The superiority of refined reservoir routing (RRR) in modelling atmospheric carbon dioxide

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Abstract Prompted by post-publication comments on my recent paper "*Refined reservoir routing (RRR) and its application to atmospheric carbon dioxide balance*", I present a multicompartment carbon balance model whose results turn out to agree with those in the paper and disagree with IPCC's official ones. I also discuss an additional approach, which is popular among mainstream sceptics, as well as several additional comments I received.

1 Introduction

In my recent paper "*Refined reservoir routing (RRR) and its application to atmospheric carbon dioxide balance*" [1], I developed a simple and effective methodology for reservoir modelling. Its application provided a full and accurate account of the atmospheric part of the carbon cycle. It enabled reliable and intuitive estimates of characteristic macroscopic quantities of the CO_2 dynamics, which contrast sharply with mainstream "climate science" estimates, typically based on imagination and climatic models full of assumptions.

Naturally, this sharp contrast caused discomfort to climate orthodoxy believers. In addition, the novelty of the methodology and in particular the fact that the RRR approach is applied to a single reservoir based on data, rather than on a system of many compartments based on models, seemed strange to many colleagues. As a result, I have received several post-publication comments on my paper, most of which focused on the appropriateness of a single-reservoir approach and the differences with a multi-compartment approach.

The comments prompted me to investigate the multi-compartment approach and compare its results to those of the RRR method. By searching the related literature I understood that it lacks clarity and therefore I tried to give clear definitions (Section 3), by adapting those of the single-reservoir case (Section 2). Following the typical assumption of the related literature, I too assume that the system dynamics of the multi-compartment system is linear, an assumption that enables analytical results (Section 4). The comparison (Section 5) shows that, whatever can be produced by the multi-compartment model, generally agrees with the RRR approach, with the latter being superior in representing observations. An additional approach, which is popular among mainstream sceptics, is discussed in Annex A. Some additional remarks on issues beyond the multi-compartment approach are also discussed in Annex B.

2 Synopsis of the RRR approach

First off, the RRR methodology, proposed in [1], provides a complete mathematical framework for a system consisting of a single reservoir with linear or nonlinear dynamics,

which receives an external inflow and produces an outflow. The framework enables finding the outflow from known inflow and obtaining macroscopic characteristics of the processes, such as response and residence times, and their distribution functions. What is more, the study in [1] critiques the ambiguity in the existing literature, proposes refined definitions of the key concepts of reservoir routing to restore clarity, and includes a glossary summarizing the definitions. From the glossary, I copy here a few entries that are relevant to this essay (with slight adaptations fit for purpose).

Linear reservoir: A reservoir in which the outflow is proportional to storage. Any other type of storage–outflow relationship defines a *nonlinear reservoir*.

Residence time (\underline{W}): The time duration that a particle (molecule) spends in the reservoir from its entry to its exit. Excepting the (unrealistic) case of a perfectly regular (laminar) flow, the residence time is different for different molecules and is therefore represented as a stochastic variable (hence the underscore in the notation).

Characteristic residence time (W_0): The time that is defined as the ratio $W_0 := S_0/Q_0$, where S_0 and Q_0 represent the initial conditions of storage and outflow, respectively, at time t = 0. In a linear reservoir, W_0 is equal to the mean residence time, μ_W , i.e., constant and independent of the initial conditions.

Impulse response function (IRF, $g_h(h)$): A system's output at a time distance (lag) h from the time in which the system is perturbed by an input that is an (instantaneous) impulse of unit mass (a Dirac delta function with total mass 1 MU, with MU denoting the mass unit).

Note that the dimensions of h and $g_h(h)$ are [T] and [M/T], respectively. An interesting property is that the IRF is identical to the probability density function of the residence time (times MU) for the case that the input is an impulse function. The integral over the positive semiaxis is $\int_0^\infty g_h(h) dh = 1$ MU. The mean and median of the function $g_h(h)$ define the *mean response time* and the *median response time*, respectively.

The study [1] also provides a novel framework for understanding the dynamics of carbon dioxide (CO_2) in the atmosphere. By application of the RRR framework, the atmospheric carbon exchanges are easily quantified, and reliable and intuitive results are obtained. Not only does this application not need to resort to complex climate models, but its results are in absolute disagreement with climate model results, while being in excellent agreement with real-world carbon dioxide concentration ($[CO_2]$) data. Specifically, the mean residence time of atmospheric carbon dioxide, as calculated by the RRR framework based on real-world data, is about four years and the response time is even smaller, but the official IPCC [2,3] estimates, based on climate models, are larger by orders of magnitude.

To achieve such long estimates, IPCC [2,3] obscures the concepts and uses an "intentionally vague" language [4]. It uses the terms *lifetime, turnover time, global atmospheric lifetime, response time, adjustment time, half-life* or *decay constant,* none of which is clear enough to allow quantification and distinction of what is referred to each time. IPCC admits that the "turnover time [of CO_2] is only about 4 years". On the other hand, it contrasts CO_2 to other substances and becomes as vague as possible about it, e.g. :

- [T]he concept of a single, characteristic atmospheric lifetime is not applicable to CO₂ [2] (p. 473).
- No single lifetime can be given [for CO₂]. The impulse response function for CO₂ from Joos et al. (2013) [5] has been used [2] (p. 737).
- *Lifetime* [for well-mixed greenhouse gases] *is reported in years: # indicates multiple lifetimes for CO*² [3] (p. 302, Table 2.2; see also p. 1017, Table 7.15).

Furthermore, IPCC insists on the weird idea that the behaviour of the CO_2 depends on its origin and that CO_2 emitted by anthropogenic fossil fuel combustion has higher residence time than naturally emitted:

- Simulations with climate carbon cycle models show multi-millennial lifetime of the anthropogenic CO₂ in the atmosphere [2] (p. 435).
- This delay between a peak in emissions and a decrease in concentration is a manifestation of the very long lifetime of CO₂ in the atmosphere; part of the CO₂ emitted by humans remains in the atmosphere for centuries to millennia [3] (p. 642, FAQ 4.2).

The evidence that is based on real-world data, the clarity of the definitions in [1] and the sound RRR mathematical framework counter the IPCC claims and show the CO_2 mean residence time in the atmosphere is:

- (a) independent of the origin (anthropogenic or not);
- (b) about 4 years on an overannual basis (there is no multi-millennial lifetime);
- (c) seasonally varying with lowest value < 2 years.

In addition, as seen in Figure 1, the RRR framework captures with good agreement:

- (a) the increase in atmospheric [CO₂];
- (b) the seasonal variation of atmospheric [CO₂]; and
- (c) the expansion of the biosphere (attributed to temperature increase).



Figure 1. Reproduction of the graphical abstract in [1].

3 Adaptation of the reservoir routing concepts to the multicompartment system with linear dynamics

In this section, I extend the definitions of the dynamics of a single reservoir to cover the case of a system comprising many interconnected reservoirs or compartments, as are more usually called. First, I give a definition of such a system:

Multi-compartment system: a system consisting of several components (reservoirs), C_i , which store mass and exchange mass with each other and with the environment.

A system component C_i outflows and receives inflows from other components C_j . I denote $S^{C_i}(t)$ the storage of mass at component C_i at a time t, $Q^{C_iC_j}(t)$ the output of the component C_i to the component C_j , and $I^{C_iC_j}(t)$ the input to the component C_i from the component C_j . Note that each input is the output of another component, i.e., $I^{C_iC_j}(t) = Q^{C_jC_i}(t)$. If two components are disconnected from each other, then $I^{C_iC_j}(t) = Q^{C_jC_i}(t) = 0$. Mass conservation implies the continuity equation in the following form:

$$\frac{\mathrm{d}S^{C_i}(t)}{\mathrm{d}t} + \sum_j Q^{C_i C_j}(t) = \sum_j I^{C_i C_j}(t) \tag{1}$$

While the study [1] examined nonlinear reservoirs, with the linear ones resulting as a special case of the former, here I limit the analysis to a system of linear reservoirs, as has been the case in typical multi-component models.

To extend the definition of a linear reservoir to a linear multi-compartment system, I provide the following definition:

Linear multi-compartment system: A multi-compartment system in which the outflow from any component to any other component is proportional to the storage of the origin component, i.e.,

$$Q^{C_i C_j}(t) = k^{C_i C_j} S^{C_i}(t)$$
⁽²⁾

Here $k^{C_iC_j}$ denotes a constant. Such a constant does not exist for inputs that are external to the system, whose magnitude should be specified independently. Since Equation (2) is valid for any time t, we can use the initial observations at time t = 0 to find the constants $k^{C_iC_j}$. Namely, we have

$$W^{C_i C_j} = \frac{S^{C_i}(0)}{Q^{C_i C_j}(0)}, \qquad k^{C_i C_j} = \frac{1}{W^{C_i C_j}}$$
(3)

I call $W^{C_i C_j}$, which has dimensions of time, the *characteristic residence time*, and $k^{C_i C_j}$ the *inverse characteristic residence time*.

I can now proceed to the adaptation of the definition of the IRF:

Impulse response function of a system's component C_i (IRF, $g_h^{C_i}(h)$): The algebraic sum of the outputs minus inputs to the component C_i at a time distance (lag) h from the time in which the system that was in equilibrium is perturbed by an external input

to its component C_i which is an (instantaneous) impulse of unit mass (a Dirac delta function with total mass 1 MU, with MU denoting the mass unit).

Mathematically, the definition is expressed as:

$$g_{h}^{C_{i}}(h) = \sum_{j} Q^{C_{i}C_{j}}(h) - \sum_{j} I^{C_{i}C_{j}}(h)$$
(4)

From the continuity equation, given that the system was initially at equilibrium and that there is no further external perturbation after the impulse of unit mass, we get:

$$g_h^{C_i}(h) = -\frac{\mathrm{d}S^{C_i}(h)}{\mathrm{d}h} \tag{5}$$

This allows an alternative definition as follows:

Impulse response function of a system's component C_i (IRF, $g_h^{C_i}(h)$): The minus derivative, $-dS^{C_i}(h)/dh$, of the storage $S^{C_i}(h)$ of the component C_i , with respect to the time distance (lag) h from the time in which the system that was in equilibrium is perturbed by an external input to its component C_i which is an (instantaneous) impulse of unit mass (a Dirac delta function with total mass 1 MU, with MU denoting the mass unit).

To avoid ambiguity, in the above definitions I have specified that the perturbation is to a single component C_i and I have set two important conditions, i.e.:

- The system was at equilibrium before the perturbation by an impulse. This means that $\sum_j Q^{C_i C_j}(t) = \sum_j I^{C_i C_j}(t)$ at all components C_i and hence $dS^{C_i}(h)/dt = 0$ for times t before the perturbation. Note that the equilibrium does depend on the total mass in the system but not on the initial conditions, i.e. the distribution of the total mass into the different components.
- The perturbation is external, that is, the unit mass input to the component *C_i* was not removed from other parts. Hence the total mass in the system was not preserved but was increased by a unit mass.

If any of these two conditions were violated, the IRF would not be uniquely identified. If the system is not at equilibrium (violation of the first condition), then the resulting $- dS^{C_i}(h)/dh$ would not represent the IRF. Furthermore, if the input to the component C_i was removed from another component C_j (violation of the second condition), the resulting $- dS^{C_i}(h)/dh$ would be different from the IRF defined as above. Yet the differences may not be visible as will be illustrated later (Section 5).

It is useful to note that in the case of violation of the first condition, we can again recover the exact IRF by the following, difference-based algorithm which finds identical results (the proof is omitted):

- Solve the system with unperturbed initial conditions.
- Solve again the system with initial conditions perturbed by an impulse.
- Find the IRF as the difference of $-dS^{C_i}(h)/dh$ between the above two cases.

4 A three-compartment model with linear dynamics applied to the atmospheric CO₂

In this section, I apply the definitions and equations of the previous section to model the atmospheric [CO₂]. In contrast to the study [1], in which I have used real-world data on monthly scale, here I make theoretical calculations on annual scale without data (but later, in Section 5.3, I will also be use data). The only empirical basis I use here is the carbon balance as given in the latest IPCC report (AR6) for the most recent reference period. The estimates given by IPCC [3] (Figure 5.12) are reproduced in summarized form in Figure 2. The schematic shows a system representing Earth's carbon with three compartments, atmosphere, land and oceans, which is modelled as will be seen below. Notice that the three-compartment system does not include fossil fuel stocks, which are regarded as external to that system. The fluxes due to fossil fuel burning and land use changes (anthropogenic changes) are modelled as an external input to the system, specifically to the atmospheric compartment as, apparently, these cannot be modelled using physical or biogeochemical laws.



Figure 2. Schematic of the three-compartment system of Earth's CO_2 with initial and equilibrium storages $(S_0 \text{ and } S_E, respectively, in Gt C)$, initial fluxes $(Q_0 \text{ in Gt } C/\text{year})$ and inverse characteristic residence times (k in years⁻¹). The initial storages S_0 and fluxes Q_0 are the "official" IPCC estimates [3] (Figure 5.12) and are representative for year 2018. The parameters k are calculated by Equation (7) from the S_0 and Q_0 . The anthropogenic component is modelled as an external input to the atmospheric component. The equilibrium storages S_E are calculated from the three-compartment model and for zero anthropogenic input.

The model presented in this section is unable to capture the seasonal variation and the overyear increasing trend of $[CO_2]$, let alone attribute it to causal factors. Furthermore, it is unable to faithfully reproduce the biosphere expansion seen in Figure 1. Yet I use it for the sake of comparison of the macroscopic characteristics it produces with those in the study [1] and IPCC [3]. Note that, while I use the IPCC carbon balance estimates, I do not use the IPCC IRF and therefore the comparison I make is meaningful.

To maximize the explanatory power of the analysis and enable better understanding of the system dynamics, in this section I only provide analytical calculations, occasionally making justified approximations that facilitate analytical expressions. In Section 5, I abandon these approximations and give precise numerical results.

At a first phase, the external input is not considered (see other cases below and Subsection 5.3 in which it is considered). The continuity equations for the three-compartment model are:

$$\frac{dS^{A}(t)}{dt} + Q^{AL}(t) + Q^{AO}(t) = Q^{LA}(t) + Q^{OA}(t),$$

$$\frac{dS^{L}(t)}{dt} + Q^{LA}(t) = Q^{AL}(t), \qquad \frac{dS^{O}(t)}{dt} + Q^{OA}(t) = Q^{AO}(t)$$
(6)

By virtue of the linearity assumption in Equation (2), we can replace the fluxes contained in Equation (6) with the storages *S* multiplied by the constants *k*:

$$k^{\rm AL} = \frac{Q_0^{\rm AL}}{S_0^{\rm A}}, \qquad k^{\rm AO} = \frac{Q_0^{\rm AO}}{S_0^{\rm A}}, \qquad k^{\rm LA} = \frac{Q_0^{\rm LA}}{S_0^{\rm L}}, \qquad k^{\rm OA} = \frac{Q_0^{\rm OA}}{S_0^{\rm O}}$$
(7)

the values of which are shown in Figure 2. The respective characteristic residence times are:

$$W^{AL} = \frac{1}{k^{AL}} = 6.1 \text{ years}, \qquad W^{AO} = \frac{1}{k^{AO}} = 10.9 \text{ years},$$

 $W^{LA} = \frac{1}{k_{LA}} = 24.5 \text{ years}, \qquad W^{OA} = \frac{1}{k^{OA}} = 498.7 \text{ years}$ (8)

Focusing on the atmospheric part, there are two different sinks that operate in parallel. As explained in [1] (Appendix B), the characteristic time in this case will be equal to the inverse sum of reciprocals (or to the harmonic mean of the characteristic times divided by the number of sinks). This results in a characteristic time of the atmosphere equal to $W^{A} = 3.9$ years, i.e., smaller than $W^{AL} = 6.1$ years and $W^{AO} = 10.9$ years. This is almost the same as the result in [1] and irrelevant to the IPCC's estimates reaching "several hundred thousand years" [3] (p. 2237).

Using the constants in Equation (7), Equation (6) is written as

$$S^{A'}(t) = -(k^{AL} + k^{AO})S^{A}(t) + k^{LA}S^{L}(t) + k^{OA}S^{O}(t),$$

$$S^{L'}(t) = -k^{LA}S^{L}(t) + k^{AL}S^{A}(t), \qquad S^{O'}(t) = -k^{OA}S^{O}(t) + k^{AO}S^{A}(t)$$
(9)

This is a system of homogenous first-order linear differential equations, which can be written in vector form as

$$\mathbf{S}'(t) = \mathbf{K} \, \mathbf{S}(t), \qquad \mathbf{S}(t) \coloneqq \begin{bmatrix} S^{\mathrm{A}}(t) \\ S^{\mathrm{L}}(t) \\ S^{\mathrm{O}}(t) \end{bmatrix}, \qquad \mathbf{K} \coloneqq \begin{bmatrix} -(k^{\mathrm{AL}} + k^{\mathrm{AO}}) & k^{\mathrm{LA}} & k^{\mathrm{OA}} \\ k^{\mathrm{AL}} & -k^{\mathrm{LA}} & 0 \\ k^{\mathrm{AO}} & 0 & -k^{\mathrm{OA}} \end{bmatrix}$$
(10)

Calculation of the eigenvalues λ_i of the matrix **K** yields:

$$\lambda_{0} = 0, \qquad \lambda_{1,2} = -\frac{\Sigma k}{2} \left(1 \pm \sqrt{1 - \frac{4(k^{AO} k^{LA} + k^{AL} k^{OA} + k^{LA} k^{OA})}{\Sigma k^{2}}} \right), \tag{11}$$
$$\Sigma k \coloneqq k^{AL} + k^{AO} + k^{LA} + k^{OA}$$

The fraction within the square root is $\ll 1$ and hence the eigenvalues are real numbers. Furthermore, thanks to this small value, we may use the approximation $\sqrt{1-a} \approx 1-a/2$ and find

$$\lambda_0 = 0, \qquad \lambda_{1,2} \approx -\frac{\Sigma k}{2} \left(1 \pm \left(1 - \frac{2(k^{AO} k^{LA} + k^{AL} k^{OA} + k^{LA} k^{OA})}{\Sigma k^2} \right) \right)$$
(12)

which after the algebraic operations yields:

$$\lambda_0 = 0, \qquad \lambda_1 \approx -\Sigma k - \lambda_2, \qquad \lambda_2 \approx -\frac{k^{AO} k^{LA} + k^{AL} k^{OA} + k^{LA} k^{OA}}{\Sigma k}$$
(13)

Substituting the values shown in in Figure 2, we find

$$\Sigma k = 0.2974, \quad \lambda_0 = 0, \quad \lambda_1 \approx -0.2835, \quad \lambda_2 \approx -0.0139$$
 (14)

with units of years⁻¹ for all constants. Note that $|\lambda_2| \ll |\lambda_1|$ with a ratio $\lambda_2/\lambda_1 \approx 1/20$. If we did not use the above approximation we would find that the exact values are

$$\lambda_0 = 0, \qquad \lambda_1 \approx -0.2828, \qquad \lambda_2 = -0.0146$$
 (15)

again with units of years⁻¹. These are very close to the approximation.

Now, from standard calculus of differential equations, the solution of the system of differential equations is:

$$S^{A}(t) = S^{A}_{E} + \kappa^{A}_{1} e^{\lambda_{1}t} + \kappa^{A}_{2} e^{\lambda_{2}t},$$

$$S^{L}(t) = S^{L}_{E} + \kappa^{L}_{1} e^{\lambda_{1}t} + \kappa^{L}_{2} e^{\lambda_{2}t}, \qquad S^{O}(t) = S^{O}_{E} + \kappa^{O}_{1} e^{\lambda_{1}t} + \kappa^{O}_{2} e^{\lambda_{2}t}$$
(16)

The constants S_E^A , S_E^L , S_E^O denote the equilibrium storages as can be seen by letting $t \to \infty$ and observing that $\lambda_i < 0$, so that $\lim_{t\to\infty} e^{\lambda_1 t} = 0$. The parameters κ_i^X ; i = 1,2; X = A, L, O depend on the initial conditions S_0^A , S_0^L , S_0^O .

Now, to find the IRF of the atmospheric $[CO_2]$ we take the equilibrium storages and modify them as follows, according to the definition in Section 3:

$$S^{\rm A}(0) = S^{\rm A}_{\rm E} + 1 \,\,{\rm MU}, \qquad S^{\rm L}(0) = S^{\rm L}_{\rm E}, \qquad S^{\rm O}(0) = S^{\rm O}_{\rm E}$$
(17)

Since the total storage has now changed by 1 MU, the equilibrium storages will also change. Given the tremendously higher storage of the oceans, compared to the other two components, we may assume that most part of the 1 MU change will end up at the ocean and approximate the new equilibrium as

$$S_E^{A^*} \approx S_E^A, \qquad S_E^{L^*} \approx S_E^L, \qquad S_E^{O^*} \approx S_E^O + 1 \text{ MU}$$
 (18)

The new solution of the differential equation for the initial conditions (17) and the equilibrium storages (18) will be

$$S^{A}(t) = S^{A^{*}}_{E} + \kappa^{A^{*}}_{1} e^{\lambda_{1}t} + \kappa^{A^{*}}_{2} e^{\lambda_{2}t},$$

$$S^{L}(t) = S^{L^{*}}_{E} + \kappa^{L^{*}}_{1} e^{\lambda_{1}t} + \kappa^{L^{*}}_{2} e^{\lambda_{2}t}, \qquad S^{O}(t) = S^{O^{*}}_{E} + \kappa^{O^{*}}_{1} e^{\lambda_{1}t} + \kappa^{O^{*}}_{2} e^{\lambda_{2}t}$$
(19)

Notice that the constants κ_i^X have changed to $\kappa_i^{X^*}$, while the eigenvalues λ_i have not changed as these depend only on the inverse characteristic residence times (Equation (13)) and not on the initial or the equilibrium storages. We can find the constants $\kappa_i^{X^*}$ by evaluating the storages and their derivatives at t = 0.

For the storages we have:

$$S^{A}(0) = S^{A^{*}}_{E} + \kappa^{A^{*}}_{1} + \kappa^{A^{*}}_{2},$$

$$S^{L}(0) = S^{L^{*}}_{E} + \kappa^{L^{*}}_{1} + \kappa^{L^{*}}_{2}, \qquad S^{O}(0) = S^{O^{*}}_{E} + \kappa^{O^{*}}_{1} + \kappa^{O^{*}}_{2}$$
(20)

which, combined with Equations (17) and (18), results in

$$1 \text{ MU} \approx \kappa_1^{A^*} + \kappa_2^{A^*}, \qquad 0 \approx \kappa_1^{L^*} + \kappa_2^{L^*}, \qquad -1 \text{ MU} \approx \kappa_1^{O^*} + \kappa_2^{O^*}$$
(21)

For the derivatives $c^X \coloneqq S^{X'}(0)$, by taking the derivatives in Equation (19) and setting t = 0, we have:

$$c^{A} = \lambda_{1}\kappa_{1}^{A} + \lambda_{2}\kappa_{2}^{A}, \qquad c^{L} = \lambda_{1}\kappa_{1}^{L^{*}} + \lambda_{2}\kappa_{2}^{L^{*}}, \qquad c^{O} = \lambda_{1}\kappa_{1}^{O^{*}} + \lambda_{2}\kappa_{2}^{O^{*}}$$
 (22)

Since the values of k are known, the values c^{X} are readily derived by combining Equations (9) and (17) and evaluating them for t = 0. Specifically, we have

$$c^{A} = -(k^{AL} + k^{AO})(S_{E}^{A} + 1 MU) + k^{LA}S_{E}^{L} + k^{OA}S_{E}^{O}$$

$$c^{L} = -k^{LA}S_{E}^{L} + k^{AL}(S_{E}^{A} + 1 MU), \qquad c^{L} = -k^{OA}S_{E}^{O} + k^{AO}(S_{E}^{A} + 1 MU)$$
(23)

On the other hand, since at the equilibrium

$$-(k^{\rm AL} + k^{\rm AO})S_{\rm E}^{\rm A} + k^{\rm LA}S_{\rm E}^{\rm L} + k^{\rm OA}S_{\rm E}^{\rm L} = 0$$

$$-k^{\rm LA}S_{\rm E}^{\rm L} + k^{\rm AL}S_{\rm E}^{\rm A} = 0, \qquad -k^{\rm OA}S_{\rm E}^{\rm O} + k^{\rm AO}S_{\rm E}^{\rm A} = 0$$
 (24)

we obtain

$$c^{A} = -(k^{AL} + k^{AO})MU, \quad c^{L} = k^{AL}MU, \quad c^{O} = k^{AO}MU$$
 (25)

It is useful to observe that mass conservation demands that the total storage:

$$S = S^{A}(t) + S^{L}(t) + S^{0}(t)$$
(26)

is constant, independent of t and its derivative is zero. From this we can obtain the following relationships:

$$\kappa_1^{A^*} + \kappa_1^{L^*} + \kappa_1^{O^*} = \kappa_2^{A^*} + \kappa_2^{L^*} + \kappa_2^{O^*} = c^A + c^L + c^O = 0$$
(27)

By solving Equations (21) and (22) (six equations with six unknowns), we find the approximate solutions:

$$\kappa_{1}^{A^{*}} = \frac{c^{A} - \lambda_{2} MU}{\lambda_{1} - \lambda_{2}}, \qquad \kappa_{2}^{A^{*}} = \frac{\lambda_{1} MU - c^{A}}{\lambda_{1} - \lambda_{2}}$$

$$\kappa_{1}^{L^{*}} = \frac{c^{L}}{\lambda_{1} - \lambda_{2}}, \qquad \kappa_{2}^{L^{*}} = \frac{-c^{L}}{\lambda_{1} - \lambda_{2}}$$

$$\kappa_{1}^{O^{*}} = \frac{c^{O} + \lambda_{2} MU}{\lambda_{1} - \lambda_{2}}, \qquad \kappa_{2}^{O^{*}} = \frac{-\lambda_{1} MU + c^{O}}{\lambda_{1} - \lambda_{2}}$$
(28)

It is readily seen that the solution in Equation (28) satisfies the conditions in Equation (27).

Once these constants have been known, the IRFs are determined by taking the minus derivatives in Equation (19) and are found to be:

$$g_{h}^{A}(h) = -\lambda_{1}\kappa_{1}^{A^{*}}e^{\lambda_{1}h} - \lambda_{2}\kappa_{2}^{A^{*}}e^{\lambda_{2}t}, \qquad g_{h}^{L}(h) = -\lambda_{1}\kappa_{1}^{L^{*}}e^{\lambda_{1}t} - \lambda_{2}\kappa_{2}^{L^{*}}e^{\lambda_{2}t},$$

$$g_{h}^{O}(h) = -\lambda_{1}\kappa_{1}^{O^{*}}e^{\lambda_{1}t} - \lambda_{2}\kappa_{2}^{O^{*}}e^{\lambda_{2}t}$$
(29)

All expressions are linear combinations of two exponential functions with exponents $\lambda_1 \approx -0.2835 \text{ years}^{-1}$, $\lambda_2 \approx -0.0139 \text{ years}^{-1}$, corresponding to time scales $1/\lambda_1 \approx 3.5$ years and $1/\lambda_2 \approx 71.9$ years. For comparison it is reminded that IPCC's [2,3] IRFs involve time scales of 4.3, 36.5, 394.4 and ∞ years with almost equal weights (from 0.22 to 0.28) and with the infinite time scale being the result of the constant term appearing in IPCC's IRFs. Obviously, there are no constant terms in the mathematically and physically consistent IRFs of Equation (29), nor terms suggesting time scales of hundreds of years.

It is interesting to compare the weights of the two time scales appearing in the IRF of the atmospheric part of Equation (29). From Equation (28) we find the ratio

$$\frac{\kappa_2^{A^*}}{\kappa_1^{A^*}} = \frac{\lambda_1 M U - c^A}{c^A - \lambda_2 M U}$$
(30)

With some algebra, using Equations (13) and (22) we find

$$\frac{\kappa_2^{A^*}}{\kappa_1^{A^*}} = \frac{k^{LA} + k^{OA} + \lambda_2}{k^{AL} + k^{AO} + \lambda_2}$$
(31)

Upon substitution of the numerical values, if we neglect λ_2 this yields $\kappa_2^{A^*}/\kappa_1^{A^*} \approx 1/6$, while the quantity is decreased if we account for λ_2 and thus, as an order of magnitude, we may take $\kappa_2^{A^*}/\kappa_1^{A^*} \approx 1/10$. On the other hand, as found before, the ratio $\lambda_2/\lambda_1 \approx 1/20$. Hence the relative importance of the two time scales is

$$\frac{\lambda_2 \kappa_2^{A^*}}{\lambda_1 \kappa_1^{A^*}} \approx \frac{1}{20} \frac{1}{10} = \frac{1}{200}$$
(32)

In other words, the second term in the IRFs, having a weight of more than two orders of magnitude smaller than that of the first term and thus becoming indiscernible from data, can be neglected. After such neglect, the atmospheric component behaves like a linear

reservoir with a characteristic residence time of 3.5 years, which will be the mean residence time of all CO_2 molecules, irrespective of their origin—anthropogenic or natural.

5 Numerical results and comparisons

5.1 Depiction of the IRF

While Section 4 includes some approximations to simplify the analytical results to a degree that is as intuitive as possible, it also contains the exact analytical solution. Based on this, the numerical results are easily calculated. First, we solve the differential equation (9) and find the solution in Equation (16) with equilibrium storages shown in Figure 2 (more precisely, $S_{\rm E}^{\rm A} = 848.695$ Gt, $S_{\rm E}^{\rm L} = 3394.67$ Gt, $S_{\rm E}^{\rm O} = 38676.6$ Gt, with $\kappa_1^{\rm A} = 25.6887$ Gt, $\kappa_2^{\rm A} = -4.3837$ Gt). Then, to find the IRF, we use the initial conditions as in Equation (17). The resulting solution for the atmospheric storage $S^{\rm A}(t)$ is described by Equation (19) with constants λ as in Equation (15) and remaining constants as follows:

$$S^{A}(t) = S_{E}^{A^{*}} + \kappa_{1}^{A^{*}} e^{\lambda_{1}t} + \kappa_{2}^{A^{*}} e^{\lambda_{2}t}$$

$$\lambda_{1} \approx -0.2828 \text{ years}^{-1}, \qquad \lambda_{2} = -0.0146 \text{ years}^{-1}, \qquad (33)$$

$$S_{E}^{A^{*}} = 848.72 \text{ Gt}, \qquad \kappa_{1}^{A^{*}} = 0.8960 \text{ Gt}, \qquad \kappa_{2}^{A^{*}} = 0.0842 \text{ Gt}$$

Consequently, the IRF is

$$g_{h}^{A}(h) = -\lambda_{1}\kappa_{1}^{A^{*}}e^{\lambda_{1}h} - \lambda_{2}\kappa_{2}^{A^{*}}e^{\lambda_{2}t}$$

$$\lambda_{1} \approx -0.2828 \text{ years}^{-1}, \qquad \lambda_{2} = -0.0146 \text{ years}^{-1}, \qquad (34)$$

$$-\lambda_{1}\kappa_{1}^{A^{*}} = 0.2534 \text{ Gt/year}, \qquad -\lambda_{2}\kappa_{2}^{A^{*}} = 0.0012 \text{ Gt/year}$$

The solution is shown graphically in Figure 3, in comparison with the IPCC AR5 [2] and AR6 [3] IRF, and the IRF by the RRR model [1] at the annual scale. We observe on this graph that:

- (a) The IPCC's IRF is totally irrelevant to reality.
- (b) The IRF produced in this essay by Equation (34) is in good agreement with that of the RRR model [1].
- (c) Neglecting the second term in Equation (34), which corresponds to the slow component, does not make a visible difference in the IRF.

Observation (b) suggests that modelling the atmosphere alone does not make any difference with modelling the three components, i.e., including the land and oceans separately. Observation (c) confirms what was already discussed in Section 5 about the insignificance of the slow component.

As a further investigation, I have produced two additional solutions with changed initial conditions. According to the definition given in Section 3, the IRF corresponds to an external instantaneous unit impulse. Indeed, this faithfully represents the case where the input is from anthropogenic CO_2 emissions, which are considered as an external input in

this modelling approach. In the additional two cases, the impulse was regarded as internal, originating either from land or from oceans and represented as a negative impulse in each of these compartments, so that the total system mass in conserved. Comparison among the three cases, as well as with the IPCC's [2,3] IRF and the IRF by the RRR model [1] is depicted in Figure 4.



Figure 3. Graphical comparison of four IRFs (standardized to unit value at lag 0), namely (a) the IPCC AR5 [2] and AR6 [3] IRF, which is taken from [5] and involves time scales of 4.3, 36.5, 394.4 and ∞ years; (b) the IRF by the RRR model [1] at annual scale, with a single time scale of 4 years; (c) the IRF of Equation (34); and (d) the same as (c) but without the slow component, represented by the second term in Equation (34).



Figure 4. Graphical comparison of five IRFs (standardized to unit value at lag 0), namely (a) the IPCC AR5 [2] and AR6 [3] IRF, which is taken from [5] and involves time scales of 4.3, 36.5, 394.4 and ∞ years; (b) the IRF by the RRR model [1] at annual scale, with a single time scale of 4 years; (c) the IRF of Equation (34) corresponding to its definition in Section 3; (d) similar to (c) but assuming that the impulse to the atmosphere is not external but internal, originating from land; and (e) similar to (d) but with the impulse originating from oceans.

The three cases are hardly distinguishable from each other, which corroborates what has been argued in [1] and in this essay about the indifference of the results to the origin of CO_2 addition to the atmosphere.

5.2 Decomposing the IRF into components

As the results of the multi-compartment model do not differ from those of the RRR model, the usefulness of the former seems questionable. Yet we may think of some usefulness in decomposing the IRF into components, corresponding to the four fluxes shown in Figure 1. We write the first part of Equation (6) for general conditions and for the equilibrium, in which $dS^A(t)/dt = 0$, in the following form

$$-\frac{\mathrm{d}S^{\mathrm{A}}(t)}{\mathrm{d}t} = Q^{\mathrm{AL}}(t) + Q^{\mathrm{AO}}(t) - Q^{\mathrm{LA}}(t) - Q^{\mathrm{OA}}(t),$$

$$0 = Q^{\mathrm{AL}^{*}}(t) + Q^{\mathrm{AO}^{*}}(t) - Q^{\mathrm{LA}^{*}}(t) - Q^{\mathrm{OA}^{*}}(t)$$
(35)

Subsequently, we subtract these two equations and get

$$-\frac{\mathrm{d}S^{\mathrm{A}}(t)}{\mathrm{d}t} = \left(Q^{\mathrm{AL}}(t) - Q^{\mathrm{AL}^{*}}(t)\right) + \left(Q^{\mathrm{AO}}(t) - Q^{\mathrm{AO}^{*}}(t)\right) - \left(Q^{\mathrm{LA}}(t) - Q^{\mathrm{LA}^{*}}(t)\right) - \left(Q^{\mathrm{OA}}(t) - Q^{\mathrm{OA}^{*}}(t)\right) - \left(Q^{\mathrm{OA}}(t) - Q^{\mathrm{OA}^{*}}(t)\right)$$
(36)

We evaluate the last equation for the case that the input is the impulse to the atmosphere and we substitute the lag *h* for time *t*. The left-hand side is then $-dS^A(h)/dt = g_h(h)$. Each of the terms in parentheses, i.e., $(Q^X(h) - Q^{X^*}(h))$, denotes the weight of the component X to the IRF $g_h(h)$. Note that as $h \to \infty$, i.e., as the system tends to the equilibrium, all terms tend to zero—as does $g_h(h)$. The decomposition is depicted in Figure 5.



Figure 5. Graphical decomposition, based on Equation (36) of the IRF $g_h(h)$ (standardized to unit value at lag 0) of the atmospheric CO_2 (**left**) into the four fluxes shown in Figure 1 and (**right**) into inflows to, and outflows from, the atmosphere.

From Figure 5 we make the following observations.

- 1. The outflow from the atmosphere to the land has the largest weight, followed by the outflow from the atmosphere to the oceans.
- 2. The inflow from the oceans to the atmosphere is practically zero, which contrasts a popular idea that it is Henry's law that controls the CO_2 emissions to the atmosphere.
- 3. On the contrary, the net result to the oceans (inflow minus outflow) is positive, i.e. an increase in the atmospheric CO_2 results in an increase of oceanic CO_2 .
- 4. At a first stage (up to a time lag of about 15 years), the impulse on the atmospheric CO_2 results in an increase of additional inflows to the atmosphere, mostly from the land. This is followed by a continuous decrease thereafter, until the system reaches a new equilibrium, at which point all components vanish.

Additional cases, in which the impulse is not external but internal, were also examined and the results (not presented in figures) did not have a notable difference from the behaviour seen in Figure 5.

The above observations are valid insofar as the system dynamics is linear and the implied concept of equilibrium is relevant. However, this is questionable as will be seen in the next subsection.

5.3 Temporal changes

When temporal changes are investigated, it turns out that the linearity assumption and the equilibrium notion are questionable. There are also some conceptual difficulties in accepting this model. The model assumes that the outflow from the land to the atmosphere is proportional to the CO₂ storage in the land, i.e., $Q^{LA}(t) = k^{LA} S^{L}(t)$. On the other hand, the outflow from the atmosphere to the land is proportional to the CO₂ storage in the atmosphere, i.e., $Q^{AL}(t) = k^{AL} S^{A}(t)$. The former represents respiration and the latter photosynthesis, i.e., closely related activities of the biosphere. We may conjecture that if one of the two increases, so will the other. However, the multi-compartment model may misrepresent this relationship. If carbon is moved from land to the atmosphere, $S^{L}(t)$ will decrease and $S^{A}(t)$ will increase. Hence, $Q^{LA}(t)$ will decrease, while $Q^{AL}(t)$ will increase. This is opposite to what is expected. Note that the RRR approach connects both $Q^{LA}(t)$ and $Q^{AL}(t)$ to the atmospheric storage $S^{A}(t)$ with relationships that are not identical but describe changes that macroscopically point in the same direction.

The poor performance of the multi-compartment model is illustrated in Figure 6 in terms of its misrepresentation of the evolution of atmospheric storage of carbon, as estimated from $[CO_2]$ data at Mauna Loa. As these data began in 1958, the simulation covers the period from 1958 to 2023. The model was formulated as in Section 4, where also its parameters *k* were estimated. The initial conditions for 1958 were set as described in the caption of Figure 7.

In addition to the evolution of the atmospheric storage of carbon, Figure 6 also depicts that of the cumulative human emissions of carbon (including land-use change) as given by reliable data bases [6]. It may be noticed that the slope in the emission curve is twice that of the storage curve. This means that natural processes have removed more carbon

from the atmosphere than they have added. This fact has been regarded by many "climate scientists", "orthodox" and "sceptic" alike, as proof that the rise in atmospheric CO_2 has human origin. This, despite being a popular argument, is a wrong conclusion. Perhaps the following line of thought explains its popularity.



Figure 6. Evolution of the cumulative human emissions of carbon (including land-use change) as given by reliable data bases, and of atmospheric storage of carbon, as estimated from $[CO_2]$ data at Mauna Loa. For comparison the evolution of the atmospheric storage of carbon as predicted by the three-compartment system with initial and equilibrium storages as described in the caption of Figure 7, is also plotted.



Figure 7. Schematic of the three-compartment system of Earth's CO_2 with initial and equilibrium storages (S_0 and S_E , respectively, in Gt C), initial fluxes (Q_0 in Gt C/year) and inverse characteristic residence times (k in years⁻¹), as in Figure 2 but for the year 1958. The parameters k are the same as in Figure 2. The initial storages S_0 are found in the following manner: (a) the total storage is smaller than in Figure 2 by 444 Gt C (the anthropogenic component between 1958 and 2018); (b) the atmospheric storage is estimated from [CO_2] data at Mauna Loa; (c) the land storage value is found so that the atmospheric component in 2018 match that in Figure 2 (this requires model runs). The equilibrium storages S_E are calculated from the three-compartment model and for zero anthropogenic input.

Let us consider the following two propositions:

- Proposition *A*: "In the last decades, natural processes have removed more carbon from the atmosphere than they have added."
- Proposition *B*: "The rise in atmospheric CO₂ has human origin."

Furthermore, let us consider the negations of the two:

- Proposition -*A*: "In the last decades, natural processes have added more carbon from the atmosphere than they have removed."
- Proposition -B: "The rise in atmospheric CO₂ does not exclusively have human origin."

We should start our analysis with propositions A and -A, which are decidable by measurements. In contrast, Proposition B is a very complicated one and requires sophisticated analysis. It is fundamentally different from: "human emissions go straightforward into the atmosphere". And is also different from: "The rise in atmospheric CO_2 has partly human origin."

It is easy to conclude that -A entails -B, symbolically, $-A \Rightarrow -B$. Indeed, if -A were true, the change in the atmospheric storage would be larger than that produced by human emissions. Note that " $-A \Rightarrow -B$ " does not mean "-A is true". It means that if -A were true, then we would have, by deduction (apodeixis), the certain result that -B would be true. The data show that in fact -A is false, or A is true.

Coming to the original proposition, the inference $A \Rightarrow B$ is a blatant error, known as the "fallacy of the inverse" or "denying the antecedent", which is very common [7]. Combining it with (a) another common fallacy, the "argumentum ad populum fallacy" [8] (also known as "appeal to popularity" or "appeal to the majority"), and (b) the general intellectual decline of our times, characterized by the replacement of rigorous logic with fuzziness and "political correctness", it is easy to explain why the false inference $A \Rightarrow B$ is so popular. Its popularity reaches a degree that correct logical arguments are regarded as "flawed logic" (an example is readily provided in the discussion comments of this essay).

In summary, $-A \Rightarrow -B$ does not imply $A \Rightarrow B$; instead, it implies either $A \Rightarrow B$ or $A \Rightarrow -B$. I illustrate the two opposite cases with two examples referring to atmospheric carbon storage, which are depicted in Figure 8.

Both examples precisely respect: (a) the historical evolution of carbon storage in the atmosphere, as derived from the measurements at Mauna Loa and depicted in Figure 6 ("observed" time series), and (b) the carbon balance equation. The latter is written by converting Equation (6) to discrete time, τ , with annual time step and incorporating the external input as:

$$\Delta S_{\tau}^{A} + SN_{\tau} = EN_{\tau} + EH_{\tau}$$

$$\Delta S_{\tau}^{A} \coloneqq S_{\tau}^{A} - S_{\tau-1}^{A}, \qquad SN_{\tau} \coloneqq Q_{\tau}^{AL} + Q_{\tau}^{AO}, \qquad EN_{\tau} \coloneqq Q_{\tau}^{LA} + Q_{\tau}^{OA}, \qquad EH_{\tau} \coloneqq Q_{\tau}^{EA}$$
(37)

with SN_{τ} , EN_{τ} , EH_{τ} standing for natural sinks, natural emissions and human emissions, respectively, at year τ , and Q_{τ}^{X} denoting the annual mass (Gt) at year τ .

In the first example, we assume constant natural emissions,

$$EN_{\tau} = EN_{2023} \tag{38}$$

without considering any model, and calculate the sinks SN_{τ} from Equation (37). In the second example, we calculate the sinks SN_{τ} from the linear reservoir model:

$$SN_{\tau} = (k^{AL} + k^{AO}) S_{\tau}^A \tag{39}$$

and determine the natural emissions EN_{τ} from Equation (37). We avoid involving the linear reservoir model (the proportionality of fluxes with storages) in calculating EN_{τ} because, if we did, we would violate the year-to-year carbon balance described by Equation (37). What we would find would be the evolution that is seen in Figure 6 marked as "3-compartment model".



Figure 8. Two examples of evolution of CO_2 inflows and outflows to the atmosphere, both consistent with the observed CO_2 storage in the atmosphere: (**upper**) assuming constant natural emissions, equal to the emissions of year 2023, namely 221.9 Gt C/year; (**lower**) assuming outflows proportional to storage, according to the basic assumption of the multi-compartment model, with natural emissions increasing from 168.1 Gt C/year for 1958 to 221.9 Gt C/year for 2023.

Now, both examples precisely respect the carbon balance and precisely reproduce the evolution that is seen in Figure 6 marked as "observed". In both examples, Proposition A is true: "In the last decades, natural processes have removed more carbon from the atmosphere than they have added." Let us now see what happens with Proposition B. Since in the first example the natural processes are more or less constant in time, the increase in atmospheric storage of carbon is a result of human emissions. Hence, Proposition B is true.

However, the second example relates a different story. There is an increase of natural emissions from 1958 to 2023, amounting to $\Delta(EN) = 53.8$ Gt. Compared to the human emissions in 2023, EH = 11.4 Gt, this is 4.7 times higher. Hence, Proposition B is false. The majority of the increase in 2023 (53.8/(53.8+11.4) = 83%) is due to the biosphere expansion.

Which of the two examples could be plausible? Clearly, the first one is not. The constancy of the natural processes is an arbitrary assumption, unjustified by any modelling means. In addition, it clashes with the empirical evidence of an expanding biosphere, as seen in Figure 1. On the contrary, the second example is plausible. Its basis on Equation (39) is as reasonable as it could be for linear multi-compartment models. It further respects the empirical evidence about the biosphere expansion and is consistent with the results found by the RRR framework.

A "climate crisis" zealot would attribute even the biosphere expansion to human emissions, the universal scapegoat. However, as has been documented in [9], the biosphere expansion is a result of the temperature increase. In turn, the temperature increase was not caused by the $[CO_2]$ rise. Instead, the causal relationship identified for the two is the inverse of that, i.e. temperature change is the cause and $[CO_2]$ change the effect [9-13].

6 Conclusions

The analyses of this essay clearly show that a multi-compartment model does not produce any notable difference from a single-reservoir model in representing the atmospheric CO_2 on annual and overyear scales. At the same time, the IPCC results differ dramatically from both the multi-compartment and single-reservoir models investigated here and in [1]. The time scales of hundreds of years to infinity, which appear in IPCC reports, are inconsistent with reality, as this is reflected in the observational data, as well as with the carbon balance presented by IPCC per se. It may thus be conjectured that the dramatically different IPCC estimates are not due to the use of one or more reservoirs in the modelling, but rather due to the IPCC's blurring of concepts and making weird assumptions.

In addition, IPCC's IRF was calibrated on climate model results. Specifically, Joos et al. [5], who produced the IRF used by IPCC (see [14]) with the so-called Bern modelling approach, in the caption of their Table 5, in which they present the time scales of their IRF and the respective weights, state:

Coefficients to fit multi-model mean responses to a pulse emission of 100 GtC [...].

Also, below their equation (11), which presents the mathematical expression of their IRF, they state:

We suggest to use numerical values as obtained by these fits for the multi-model mean in future studies.

The fact that they used climate model outputs (multi-model means), rather than measurements, in the fitting of their IRF reflects a circular logic that leads to the absurd.

For these reasons, the IPCC approach is deemed non-scientific. From the other two modelling approaches, it turns out that the RRR approach in [1] is clearly superior to the multi-compartment approach for several reasons including the following:

- 1. The RRR approach promotes the epistemological principle to build, calibrate and validate a model based on observational data. Instead, the multi-compartment approach builds on an annual carbon balance which is less reliable than observational data and hides the variability of the involved processes.
- 2. The RRR approach captures the seasonality of the related processes, which is substantial, and provides a clear picture of the substantial changes in the atmospheric part of the carbon balance, both seasonally and in the succession of the years. In contrast, the multi-compartment approach is mostly appropriate in static terms.
- 3. The RRR approach shows the dramatic changes in the residence time within a year, which are due to the biosphere processes, namely photosynthesis and respiration, while the multi-compartment approach hides these changes and hence the role of the biosphere. As seen in Figure 1, when the processes are viewed on a monthly basis, the mean residence time varies from less than 2 years to about 10 years. This is very substantial information for understanding the carbon cycle, but it disappears when the system is studied on an annual basis.
- 4. The RRR approach supports a nonlinear setting, in which linearity is obtained as a special case of the general case of nonlinearity. In contrast, in the typical multi-compartment approach linearity is assumed from the outset. We note that a power-law relationship between storage and outflow, which is the basis of the RRR approach, may potentially capture a heavy tail of the IRF, if this is supported by the data—which however turns out not to be the case. In contrast, the multi-compartment approach by construction results in an exponential tail.
- 5. The assumption of proportionality in the multi-compartment model entails inconsistencies with respect to the function of the biosphere and, as a result, it has poor performance in representing the temporal evolution of the related processes.
- 6. A key notion in the multi-compartment model is that of an equilibrium state. This notion is problematic and can hardly capture what happens in nature.

With respect to the last point, it should be emphasized that the dynamics of the atmospheric CO_2 is dominated by the biosphere function as evidenced by the substantial seasonal changes. When studying carbon balance, we are not dealing with a purely physical phenomenon that could be described by the laws of physics alone. Rather, we have a complex biogeochemical system driven by the changes and the evolution in the

biosphere, which have their own dynamics. Such dynamical behaviour is not easy to infer by deduction. Therefore, stochastic approaches based on observational data provide the only reliable scientific method to confront such a complex system.

Annex A: Notes on the mainstream sceptic approach

According to the theoretical framework developed in [1] and summarized in section 2 of this essay, the characteristic residence time of the atmospheric CO_2 reservoir,

$$W_0 := \frac{S_0}{Q_0} \tag{A1}$$

is about 4 years. As the data examined supported the almost linear behaviour of that reservoir, the characteristic time W_0 is equal to the mean residence time, μ_W , and the mean response time, μ_h , i.e., $W_0 = \mu_W = \mu_h = 4$ years. In this framework, the atmospheric reservoir was considered alone, without reference to the other reservoirs. The study [1] also criticized IPCC for using obscure or undefined concepts such as "adjustment time", and many more.

Obscure concepts are also popular among mainstream sceptics. The most popular approach among them is to adopt the IPCC's *adjustment time*, τ , and promote it as the truth in atmospheric CO₂ dynamics, even though they do not accept the extraordinarily high values promoted by IPCC. Their framework resembles that of the RRR framework in my study [1] only in that it is based on the atmospheric reservoir alone. In my understanding, the mainstream sceptic approach is based on the following premises:

- 1. There is an equilibrium storage, S_E^S , such that, if there are no human CO_2 emissions, then the storage retains this value.
- 2. Only human emissions EH(t) at time t count in the CO_2 dynamics. Without them, everything would be stable, at the equilibrium state.
- 3. Human emissions provoke the system to move from the equilibrium and produce a net sink in land and oceans, $NS^{S}(t) \coloneqq SN(t) EN(t)$, with dynamics described as:

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = \mathrm{EH}(t) - \mathrm{NS}^{\mathrm{S}}(t) \tag{A2}$$

4. The net sink is proportional to the difference of the storage from the equilibrium, $S(t) - S_{\rm E}^{\rm S}$, that is:

$$NS^{S}(t) = k^{S} \left(S(t) - S_{E}^{S} \right)$$
(A3)

The superscript "S" in the above quantities, standing for (mainstream) "sceptic", is used to distinguish them from the respective quantities of a formal approach. Every one of these four premises can be criticized for problematic elements:

1. The notion of the equilibrium state is incompatible with the recent biosphere expansion, which was documented in the main part of this essay and elsewhere [1,9,25].

- 2. The idea that human emissions are the only or the main agent of change is incompatible with the ever-changing Earth's states, including the atmosphere and climate [13].
- 3. The importance given to human emissions is disproportional to its share in total (4%).
- 4. While the proportionality relationship in Equation (A3) has an empirical basis (see below), it may be a result of the coincidence of increasing human and natural CO₂ emissions. The latter, caused by the biosphere expansion, are totally neglected in this relationship.

However, this approach may have additional problems that need to be identified by working with the premises and analysing their implications. I will attempt to do this below.

In Equation (A3), k^{S} is the proportionality coefficient, whose inverse represents the adjustment time:

$$\tau = \frac{S(t) - S_{\rm E}^{\rm S}}{\rm NS^{\rm S}(t)} \tag{A4}$$

which in that "sceptic" approach is assumed to be constant, independent of time *t*.

As both S(t) and $NS^{S}(t)$ can directly be calculated from measurements, while Equation (A3) represents a linear dependence thereof, the unknown parameters k^{S} and S_{E}^{S} can be determined by linear regression. A related graph by Burton [15] (also appearing in other internet sources) is reproduced in Figure 9. Engelbeen [16] followed an analogous approach with slight differences. Harde [17] and Stallinga [18] also examined this approach in their papers. Spencer [19] provided the theoretical basis of this approach and estimated the parameters as $k^{S} = 0.0202$ years⁻¹ and $S_{E}^{S} = 625.2$ Gt C (corresponding to 293.6 ppm of [CO₂]).



Figure 9. A popular graph among mainstream sceptics, correlating the "net natural CO_2 removal rate" with Mauna Loa CO_2 annual data, where the values of the former are estimated as the difference between the previous year's and next year's average CO_2 levels divided by two, subtracted from the year's emissions. Source: Burton [15].

Now, combining Equations (A2) and (A3) and following Spencer [19] who, in an attempt to find the adjustment time, sets EH(t) = 0, we get the differential equation:

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} + k^{\mathrm{S}} \left(S(t) - S_{\mathrm{E}}^{\mathrm{S}} \right) = 0 \tag{A5}$$

Its solution for the initial condition $S(0) = S_0$ is readily found to be

$$S(t) = S_{\rm E}^{\rm S} + \left(S_0 - S_{\rm E}^{\rm S}\right) e^{-k^{\rm S} t}$$
(A6)

and from Equation (A3) the net sink is

$$NS^{S}(t) = k^{S} (S_{0} - S_{E}^{S}) e^{-k^{S} t}$$
(A7)

so that Equation (A4), with Spencer's [19] estimate of $k^{S} = 0.0202$ years⁻¹ yields

$$\tau = \frac{1}{k^{\rm S}} \approx 50 \text{ years}$$
(A8)

(more precisely, 49.5 years). The same result can be found in a different approach, i.e. by defining a transformed storage index:

$$R(t) = \frac{S(t) - S_{\rm E}^{\rm S}}{S_0 - S_{\rm E}^{\rm S}}$$
(A9)

and setting $R(t) = e^{-1}$. The thus estimated time lag is known as e-folding time. In this case

$$R(t) = e^{-k^{S}t} \tag{A10}$$

and hence the condition $R(t) = e^{-1}$ results in $t = 1/k^{S}$.

Now let us try to compare this approach with a formal one that does not lump the natural emissions EN(t) and the natural sinks SN(t), which represent totally different natural processes. In this case, the mass balance is written as

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} + \mathrm{SN}(t) = \mathrm{EH}(t) + \mathrm{EN}(t) \tag{A11}$$

Note that the total emissions are the sum EH(t) + EN(t) while the total sinks equal the natural sinks, SN(t). We assume again linearity as in all the above cases, i.e.,

$$SN(t) = k S(t)$$
 (A12)

The proponents of the above "sceptic" approach may have difficulties to accept this relationship, preferring Equation (A3), but Equation (A3) is for the net sink $NS^{S}(t) \coloneqq SN(t) - EN(t)$, while Equation (A12) is for the natural sink SN(t) without subtraction of the emission. It would be absurd to assume that SN(t) would be related to a portion of the atmospheric storage, such as the surplus above the assumed equilibrium state.

Given that the mean residence time is $\mu_W = W_0 = 4$ years, as determined in [1] and accepted even by IPCC (as "turnover time"; see Section 2), the proportionality coefficient is necessarily $k = 1/W_0 = 0.25$ years⁻¹. This value also agrees with that determined by the three-compartment model (Sections 4 and 5.1). Hence, omission of Equation (A12) or use of a different value of k would be incompatible to the fact that the mean residence

time is 4 years. Compatibility of Spencer's Equation (A5) with the formal Equation (A11) for EH(t) = 0 entails:

$$k^{\rm S}(S(t) - S^{\rm S}_{\rm E}) = k S(t) - {\rm EN}(t)$$
(A13)

Solving this for EN(t) we find

$$EN(t) = k^{S}S_{E}^{S} + (k - k^{S})S(t)$$
(A14)

and substituting the solution of Equation (A6) for S(t) we obtain

$$EN(t) = kS_E^S + (k - k^S)(S_0 - S_E^S)e^{-k^S t}$$
(A15)

This shows that, behind the Spencer's [19] model, a hidden assumption lies that the evolution of the natural emission follows Equation (A15). To further validate this result, we rewrite the formal mass balance Equation (A11) with EH(t) = 0 and EN(t) as in Equation (A15):

$$\frac{dS(t)}{dt} + k S(t) = kS_{\rm E}^{\rm S} + (k - k^{\rm S}) (S_0 - S_{\rm E}^{\rm S}) e^{-k^{\rm S} t}$$
(A16)

We will show that Equation (A6) is a solution of this formally derived differential equation (16). Indeed, from Equation (A6) we have

$$\frac{dS(t)}{dt} = -k^{S} (S_{0} - S_{E}^{S}) e^{-k^{S} t}$$
(A17)

The left-hand side of Equation (A16) is

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} + k\,S(t) = -k^{\mathrm{S}} \left(S_0 - S_{\mathrm{E}}^{\mathrm{S}}\right) \mathrm{e}^{-k^{\mathrm{S}}\,t} + kS_{\mathrm{E}}^{\mathrm{S}} + k\left(S_0 - S_{\mathrm{E}}^{\mathrm{S}}\right) \mathrm{e}^{-k^{\mathrm{S}}\,t} \tag{A18}$$

Upon inspection, we find that it is identical to the right-hand part of Equation (A16).

The evolution of storage S(t), natural emissions EN(t) and transformed storage R(t) for the Spencer model case, in which EN(t) is given by Equation (A15), is depicted in Figure 10. For richer investigation and better illustration, Figure 10 shows five additional cases as follows:

- 1. Zero EN(t), as is the formal assumption to determine the IRF.
- 2. EN(*t*) constant, equal to the natural emissions of the year 2023, namely 221.9 Gt C/year (see Figure 8, upper).
- 3. EN(*t*) linearly increasing with a rate of 0.827 Gt C/year², which corresponds to that seen in Figure 8 (lower), where EN(*t*) has increased from 168.1 Gt C/year for 1958 to 221.9 Gt C/year for 2023.
- 4. EN(t) linearly decreasing with a rate arbitrarily set to -2 Gt C/year² for illustration.
- 5. The solution of the three-compartment model as given by Equation (16) with $S_{\rm E}^{\rm A} = 848.695$ Gt, $\kappa_1^{\rm A} = 25.6887$ Gt, $\kappa_2^{\rm A} = -4.3837$ Gt (see beginning of Section 5.1).



Figure 10. Evolution of (**upper**) storage S(t), (**middle**) natural emissions EN(t) and (**lower**) transformed storage R(t) for the five cases defined in text and for the Spencer model. Notice that the R(t) is not defined for the case of increasing EN because in this case $S(\infty) = \infty$.

The initial storage S_0 is set equal to the value of the year 2023, $S_0 = 895$ Gt, for all cases except case 5, where it is set equal to that of the three-compartment model as in Section 5.1, i.e., $S_0 = 870$ Gt. The initial natural emission is EN(0) = 0 for case 1, EN(0) =221.9 Gt/year in cases 2-4, as in Section 5.3, EN(0) = 213.3 Gt/year for case 5, as resulting from the three-compartment model, and EN(0) = 218.3 Gt/year for the Spencer model, as resulting from Equation (A15). The limiting storage values as $t \to \infty$, are $S(\infty) = 0$ for cases 1 and 4, $S(\infty) = EN(0)/k = 887.7$ Gt for case 2, $S(\infty) = \infty$ for case 3, $S(\infty) = S_E^A =$ 848.695 Gt for case 5 and $S(\infty) = S_E^S = 625.2$ Gt for the Spencer model.

Based on the limit $S(\infty)$, we generalize Equation (A9) to read:

$$R(t) = \frac{S(t) - S(\infty)}{S_0 - S(\infty)}$$
(A19)

One may tend to think that the additional cases examined do not satisfy the observationbased curve shown in Figure 9. This may be true for the three-compartment model (cf. the deviation from reality in Figure 6) but not for the other cases. Specifically, the results from the IRF consistently determined for case 1 perfectly agree with observations as shown in [1] and reproduced in Figure 1. Cases 2 and 3 have already been examined in Section 5.3, where it was stressed that they fully respect (a) the historical evolution of the carbon storage in the atmosphere, as derived from the measurements and (b) the carbon balance equation. In other words, they precisely reproduce the data points in Figure 9, rather than the regression line on them.

We may observe in Figure 10 that the method based on the notion of e-folding time gives a variety of "adjustment time" values. Cases 1 (zero EN) and 2 (nonzero constant EN) result in an e-folding time of 4 years, equal to the mean residence time and the mean response time. The three-compartment model gives a lower e-folding time of 2.7 years. The Spencer model gives a value of 50 years that is popular among mainstream sceptics. The case of linearly decreasing EN gives an e-folding time of 73.8 years. However, we can make this value as high as we wish by decreasing (in absolute value) the arbitrarily set rate of -2 Gt C/year². In the case of linearly increasing EN the e-folding time becomes infinite.

All these reflect the fact that the method based on the e-folding time is inconsistent. A consistent method would determine a finite time lag even if the natural emissions are increasing. The consistent method is the difference-based algorithm described at the end of Section 3.

Specifically, the examined cases 1-4 and the Spencer model obey the general differential equation

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} + k\,S(t) = \mathrm{EN}(t) \tag{A20}$$

with EN(t) depending on the particular case examine. The general solution of this equation is:

$$S(t) = ce^{-kt} + \int_0^t e^{k(u-t)} EN(u) du$$
 (A21)

where *c* is an integration constant, with the obvious property S(0) = c. The natural system described by this equation is not at an equilibrium state and the input is not zero, as it should in order to determine the IRF directly. Therefore, to find the IRF we apply the difference-based algorithm. Writing Equation (A21) for two cases with initial conditions $S_1(0) = S_0$ and $S_2(0) = S_0 + 1$ MU, where $S_1(t), S_2(t)$ denote the storages in the two cases, and subtracting by members, we find:

$$S_2(t) - S_1(t) = e^{-kt} MU$$
 (A22)

where we have utilized the fact that the integral in the right-hand side of Equation (A21) is identical in the two cases. Taking the derivative and substituting the lag *h* for *t*, we find that the IRF is

$$g_h(h) = -\frac{d(S_2(t) - S_1(t))}{dt} = ke^{-kt}MU$$
 (A23)

In other words, the IRF $g_h(h)$ does not depend on the assumed variation of the natural emissions EN(t) and hence what is depicted in Figure 10 has no relationship with response times.

For an additional illustration, we repeat the calculation, now specifying EN(t) to that corresponding to the Spencer model, as given in Equation (A15):

$$\frac{dS(t)}{dt} + k S(t) = kS_{\rm E}^{\rm S} + (k - k^{\rm S})(S_0 - S_{\rm E}^{\rm S})e^{-k^{\rm S}t}$$
(A24)

Its general solution is

$$S(t) = c e^{-kt} + (S_0 - S_E^S) e^{-k^S t} + k S_E^S$$
(A25)

where obviously, only the first term depends on the integration constant *c*. Hence

$$S_2(t) - S_1(t) = (c_2 - c_1)e^{-kt}$$
(A26)

Since $S_2(0) - S_1(0) = 1$ MU, we must have $c_2 - c_1 = 1$ MU, and we conclude again in Equation (A22) and in the IRF of Equation (A23)

Hence, regardless of the assumed EN(t), the IRF for cases 1-4 as well as the Spencer model is that in Figure 3 marked as "Koutsoyiannis (2024), μ h = 4 years" and the IRF for case 5 is the curve marked as "three-compartment model" in the same figure.

The results of the above analyses can be summarized in the following final points:

1. The approach of mainstream sceptics, as well as that of the IPCC, suffers from an emphasis on the minor human part in the CO_2 dynamics and a neglect of the major natural part, which is governed by physico-bio-geo-chemical processes that are not in equilibrium (and never have been).

- 2. This widespread approach is based on the concealed assumption of a specific temporal evolution of the natural emissions, which is subjective, arbitrary and incompatible with the IRF concept.
- 3. The methodology for deriving an adjustment time in this approach suffers from inconsistency with the concept of the IRF, which is the basis for estimating time lags in the response of the system state to perturbations.
- 4. The adjustment time of about 50 years derived by that methodology has no physical or mathematical basis, and is subjective and arbitrary. By changing the evolution of the natural emissions one can infer any adjustment time from about 4 years to infinity.
- 5. By the correct use of the IRF concept, including its rigorous definition, and the application of a correct methodology for determining IRF, we infer that it is independent of the evolution of natural (or human) emissions.
- 6. The instrumental data of the modern period, combined with the RRR methodology, support the conclusion that the atmospheric CO_2 residence time is about 4 years on an overannual basis (with marked seasonal variation). This does not change even if we use the three-compartment approach or even the mainstream sceptic model with a correct estimation methodology.
- 7. This value of 4 years represents both the mean residence time and the mean response time, whereas time lags with other names and obscure definitions do not represent anything objective.

Annex B: Notes on additional post-publication comments

Some of the post-publication comments I received were posted online on ResearchGate [20-22], while others were contained in email exchanges. Some of them criticize specific phrases of my paper, while others refer to my paper in general or to my entire work on climate. I discuss these comments in the three subsections below.

B1. Comments on specific statements in my paper

My following statement (Section 3.2 in [1]) was criticized as constituting a serious misunderstanding:

The ambiguity is accompanied by inappropriate assumptions and speculations, the weirdest of which is that the behaviour of the CO_2 in the atmosphere depends on its origin and that CO_2 emitted by anthropogenic fossil fuel combustion has higher residence time than when naturally emitted.

My reply (see [23]) was that I am not interested in discussing understanding and misunderstanding as, contrary to popular belief, understanding is subjective, not objective. Instead, I am interested in errors that a commentator thinks I have made. My above-quoted statement contains no errors—it is 100% correct. It is supported by quotations from IPCC reports that follow this statement in my paper [1], some of which are also reiterated in this essay (Section 2). The inappropriate assumption I mention is

indeed weird and has a long history, as it was thought from the beginning of climate modelling that the fate of anthropogenic CO_2 , is different from that of natural CO_2 .

My following statement (Section 2.1 in [1]) was also criticized:

Different equations can be formulated, depending on the system dynamics, but the following power type (combined) equation is representative for most problems: $Q(S)=Q_0(S/S_0)^b$ where b is a dimensionless parameter (exponent), and S_0 and Q_0 are parameters with units of mass and mass flow, respectively.

The critique is that this is erroneous and misleading in the context of a paper about atmospheric CO_2 , because the power type equation is not a valid or a plausible approximation to describing the system dynamics in the carbon cycle.

It is interesting that the commentator finds the linear differential equations more appropriate for the carbon balance. However, these are based on the assumption that the outflow is proportional to storage, and proportionality is a special case of the power law, in which b = 1. Furthermore, the more general power-law relationship can be used as an approximation for more complex equations (including the sum of exponentials), to avoid non-parsimonious modelling with many parameters, whose estimation cannot be supported by the usually poor data sets.

My following statements (Section 3.3 in [1]) have been interpreted as my failure to recognize the multi-compartment modelling:

IPCC's methodology in modelling the atmospheric CO_2 exchange is based on the socalled Bern modelling approach [...] It is reflected in the following expression of the *IRF* as the sum of three exponential functions and a constant term: $g(h) = a_0 + \sum_{i=1}^{3} a_i \exp(-h/W_i)$.

[...] the form of the equation is arbitrary and does not correspond to a reservoir's dynamics.

Furthermore, the commentator correctly states that multi-compartment models are the "standard approach among climate scientists modelling the carbon cycle and changes in atmospheric carbon" and praises the so-called Bern model and the paper by Strassmann and Joos[24]. The latter, however, is already cited in [1] and is criticized for its wrong results.

My critique of these results (including my analysis in Appendix B in [1], which the commentator criticizes too) is valid but perhaps too brief. For example, I should have stressed, in addition to the problems I mentioned, the near-equality of the coefficients *a*^{*i*} in the sum of exponential functions (not excluding the constant term), which further corroborates my assertion that this sum represents a cascade of linear reservoirs in a series, rather than in parallel, as one would expect.

Apparently, I did not follow the "standard approach among climate scientists" in [1], but I have tried to create a novel framework based on the "standard approach among scientists", i.e. the use of the scientific method and, in particular, the clarification of concepts and the inference based on observational data. As my approach is based on

observed data of the atmospheric part of the carbon cycle, it does not necessitate the modelling of the other carbon storages.

In this essay, the modelling of the other carbon storages in a multi-compartment setting gave virtually the same results as the single-reservoir approach. Indeed, there are no essential differences between the single-reservoir and the multi-compartment approaches. The difference appears to be between a correct data-based approach and the IPCC climate model-based approach. In particular, the correct multi-compartment model results in negligible weights for scales larger than the basic one (< 4 years), as would be expected for mechanisms acting in parallel. This is in sharp contrast to the IPCC approach, which involves time scales of tens, hundreds and even infinite years, all with almost equal weights. These are blatantly wrong.

For these reasons, my following statement (Section 3.3 in [1]), also criticized by the commentator, is absolutely correct:

[...] even if we replace the nominal upper limit of integration, which is infinity, with 1000 years (the duration considered by Joos et al. [5] for their model fitting), the mean response time is no less than 432 years. These values can hardly be reconciled with the fact that the residence time of CO_2 is no more than 4 years, as admitted even by IPCC [3] (p. 2237).

Finally, my following statement (Appendix C in [1]), was criticized as if it "implicitly assumes that it is appropriate to apply the same dynamical model to ¹⁴C and to total C" and as indicating my failure to differentiate ¹⁴C and total carbon dynamics:

The precise quantification of these factors is not easy and does not belong to the scope of this paper. Nonetheless, the ¹⁴C analysis offers an indirect validation of the RRR results by determining an upper bound of the response time, which the RRR model respects, while the IPCC model blatantly violates it.

Even this very statement of mine falsifies the commentator's claim about my "failure to differentiate ¹⁴C and total carbon dynamics". If the model was identical in the two cases, I would not note that the ¹⁴C model represents an upper bound of the response time of the total CO_2 . I clarify the reasons for the difference in another statement (Appendix C in [1]):

The absorption of the heavier isotope ${}^{14}C$ is subject to a function known as fractionation, that is, isotope discrimination. In particular, photosynthesis, during the exchange of O_2 and CO_2 , discriminates against the heavier isotopes, and, as a result, ${}^{14}C$ remains in the atmosphere for longer periods.

The commentator involves the so-called Revelle factor to justify the difference between the ¹⁴C and total C, in an obscure context related to whether this applies or not. I have not used this factor at all and I do not find it relevant, as it refers to the absorption of the atmospheric CO_2 by the ocean surface layer. On the contrary, my framework shows that it is the action of the biosphere that controls the atmosphere's outflow of CO_2 . Hence it is the fractionation that drives the delay in the absorption of ¹⁴C, as stated in the above quoted phrase.

B2. More general comments on the CO₂ dynamics

An interesting critique I received from another commentator is that I (and others) "do not understand the vocabulary of the science they challenge". I admit that this critique is valid. Indeed, I have difficulties understanding that vocabulary, as it is "intentionally vague" (see example in Section 2), that is, non-scientific. If it were scientific, it would not serve its nonscientific purpose.

The argument that this commentator uses is the following:

The mass-balance (or carbon conservation) argument makes clear that the rise in atmospheric CO_2 over the last century has human origins. (Atmospheric carbon accumulation is less than half of human emissions on annual or longer time scales, definitively demonstrating that natural processes have on balance been removing carbon from the atmosphere for the last century, not adding it.) Therefore it is customary to loosely refer to the atmospheric carbon excess compared to pre-industrial times as "anthropogenic carbon."

This is complemented by another commentator's opinion, who, likewise, asserts that the annual increment in atmospheric concentration is roughly half of the annual increment in emissions and, hence, there is no question over the origin of the increase of $[CO_2]$.

I had enormous difficulty understanding this argument and I tried hard to show that it is unfounded. The mass balance entails degrees of freedom and one cannot infer the origin of the increase from mass balance alone. In my presentation [25] (slide 23), I provided hypothetical examples, in which $[CO_2]$ increases at a rate higher than human emissions, or even decreases, while the balance is accurately preserved. I also gave an example in which I substituted the soil respiration in Brazil for human emissions (slides 25-26) to show that the latter are not more important than the former.

Therefore, it is my opinion that this stereotype reflects a dominant dogma, rather than anything related to science. There has not been any proof that the rise in atmospheric CO_2 has human origin. In Section 5.3 I have tried to understand why the argument, despite being unproven, is so popular. Hopefully, this will be my last involvement with this issue.

An additional critique of my work is that I mix up the residence time with

the time it takes to change "anthropogenic carbon", a time scale often called "adjustment time". That would be a time scale describing what would happen to atmospheric CO_2 levels if human emissions were abruptly switched off. There clearly could be more than one such adjustment time, as there is no reason to think pre-industrial levels would be approached by a single exponential decay.

This has led me to despair as my attempt to clarify the concepts and provide a glossary with scientific definitions that could potentially replace the IPCC's "intentionally vague" vocabulary (see Section 2) seems futile. But I do not regret it, despite futility. The Annex A provides a clarification of this issue.

Related to this is another commentator's statement that he would welcome any thoughts of mine about the following issue:

[I]f, in absence of anthro[pogenic] emissions, the anthro[pogenic] increment in CO_2 atmos[pheric] conc[entration] would decrease on a time scale of a few years, say 5 yr, or removal rate of the whole corpus of excess CO_2 of about 20% yr⁻¹, I think it is hard to come up with a model in which the annual increment of anthro[pogenic] atmos[pheric] CO_2 is as great as about 50% under current emissions. To put some numbers on that. Round numbers. Say (ppm) PI [preindustrial] atmos[pheric] CO_2 is 280 and present is 420, or a difference of 140 ppm or 300 Pg, what I am calling the corpus. 20% of that per y[ea]r would be 60 Pg yr⁻¹ or 5 times present anthro[pogenic] emissions of 12 Pg yr⁻¹. Hard (impossible) to reconcile that with obs[erve]d increase in corpus over anthropocene and with annual growth of atmos[pheric] CO_2 of roughly 5 Pg yr⁻¹.

I am afraid I cannot continue this thought experiment in a manner that would satisfy the commentator. If we accepted the multi-compartment model results and the notion of the equilibrium it implies, then based on Figure 3, we would expect that, in absence (switching off) of anthropogenic emissions, any anthropogenic perturbation would disappear in a couple of decades. This however does not mean that $[CO_2]$ would return to the preindustrial level of 280 ppm. Most probably it would continue to increase if temperature continued to increase (see Figure 10, upper, curve marked as "Increasing NE"), because $[CO_2]$ is mainly driven by the biosphere processes, responsible for 96% of total emissions, and the biosphere processes depend on temperature. The rather fast decay of the anthropogenic signal, in the imaginary case of switching off anthropogenic emissions, should not surprise us, as we have evidence of fast changes in the case of the ¹⁴C isotope. Indeed, the ¹⁴C data show that the changes even in this heavy isotope, which is discriminated by plants and remains in the atmosphere for longer periods, occur on time scales of decades, rather than hundreds or even thousands of years and beyond.

I started this subsection with a comment that I agree with and I am ending it with another comment that I agree with too, which is that the

rate of decrease of ${}^{14}CO_2$ following bomb perturbation is a measure of gross rate of removal of individual molecules. For cold CO_2 indiv[idual] molecules get removed, say by photosynthesis, but they are replaced (say by respiration) so it is not the gross rate of removal but net that is relevant.

If I understand well the comment, it agrees with what I have done in my paper [1] (Appendix C), as described in the following quotation from it:

We wish to investigate the time evolution of the radiocarbon fraction in the atmosphere, say, $F^{14}C$, and, in particular, how fast this fraction converges to the prebomb testing minimum value $(F^{14}C)_{min}$, which can be assumed to be the naturally occurring one. We clarify that this differs from examining the concentration $[^{14}CO_2]$ per se, because the latter also depends on the total $[CO_2]$ in the atmosphere, which has been increasing for more than a century. Here, the question we deal with is how fast the excess ^{14}C was removed by the biosphere, and, therefore, we should isolate the study of that question from the modern increase of the total $[CO_2]$. To see that this is the reasonable approach, let us consider the imaginary case that throughout the examined

period, the concentration of $[{}^{14}CO_2]$ was constant, while the fraction $F^{14}C$ was decreasing, e.g., at the observed rate. This would happen if the ${}^{14}CO_2$ absorbed by the biosphere, $[{}^{14}CO_2]_{ABS}$, was replaced by that added through the total CO_2 inflow, $[CO_2]_{IN}$, that is, if $[{}^{14}CO_2]_{ABS} = [F^{14}CO_2]_{IN} \times [CO_2]_{IN}$, where the $[F^{14}CO_2]_{IN}$ is the isotope-14 fraction in the input CO_2 . Clearly, if in this imaginary case we considered the concentration $[{}^{14}CO_2]$ in our calculations, we would conclude that the residence time of ${}^{14}CO_2$ would be infinite, because $[{}^{14}CO_2]$ would be constant. This is absurd, because the biosphere in fact removes ${}^{14}CO_2$, as shown by the decrease in $F^{14}C$.

B3. More general comments on more general issues

Some of the comments I received on the occasion of the publication of my recent paper are related to the general behaviour of the climatic system.

A first set of comments of this type diagnosed my "misrepresentation of CO_2 's role in climate forcing". It repeated the stereotype (mostly based on Lacis et al. [26]) that water vapour and clouds act primarily as feedback mechanisms, while CO_2 is a forcing agent that initiates warming, which then amplifies the concentration of water vapour due to higher evaporation rates. I have overturned this stereotype in another work, [27], a paper that arguably is a good one as it was rejected by three journals before it was published in a fourth one. Here, I limit myself to note that the history of changes in climate is not 100 years old but 4.5 billion years old, and this history can hardly be reconciled with the stereotype that CO_2 is the principal control knob governing Earth's temperature (see also discussion on other comments below). As per the "higher evaporation rates", these happen in the imaginary world of climate models. As I have shown in my study [28], when I examined rich data sets from reanalyses, the evaporation rates show a fluctuating behaviour, rather than a consistently increasing trend.

Similar to this is a comment stating that the energy balance is foundational to understanding climate (which in principle I do not disagree with) but continuing to diagnose my

misunderstanding and underestimation of CO_2 's long-term forcing role and its amplification effect on water vapor, as well as the oversimplified treatment of cloud dynamics and their complex role in the climate system.

This comment has already been replied to in my recent paper "Stochastic assessment of temperature – CO_2 causal relationship in climate from the Phanerozoic through modern times" [13]. If CO_2 had the presumed forcing role, this would be reflected in the time precedence of changes in it from changes in other variables, such as temperature. However, both instrumental and proxy data over the entire Phanerozoic or parts of it—and most importantly over the modern period of instrumental data—suggest the opposite behaviour, i.e. changes in CO_2 follow temperature changes.

In another comment this commentator states:

Coefficient of correlation between CO_2 and global temperature are higher than any other variable (at least over the past 50 years).

I agree that there is a high correlation between CO_2 and global temperature, but, as is well known, correlation does not imply causation. Nor even does it imply *potential* causation, because it is the time precedence that determines the latter. In a series of papers [9-13], my coauthors and I have:

- (a) confirmed this high correlation between CO_2 and global temperature (albeit not in all periods of the Phanerozoic);
- (b) formulated necessary (but not sufficient) conditions for causation;
- (c) excluded the case that $[CO_2]$ changes cause temperature changes, as such a proposition violates the necessary condition of time precedence of the cause;
- (d) proposed a potential causality with temperature as the cause and the $[CO_2]$ as the effect [9-13];
- (e) suggested a mechanism that explains the latter potential causality, which is the action of the biosphere—the principal driver of climatic changes, as well as geological changes [29,30].

Funding information. The research contained in this essay, as well as that in my climate papers including [1], received no external funding but was conducted out of scientific curiosity, in an attempt to restore science to what is called "climate science", but is in fact climate sophistry serving a politico-economic agenda.

Acknowledgements. I am grateful to Camille Veyres for his interest in my works, and in particular for his discussions on my paper [1] and the concept of multi-compartment models. I thank Robert Wentworth for his criticism of my paper [1] in relation to multi-compartment models. The remarks of both colleagues encouraged me to delve into the latter models and to clarify some issues about them which were obscure in the existing literature. I acknowledge the criticism and the remarks about my works by Alain Robichaud, Dave Andrews and Stephen E. Schwartz, which prompted me to reexamine my works and ultimately corroborate them and increase my confidence in their results. Dave Andrews and Ed Berry posted comments on ResearchGate on a previous draft of this essay, which inspired me to produce Section 5.3. I thank Frans Schrijver and Vassilios Zoukos for their comments and suggestions on a previous draft of this essay. I thank Ferdinand Engelbeen for his emails, which motivated me to make the apodeictic part in Section 5.3 as elementary as possible. Finally, I especially thank Mark Johnson, who checked all mathematical derivations and found errors, which I corrected.

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