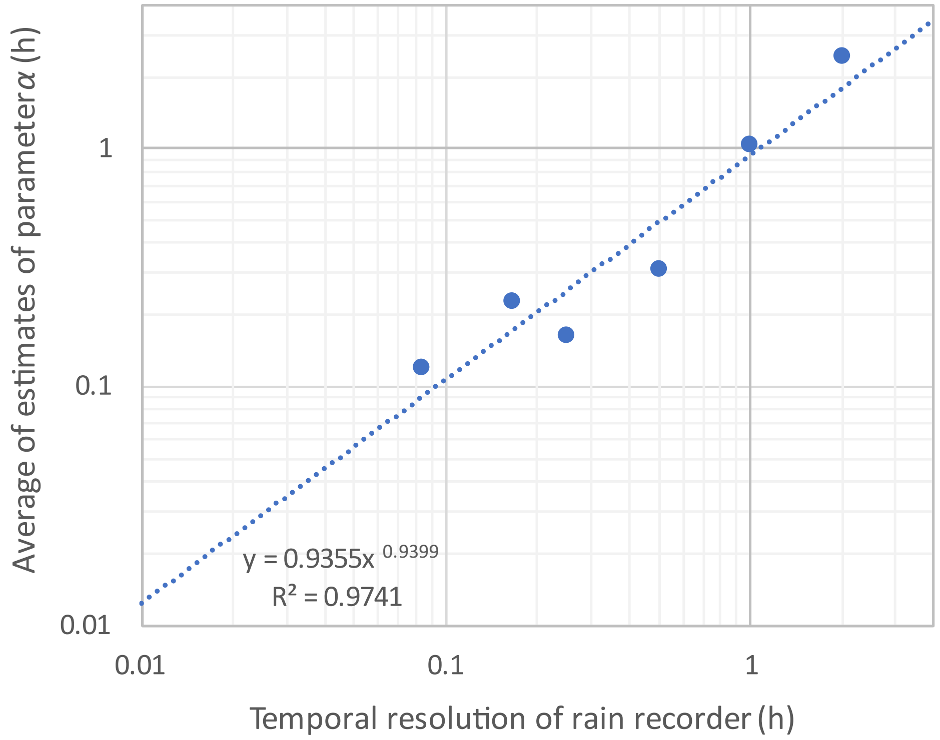
Workflow of the regionalization procedure



Regionalization of timescale function parameters – α (I)

- We find that the estimation of the parameter α greatly depends on the temporal resolution of the measuring instrument. Specifically, in stations with fine temporal resolution (5 or 10 min) resulting values of the parameter α are small—and vice versa.
- This is interpreted as an artificial statistical effect rather that as representing some physical reality.



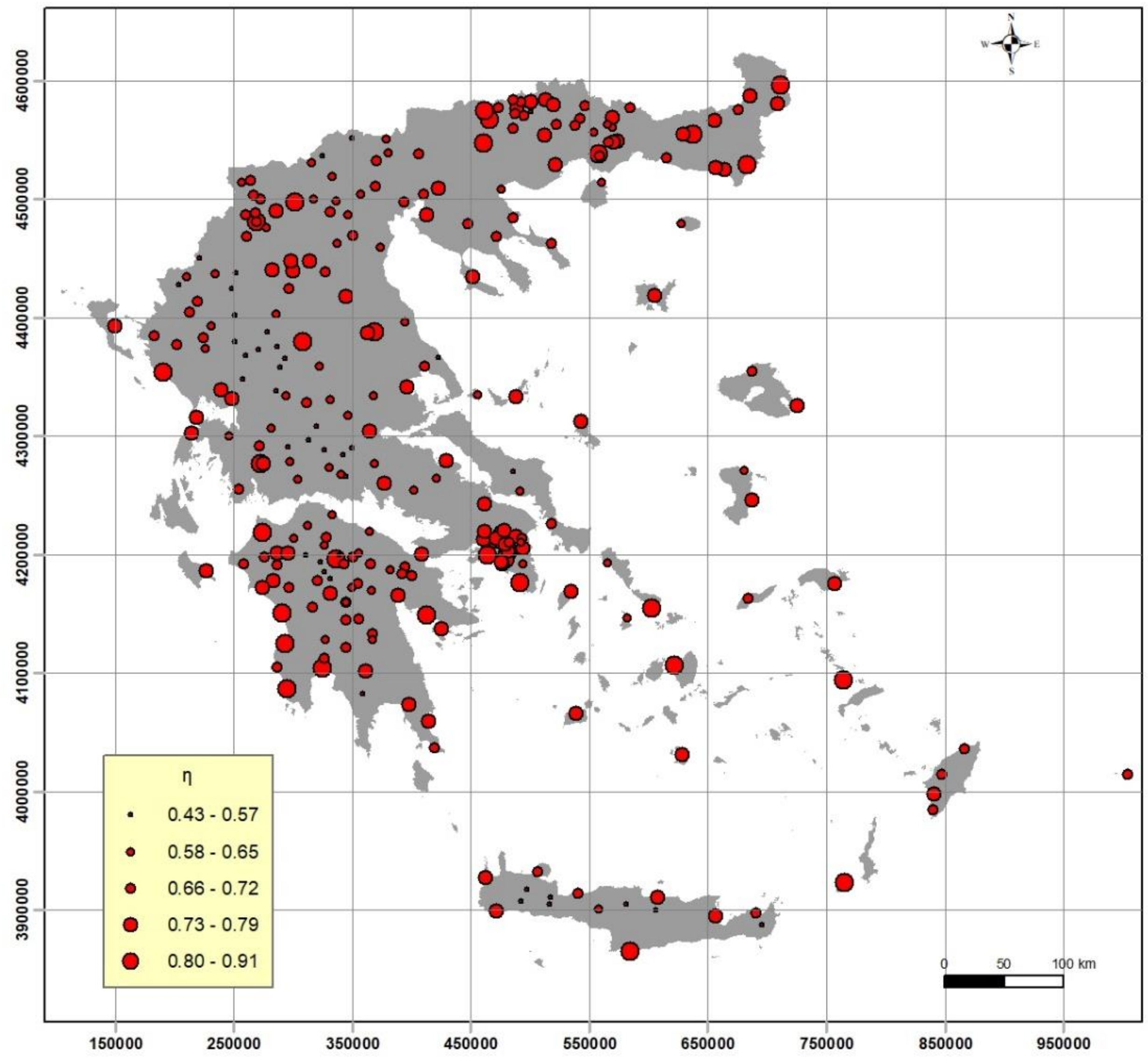


Regionalization of timescale function parameters – α (II)

- To compensate for the great sensitivity of the α parameter to time resolution of the data, we identify a single value of this parameter for all of Greece, by the following procedure:
  - We select the 53 stations with the longest records having temporal resolution 30 min or finer, distributed over all water districts.
  - We re-estimate the parameters of the equation $a(k)$, α and η, through optimization in which we set as a constraint that the value of the α parameter is the same among all stations.

- As a result of this methodology, the common value of $\alpha = 0.18$ h is obtained, which is used in all further analyses.

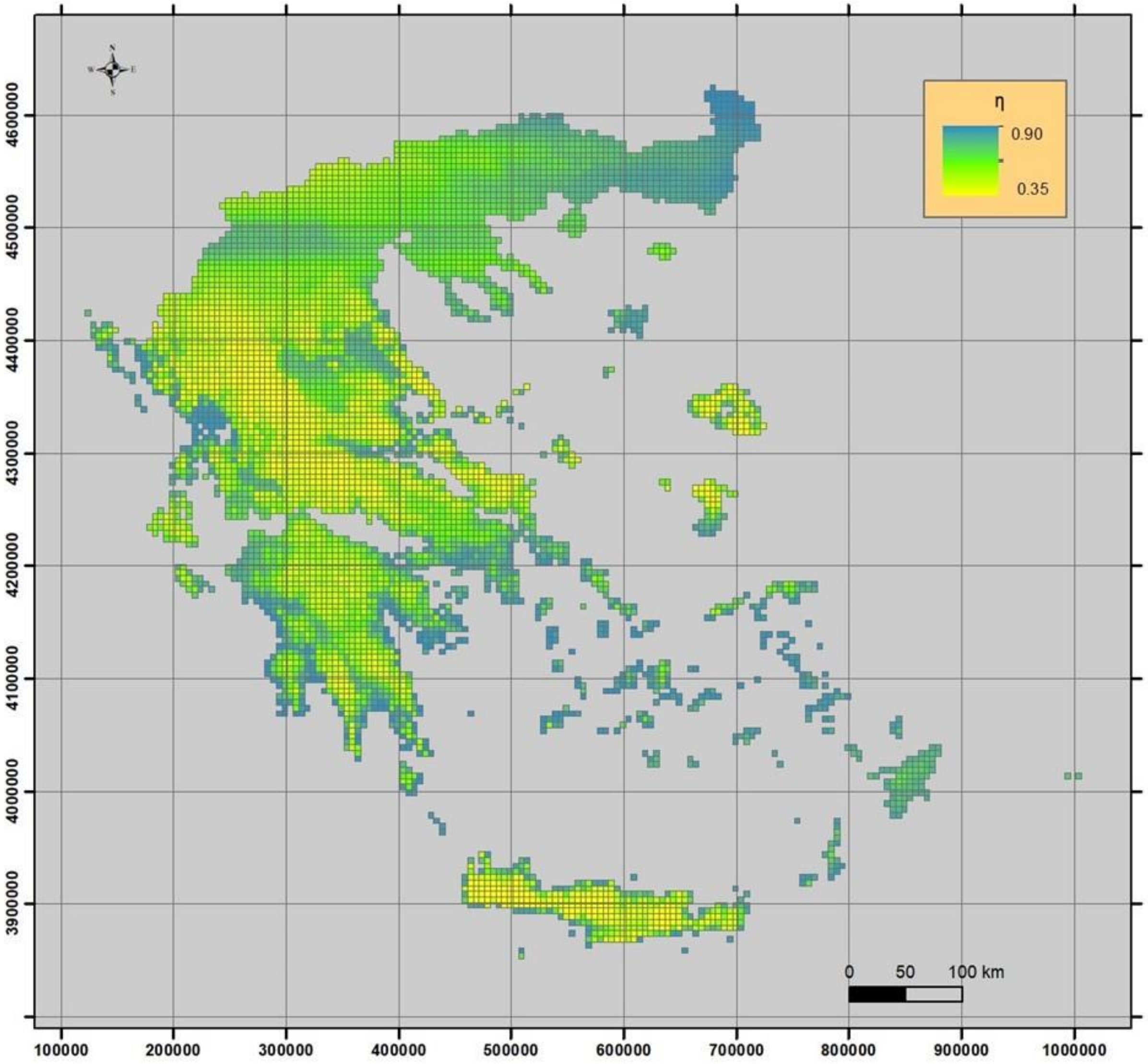
Regionalization of timescale function parameters - η (I)



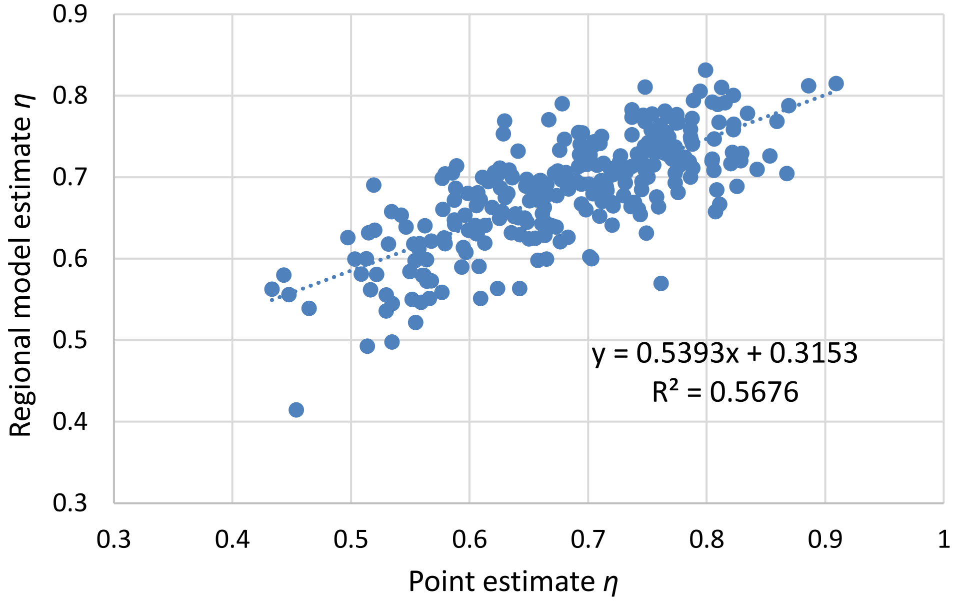
The map shows point estimates of the η parameter conditional on a common α parameter, $\alpha = 0.18$ h:

- Presence of clusters of low and high values of the η parameter in space.
- Emergence of an inverse relationship η with the altitude (i.e. lower values of the parameter are more likely at high altitudes).

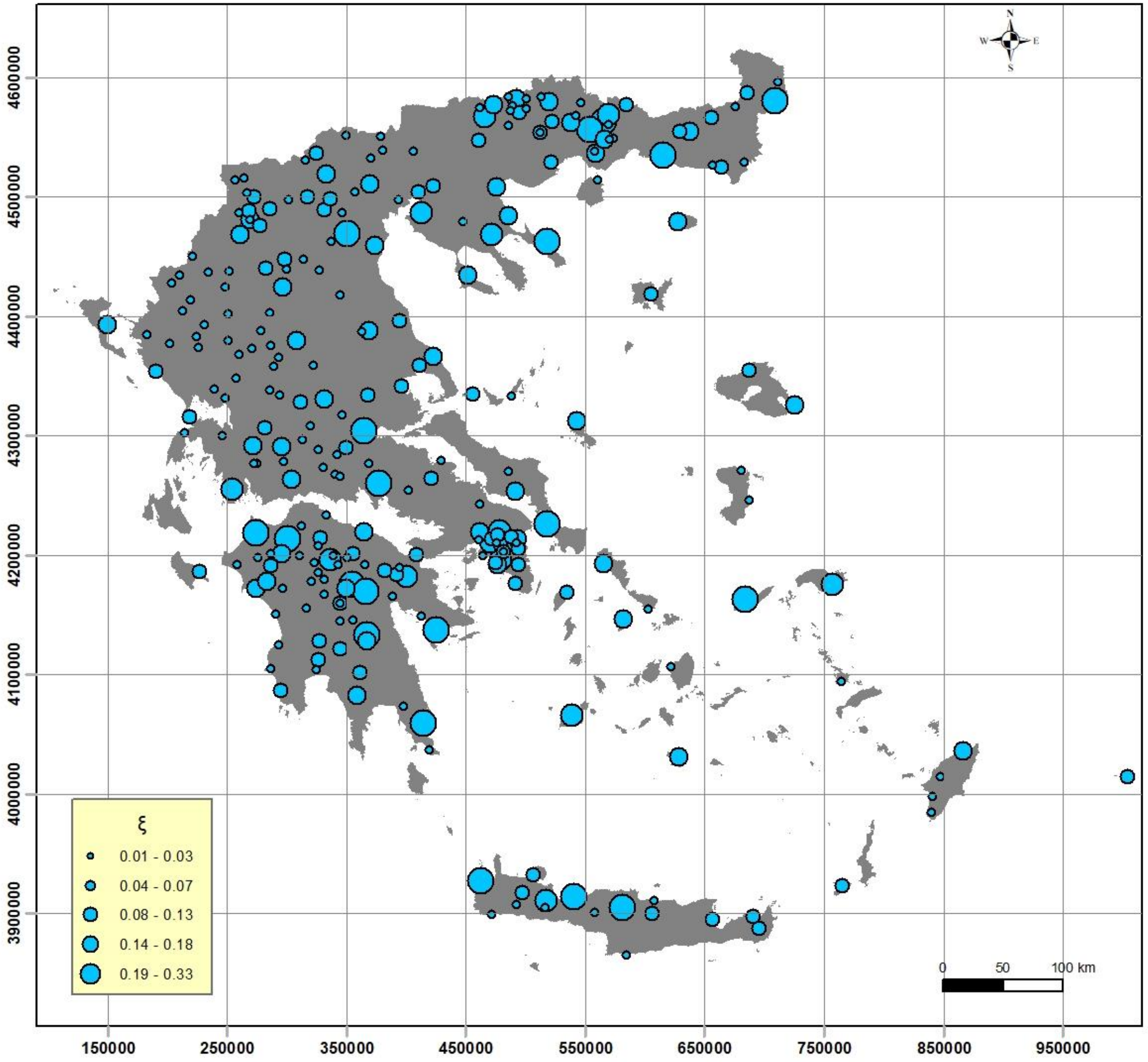
Regionalization of timescale function parameters - η (II)



In the case of the η parameter, the best statistical characteristics of the fitting in terms of leave-one-out cross-validation resulted from the application of the spatial BSSE smoothing model with the altitude (derived from SRTM) as an additional explanatory variable.



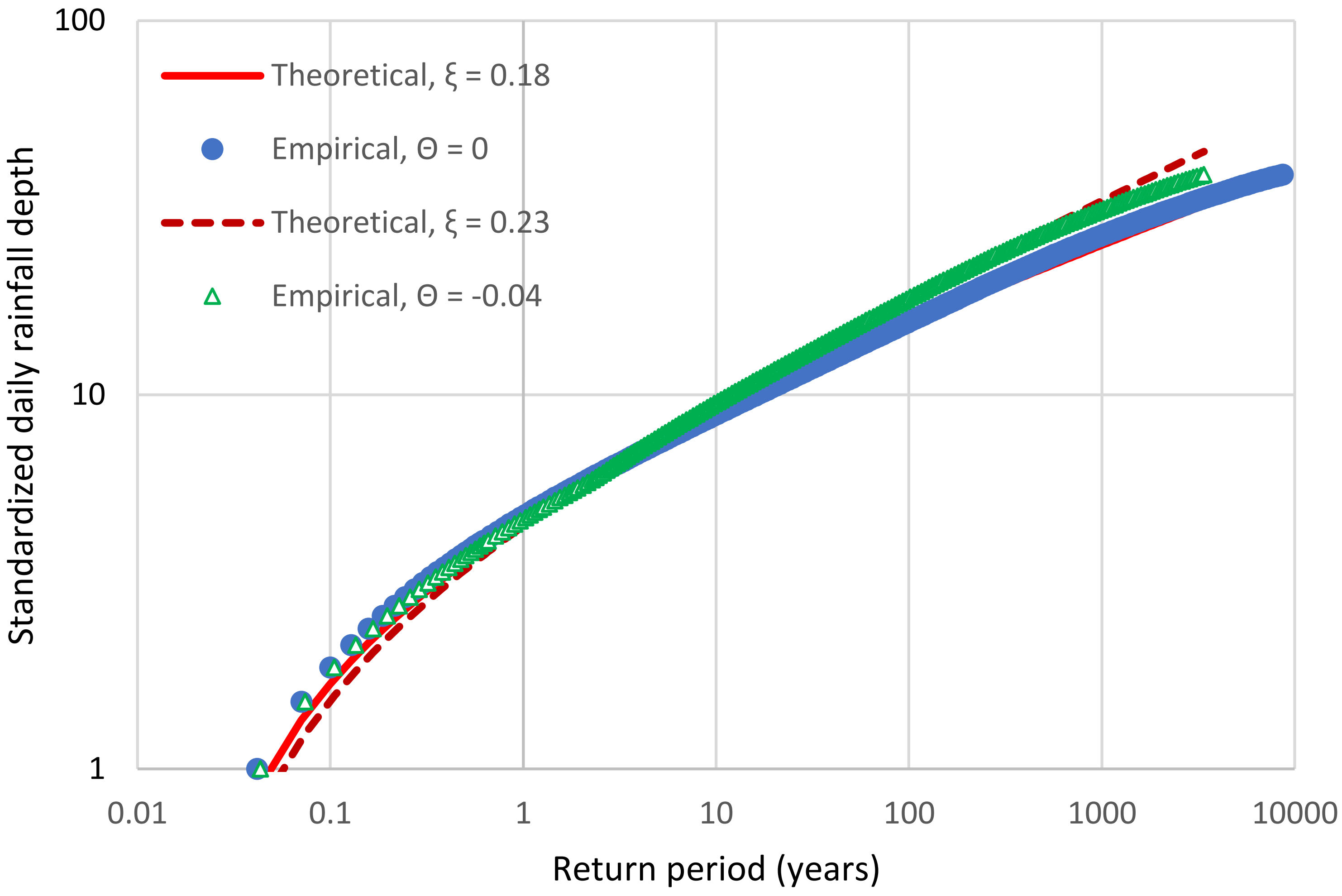
Regionalization of distribution function parameters – ξ (I)



- The parameter ξ (tail index of the distribution) was estimated individually per station and per instrument, and simultaneously with the optimization of the other parameters of the rainfall curves.

- We observe the large spatial variability of the parameter estimates, which reflects both the measurement uncertainty of maximum rainfall as well as the typical absence of systematic patterns in the realization of extreme rainfall.

Regionalization of distribution function parameters – ξ (II)

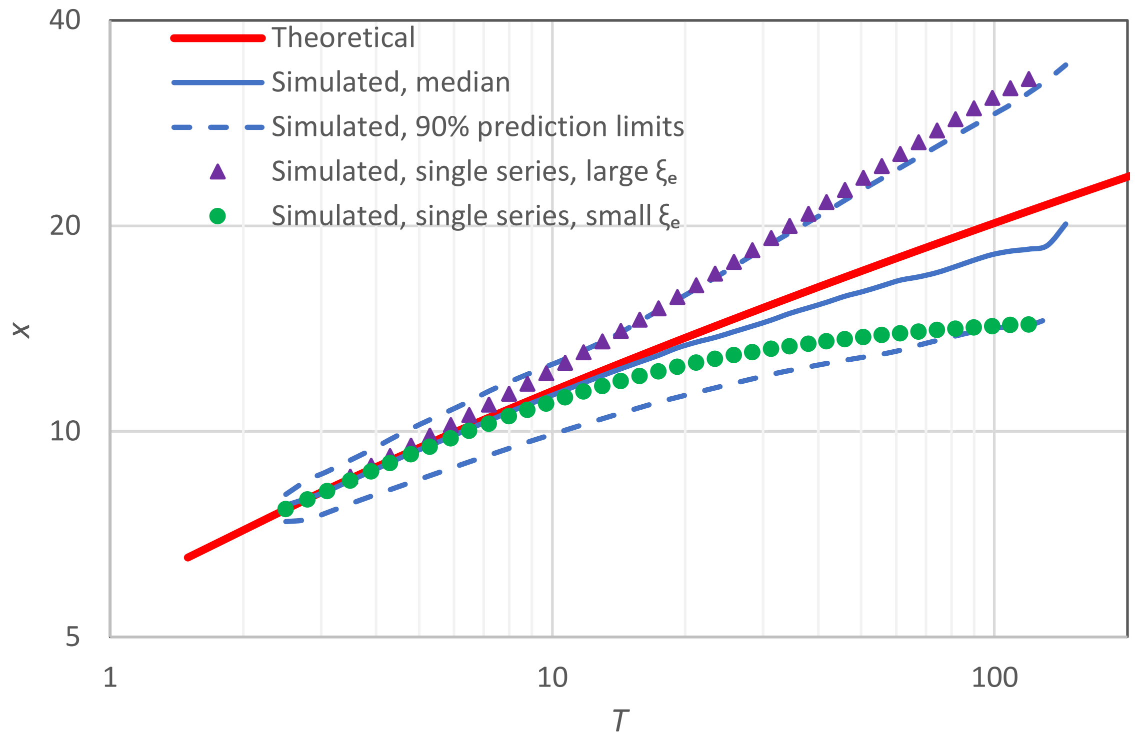


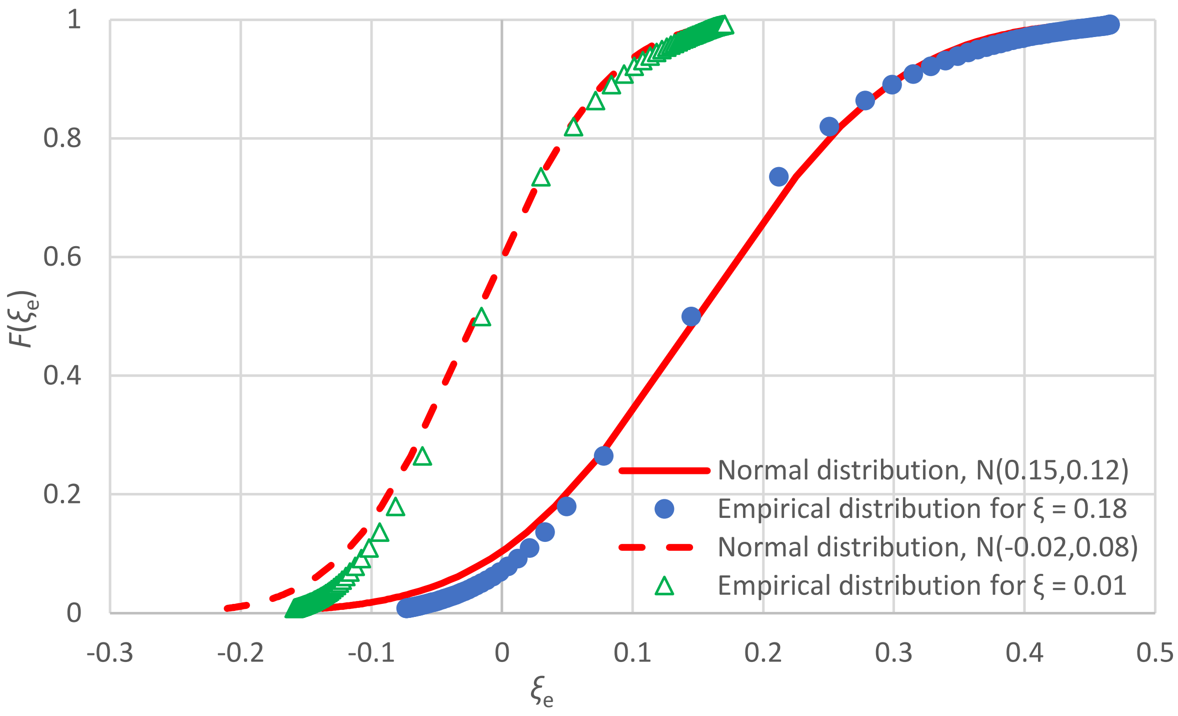
- If we assume that the entire variability of ξ estimates is a statistical effect, then:
  - We can unify (merge) all records at a certain timescale after standardizing with the mean;
  - We can estimate a unique value of ξ from the unified record.
- We have used 61 stations across the Greek territory which have at least 60 years of complete daily timeseries.
- These form a large sample of 299 481 (standardized) nonzero daily rainfall values.
- The resulting ξ is estimated to 0.18 if the different stations are assumed independent (Θ = 0) or larger if dependence is assumed (ξ = 0.23 for Θ = -0.04, where Θ denotes bias; see Koutsoyiannis, 2022 for details).
- The minimal value if ξ = 0.18 is finally chosen.

Regionalization of distribution function parameters – ξ (III)

Monte Carlo simulation results (70 simulations with Pareto distribution, each corresponding to 70 years of rainfall):

- Show the large variability of the estimated value of ξ (ξ_e), spanning from ~-0.1 to ~0.5, when the true value is ξ = 0.18.
- Verify the consistency of the assumption of a single ξ = 0.18 for the entire Greece.





Investigation of alternative options for regionalization of parameters β and λ

In addition to the direct regionalization of parameters β and λ, we investigated the use of alternative quantities linked to characteristic rainfall intensities, since the statistical behaviour of the latter is more robust and better suitable for regionalization (no boundary issues and better spatial coherence).

Specifically, we express parameters β and λ as functions of either the rainfall intensities x_1 and x_2 corresponding to return periods T_1= 2 years and T_2= 100 years, respectively, or equivalently, of x_1 and the ratio $r_x \coloneqq x_2/x_1$, as follows:

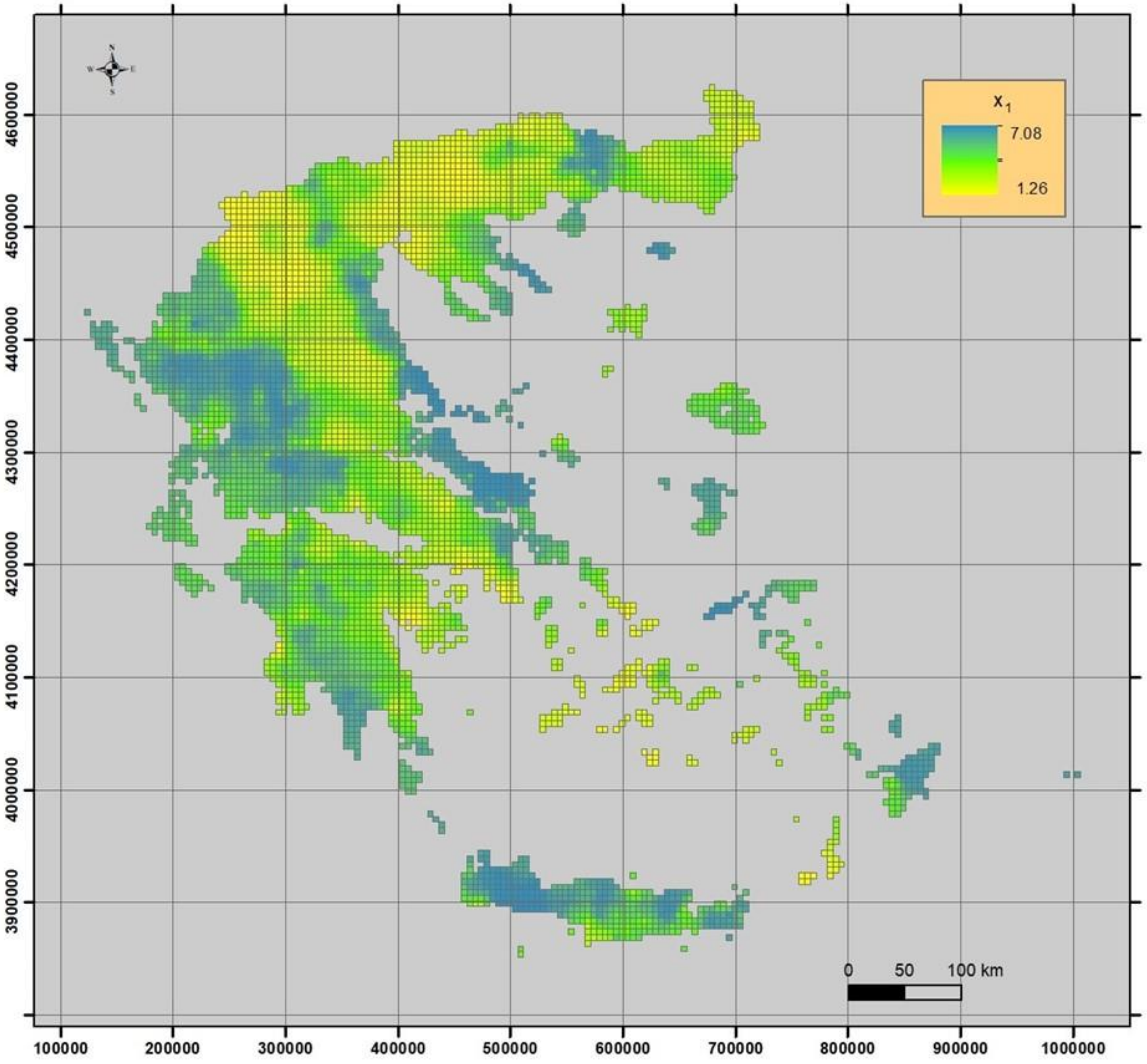
$$\beta = \left(\frac{r_x - r_T}{r_x - 1}\right)^{1/\xi} T_1, \qquad r_T \coloneqq (T_2/T_1)^{\xi}, \quad r_x \coloneqq x_2/x_1$$

$$\lambda = b\,\frac{r_x - r_T}{r_T - 1}\,x_1, \qquad b \coloneqq (1 + k/\alpha)^{\eta}$$

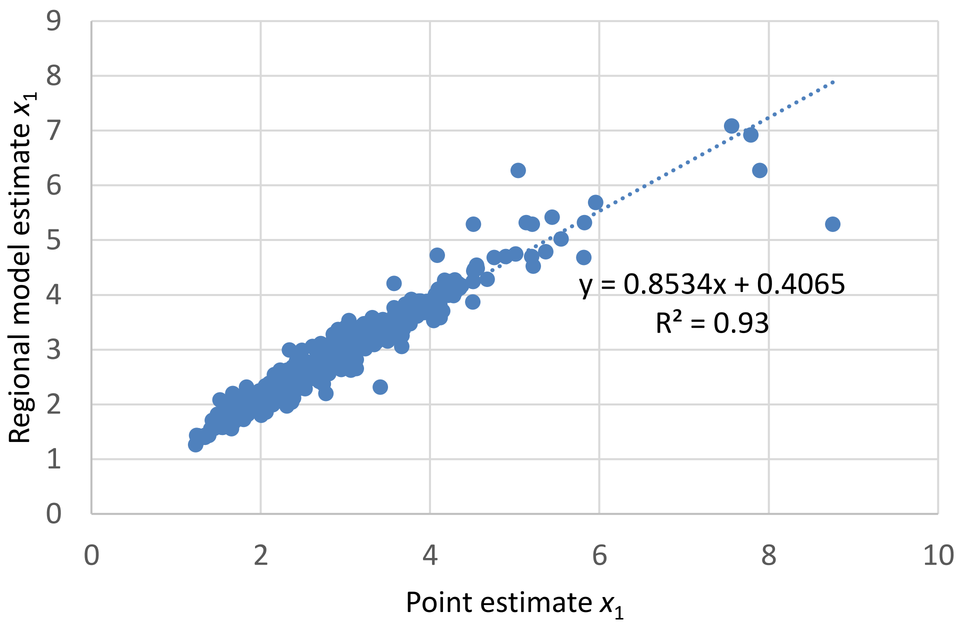
After examining the correlations between the alternative parameters sets, we chose to use the pair of parameters x_1 and r_x, since they are found uncorrelated with each other and thus the pair's information content is not affected by redundancy.

The intensities x_1 and x_2 are modelled at the 24 h scale (k = 24 h).

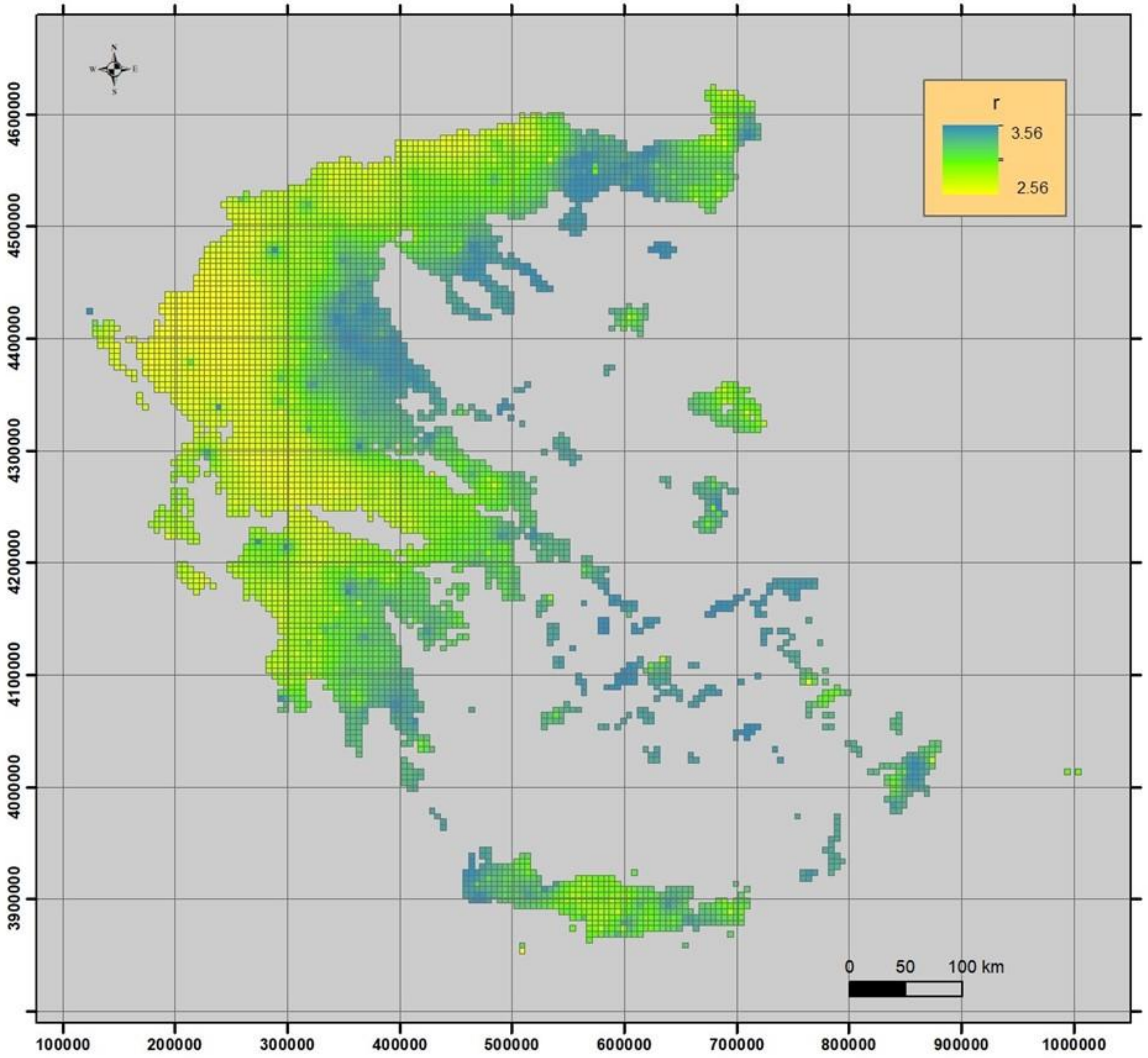
Regionalization of x_1



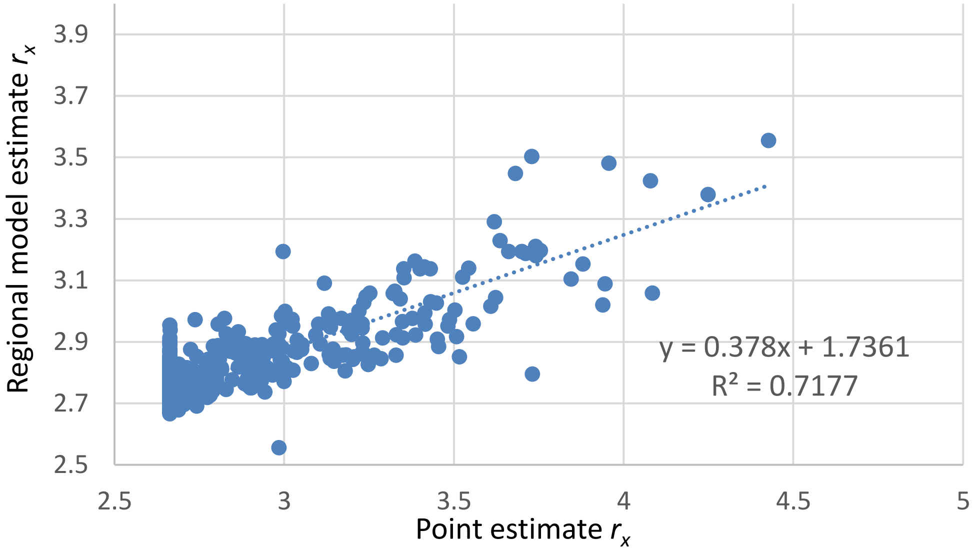
In the case of the x_1 parameter, the best statistical characteristics of the fitting in terms of leave-one-out cross-validation resulted from the application of the spatial model of the IDW method.



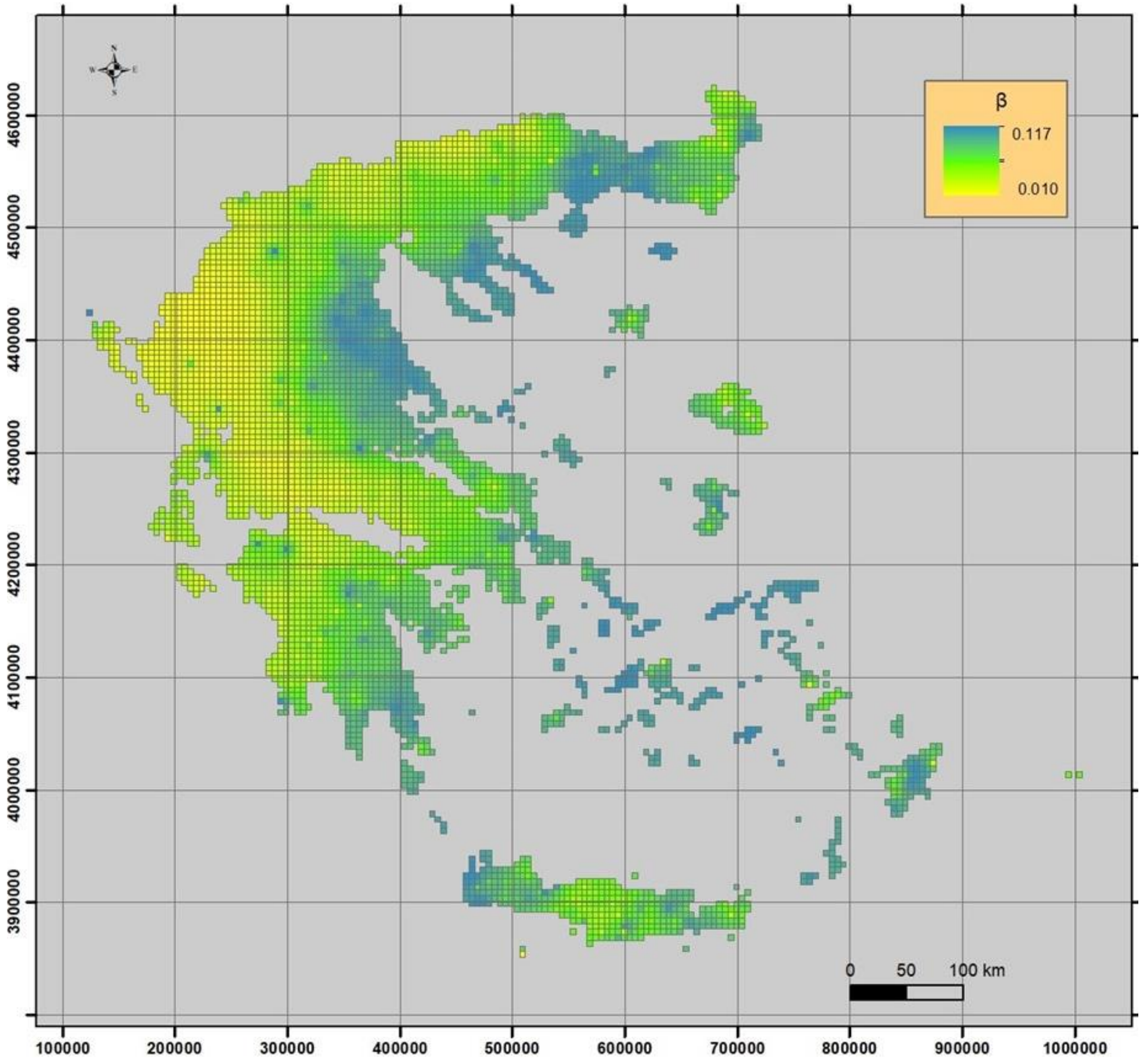
Regionalization of r_x

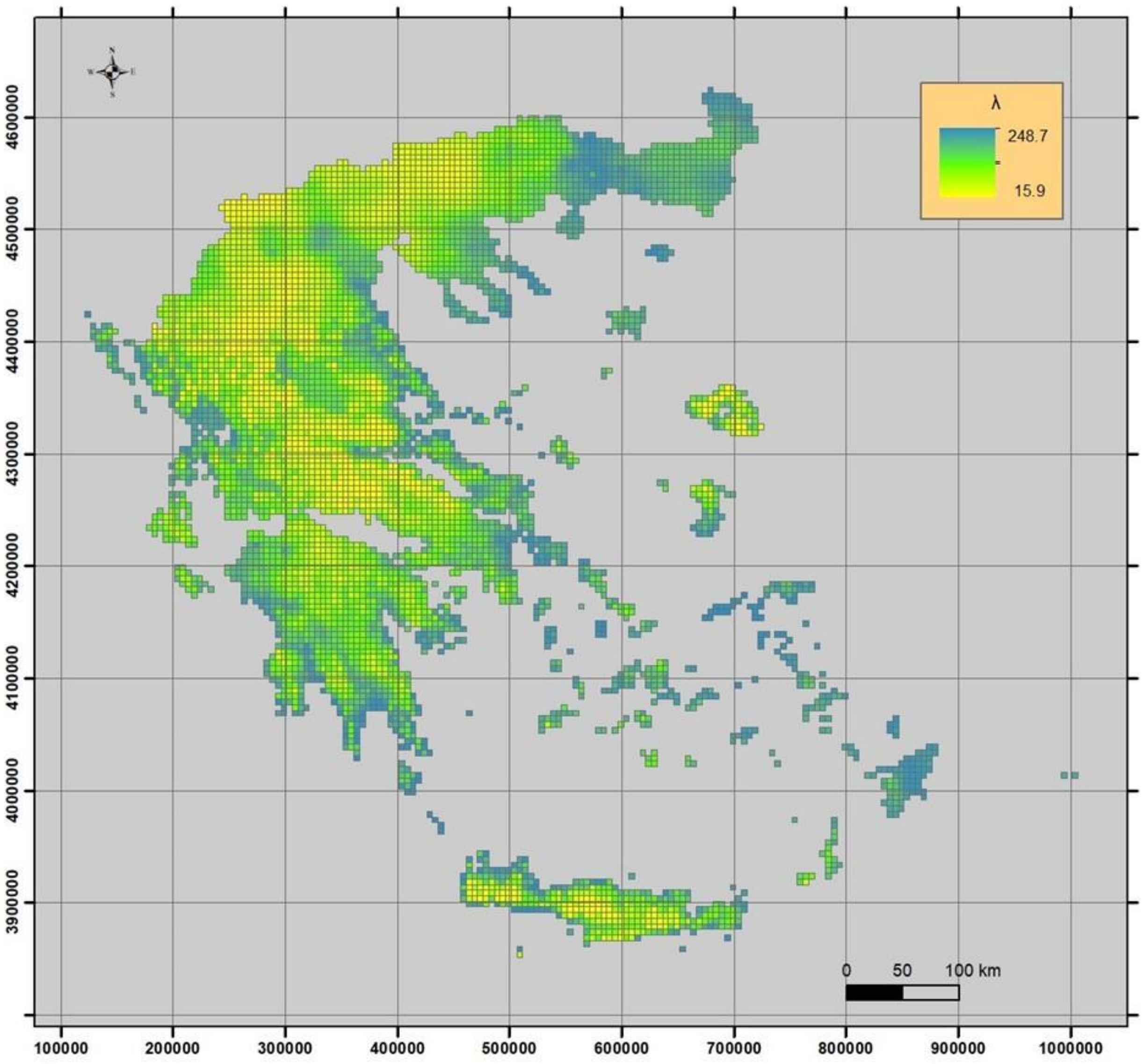


In the case of the r_x parameter, the best statistical characteristics of the fitting in terms of leave-one-out cross-validation resulted from the application of the spatial model of the IDW method.



Regionalization of distribution parameters – β & λ





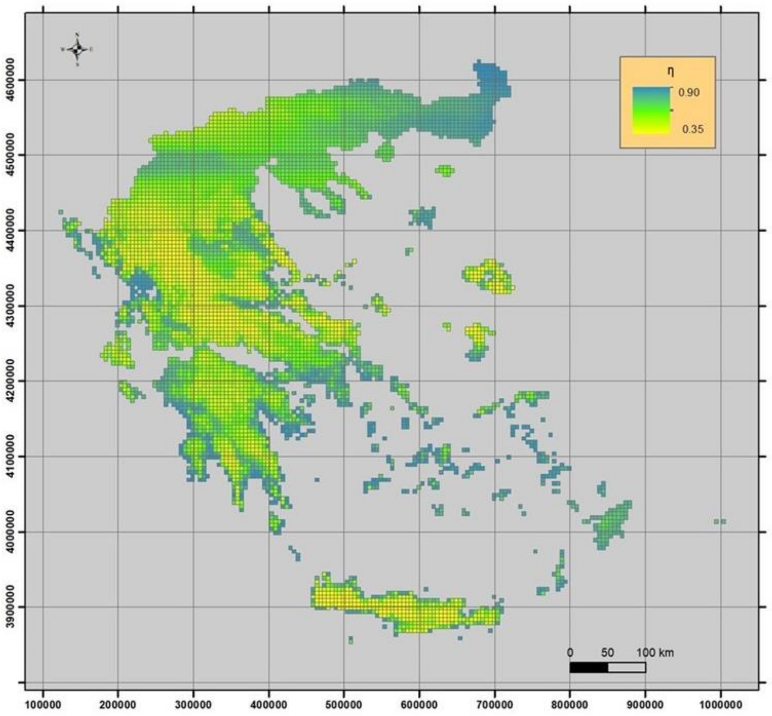
Final parameterization

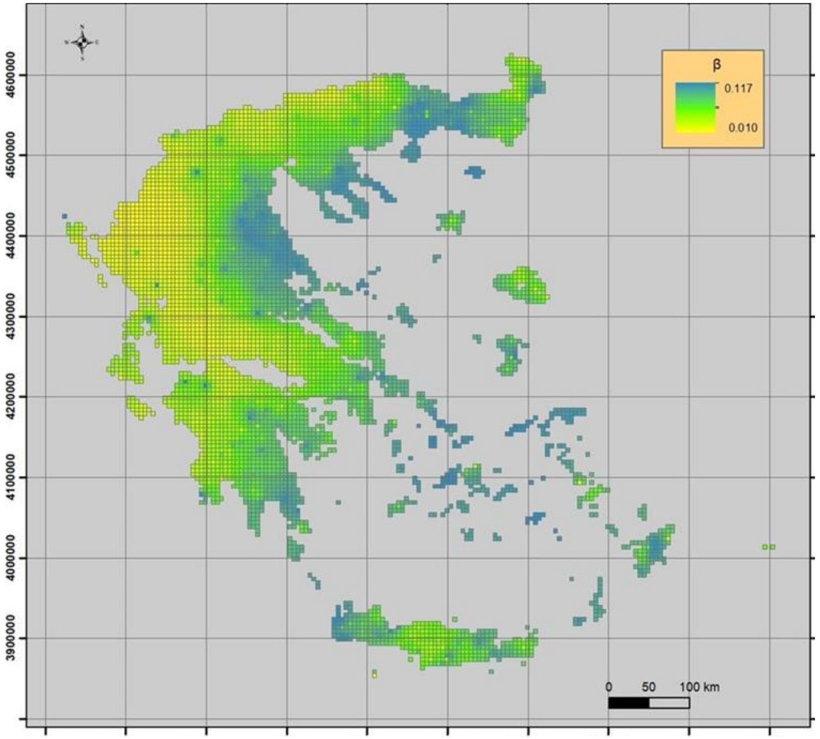
The following generalized form of ombrian curves is derived for rainfall intensity *x* (mm/h), return period *T* (years) and temporal scale *k* (h):

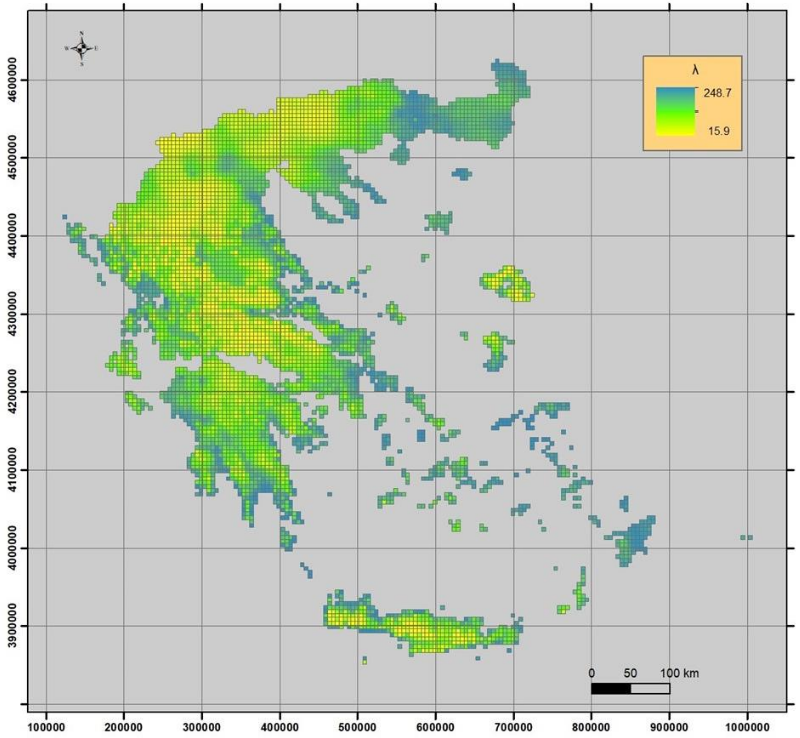
$$x = \lambda_* \frac{(T/\beta_*)^{\xi} - 1}{(1 + k/\alpha)^{\eta_*}}$$

with the following five parameters

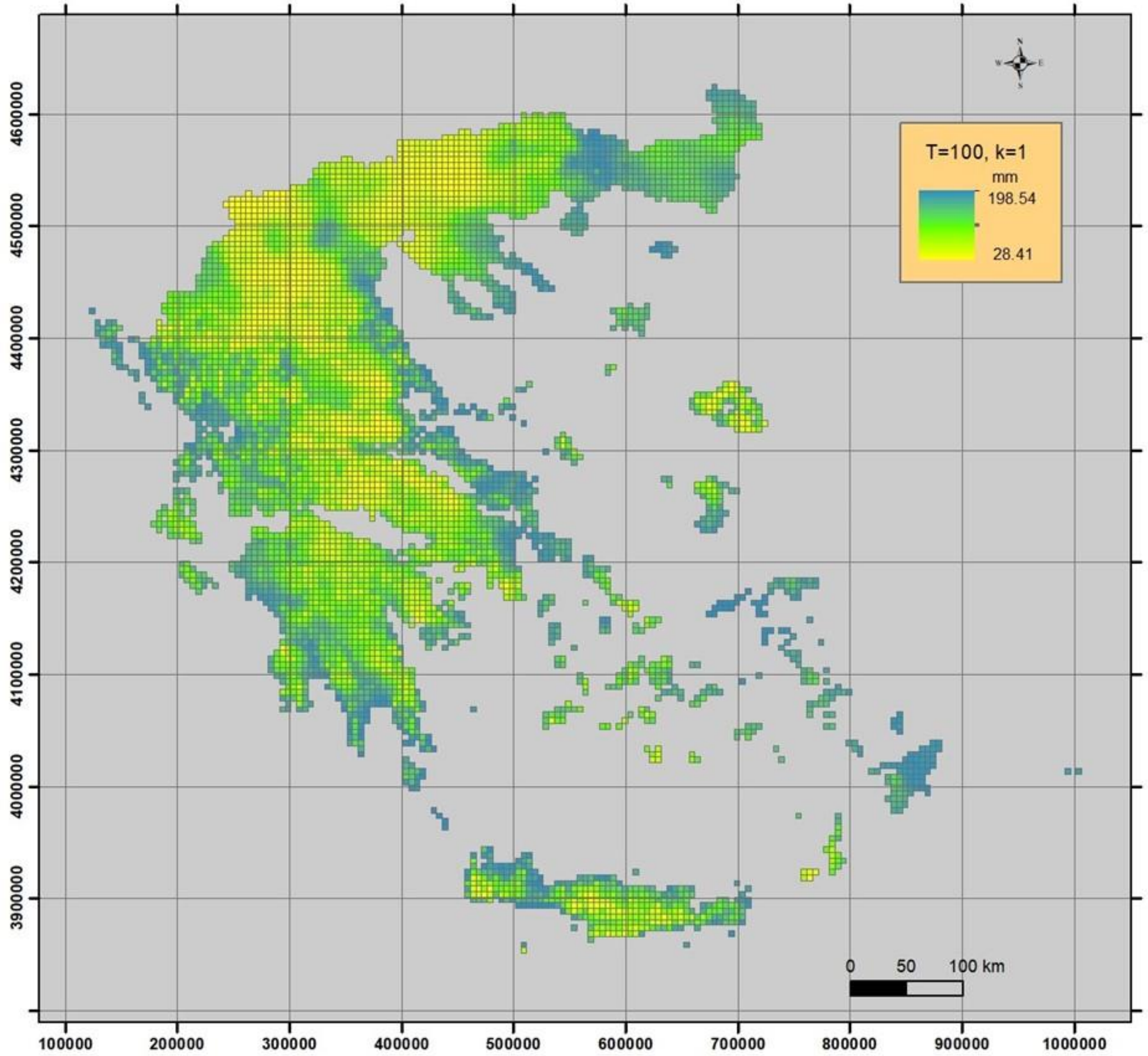
- characteristic timescale α = 0.18 h
- tail index ξ = 0.18,
- three spatially varying parameters $\eta_*[-]$, β_* (years) and λ_* (mm/h) :

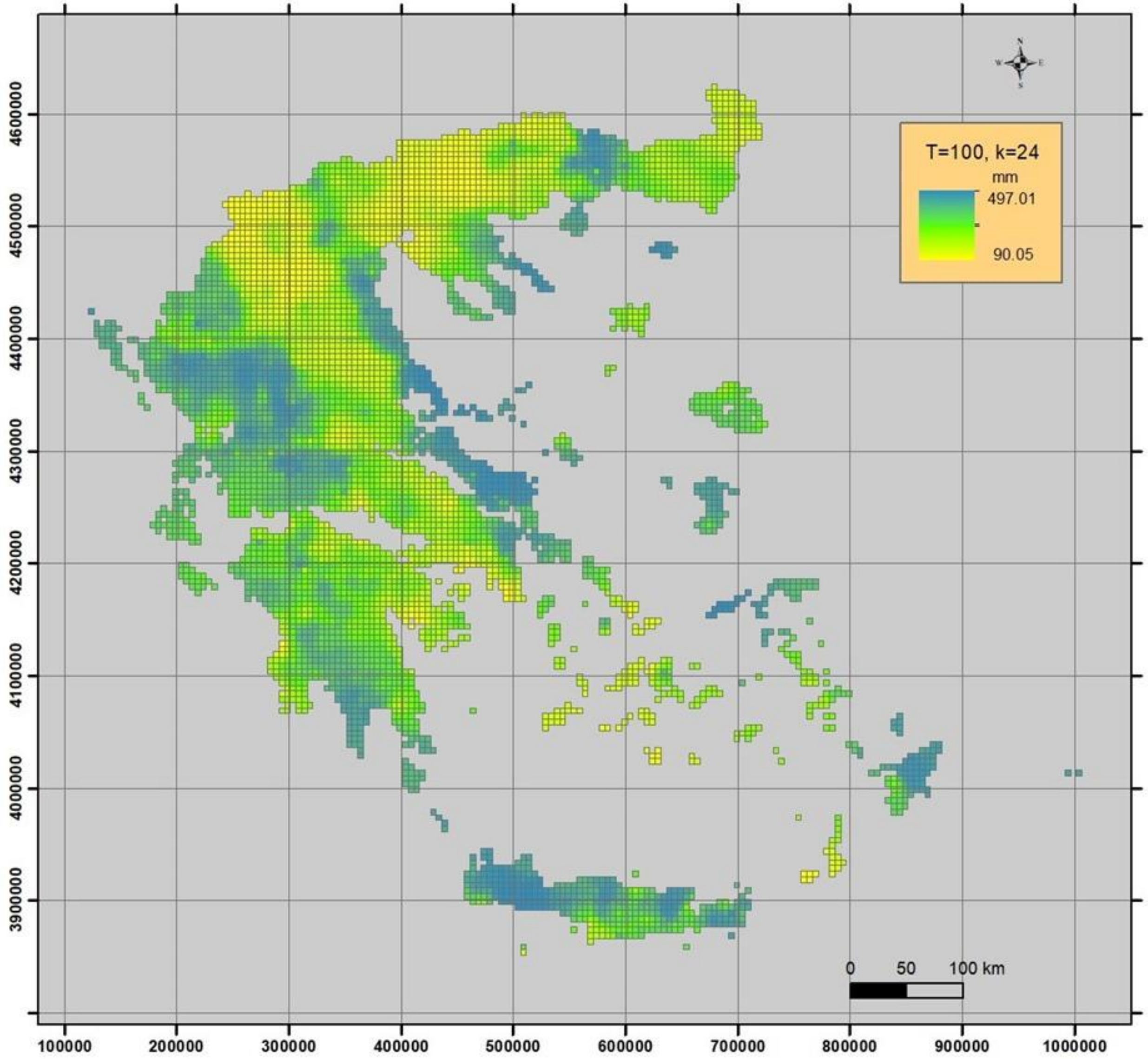




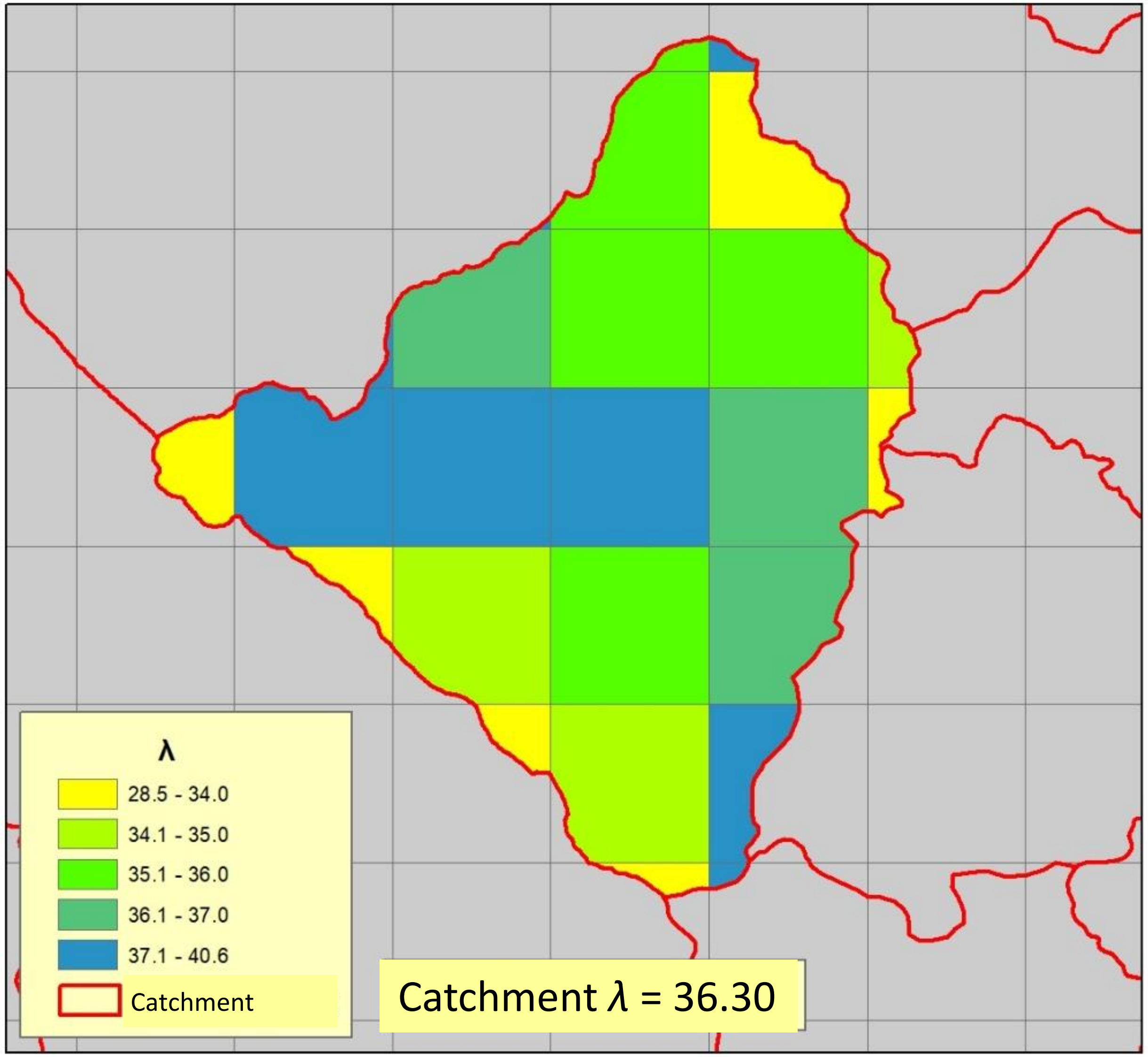


Final design rainfall depth maps





Design rainfall for the catchment scale



The ombrian curves for any region within the Greek territory are derived based on the two constant-value parameters and the three regionally varying parameters which are obtained as a weighted average of the grid points falling within the area.

Conclusions

- The approach followed incorporates an advanced framework for regional frequency analysis employing knowable (K-) moments that allow:
  - ✓ reliable high-order moment estimation; significantly increasing the number of moments that can be justifiably employed in regional analyses of extremes; and
  - ✓ handling of temporal and spatial dependence, which is non-negligible.
- The regionalization approach enables integration of smoothing and interpolation methods and flexible usage of different data sources.
- The final product is a powerful tool, easy to apply for engineering tasks, covering the entire territory of Greece.
- The methodology can be readily applied to other countries or parts thereof.

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