A SIMPLE STOCHASTIC RAINFALL DISAGGREGATION SCHEME FOR URBAN DRAINAGE MODELLING

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ABSTRACT

An alternative method to both the design storm approach and the continuous simulation of historic or synthetic storms is presented. The method is based on, and uses as the only input, the intensity-duration-frequency (IDF) curves of the particular urban catchment of interest. The main concept is to keep the design storm approach for the determination of the total characteristics of the design storm event, i.e. duration and depth extracted from the IDF curves of the particular region, and use a disaggregation technique to generate a ensemble of alternative hyetographs (instead of adopting a unique arbitrary design time profile). The stochastically generated hyetographs are then entered into a rainfall - runoff model and then routed through the sewer network in order to simulate the hydraulic performance of the sewer network. This enables the determination of the conditional distribution of the outflow peak, which can then be utilised for studying the design characteristics and the behaviour of the sewer network.

KEYWORDS

Design storms; disaggregation techniques; IDF curves; peak flows; stochastic modelling; urban drainage.

INTRODUCTION

The concept of the design storm has been, and probably will be for a long time, a crucial topic of research and engineering within the urban drainage context. This is due to the relative ease of extracting and using the design storms which have been developed by various researchers the past few decades (Keifer and Chu, 1957; Hershfield, 1962; Huff, 1967; SCS, 1973; Sifalda, 1973; NERC, 1975; Pilgrim and Cordery, 1975; Yen and Chow, 1980; Cordery et al., 1984; Lambourne and Stephenson, 1987). Recently new approaches towards a new concept of designing urban storm drainage systems have been developed, which are based on the continuous simulation of historical or synthetic rainfall events (e.g., Packman and Kidd, 1980). However, the absence (even in developed countries), of long, continuous and reliable rainfall records with fine temporal resolution (necessary for urban areas that are generally characterised by fast response), coupled with the requirement of simulating a vast number of synthetic events to calculate the flood peak for a given exceedance probability have become a barrier to the use of such approaches. Therefore, the use of synthetic design storms based on local intensity-duration-frequency (IDF) curves remains at present the most popular method not only for its computational simplicity but mainly because most frequently the IDF curves represent the only available information on local rainfall.

The design storm approach

A design storm is a precipitation pattern for use in the design of a hydrologic system (Chow et al., 1988). It mainly serves as the system input and the resulting flow rates are computed utilising rainfall-runoff and flow routing procedures. A design storm is generally determined as a synthesised rainfall event with certain characteristics such as the return period, the total depth of precipitation, the duration of the storm and the temporal distribution or the storm hyetograph. The interested reader may find comprehensive studies on design storms such as those by Marsalek (1978); Arnell et al. (1984); Wenzel (1988); among others.

Some of the design storm methods existed in literature have been based on particular rainfall data of the area of interest (e.g., Chicago storm), and therefore cannot be directly applied to other regions. To this category we can include the well-known Huff's curves for Illinois (Huff, 1967) although they have been used in several studies for other regions, as well. The design storms that are based on the well-known intensity-duration-frequency (IDF) curves of the particular area of interest overcome this drawback. The main features of the IDF-based procedures is that they lead to a unique hyetograph with no other inherent assumptions and they use data only from the area of interest. The alternating block method (e.g, Chow et al., 1988) is the most representative, yet powerful, procedure of this category. The hyetograph synthesised by this method exhibits the property that the maximum rain depth for every duration equals the depth given by the IDF curve for that duration and for a specified return period. This is equivalent to the assumption that the probability of rainfall observed for various durations within a storm is constant, which obviously is unrealistic as variations in rain severity during storms are ignored (Arnell et al., 1984; Qian, 1987); also, the method does not take into full account the correlation between short-term rainfall intensities (Zhang et al., 1996). The major criticism to the design storm approach is concerned with the assumptions of an arbitrary temporal distribution and a one-to-one correspondence between the return periods of the storm and the peak discharge. This is due, among other reasons, to the unique temporal distribution of the rainfall event profile and the primary assumption of system linearity, which may be unrealistic in determining catchment response, particularly for urban catchments and where local storage is important.

The continuous or event-based simulation approach

Continuous simulation utilises long, non interrupting, rainfall series with a fine temporal resolution and a computer model to convert rainfall to runoff and route the flow through the sewer network. Continuous simulation is probably the most reliable method for estimating the flow peak or volume for a given exceedance probability and it is free of subjective assumptions. Calibration of the involved models is possible in case where fine resolution rainfall and flow data in the sewer network are available. Since those records are rarely available, simulation is usually accompanied with sensitivity analysis of the parameters affecting runoff in urban areas (e.g., imperviousness, infiltration, antecedent conditions). If historical meteorological records do not exist, then stochastic techniques may be incorporated to generate hypothetical storm records. This task is extremely time consuming and copious especially for large sewer systems. The event-based simulation, which analyses only the floods produced by individual large storm events, is an alternative to the continuous simulation approach. This allows a considerable simplification of continuous simulation, primarily in computational time, but it does not account for conditions prior to storms nor it assigns an objective risk measure. A combination of the design storm and continuous simulation methods was carried out by various researchers (Voorhes and Wenzel, 1984; Cao et al., 1994; Zhu et al., 1996) who attempt to calibrate the design storm parameters by applying continuous or event-based simulations. This undoubtedly marks an improvement of the design storm concept but it suffers from most of the disadvantages of the original methods.

BASIC ELEMENTS OF THE PROPOSED METHOD

The method proposed hereafter is a compromise between the traditional design storm approach and the simulation method. The main idea is to keep the external rainfall characteristics (the total duration and depth) from the IDF design storm and then use a conditional stochastic technique to disaggregate the rainfall depth in shorter time intervals. This method is actually an implementation to urban sewer networks of a recent generalised framework proposed by Koutsoyiannis (1994).

The general outline of the procedure includes the following steps: (a) the total duration and depth of the design storm are estimated based on a selected return period and using the IDF curves of the area of interest; (b) a stochastic disaggregation model is set up and its parameters are estimated; (c) the stochastic model is run and produces a series of independent and random storm events, all having the total duration and depth estimated from step (a); (d) the hyetographs are then entered into a rainfall - runoff and flood routing model for the specified urban catchment and sewer network; (e) the output flood peak (or flood volume) from every run is computed, (f) the empirical distribution of the flood peaks are determined; (g) a probability level is selected and the corresponding outflow peak is determined.

The main differences from the empirical design storm methods are summarised as follows: (a) the proposed method uses a stochastic disaggregation model to derive the temporal distribution of the design storm; (b) instead of determining a single design hyetograph as the model input, it produces a number of alternative, random, and thus realistic, time profiles, and (c) instead of having a single design hydrograph as the model output, the proposed method gives the conditional probability distribution function of the outflow peak.

Onof et al. (1996) have proposed a similar method based on a Bartlett-Lewis model. The parameters of the models are calibrated using the Depth Duration Frequency (DDF) curves given in the Flood Studies Report (National Environmental Research Council, 1975) so that the model reproduces the mean and variance of the total storm depth together with the correlation structure of the rainfall process.

Rainfall process

The method adopted in this study is based on stochastically generated storm events; details of the method are given by Koutsoyiannis (1994). The storm event is considered as an entity having a stochastic structure described by the instantaneous intensity $\xi(t, D)$, $0 \le t \le D$, where t denotes time and D is the definite duration of the event. However, significant simplifications of the actual rainfall process are performed by the introduction of a surrogate rainfall process, which takes into account only the flood producing characteristics of the actual process. The concept is to construct a stochastic process in discrete time with simple structure and few parameters, which are estimated so as the actual IDF curves of the corresponding region are matched. The simplest and most convenient structure of the surrogate process is the gamma-autoregressive (GAR) structure (Lawrance and Lewis, 1981; Fernadez and Salas, 1990), which is similar to the well-known AR(1) model but it is appropriate for gamma distribution functions. The gamma distribution sufficiently fits to the short time steps rainfall sequences (Haan, 1977; Zhang et al., 1996). Additionally, the gamma distribution not only covers a wide variety of distribution shapes (from symmetrical to inverse J-shaped), but also has a lower bound (zero in our case), which is definitely an advantage to rainfall distribution. The GAR surrogate rainfall process can be represented by three parameters, namely the mean, the variance and the lag-one autocorrelation of the incremental depths. Symbolically, if X_i denotes the incremental rainfall depth for the ith time increment of length Δ (5 minutes), where $i = 1,..., k \equiv D/\Delta$, the three parameters are: $\mu = E[X_i]$, $\sigma^2 = \mathrm{Var}[X_i]$, $\rho = \mathrm{Corr}[X_{i-1}, X_i]$. The estimation of parameters, which are conditional on a certain duration D, is based on a simple trial-and-error procedure, including generation of synthetic events, extraction of simulated IDF curves and comparison of the latter with the given ones (Koutsoyiannis, 1994). The introduction of the surrogate process instead of the actual one has some advantages (Koutsoyiannis, 1994). For example, it uses the IDF curves and thus it does not require any complicated analysis of the rainfall data; it avoids the use of complicated rainfall models and minimises the model parameters; and it is flexible and quick in application.

The disaggregation model

Once the parameters of the surrogate rainfall process are estimated, the subsequent step is the generation of a series of storm events that will later be used as an input to the rainfall-runoff and flood-routing model. To avoid a huge number of generated events, most of which would not produce flood, all events are conditioned for their total duration and total depth. The total duration D is considered as a multiple of the basin response time. The total storm depth H is determined from the given IDF curve for the chosen duration D and return period T. The subsequent problem is to disaggregate the total depth into increments $X_1, X_2, \ldots, X_k$ ($k \equiv D/\Delta$), so that $X_1 + X_2 + \ldots + X_k = H$. Given the theoretical developments by Koutsoyiannis (1994) and Koutsoyiannis and Manetas (1995) this is a rather simple task, which can be performed in two steps. First, an initial sequence of incremental rainfall depths is generated using the GAR model without reference to the known total depth H. Second, the initial sequence is adjusted to add up to H using an exact adjusting procedure, which does not induce bias to the distribution function of the incremental depths. Comprehensive description of the disaggregation technique is given by Koutsoyiannis (1994) and Koutsoyiannis and Manetas (1995).

The rainfall-runoff and flood routing model

For the calculation of rainfall-runoff and flood routing the HAuSS model has been applied (Jakobs et al., 1997). HAuSS uses a hydrologic module for the simulation of the movement of precipitation and pollutants on both pervious and impervious areas. For the simulation of the unsteady flow in sewer networks HAuSS uses a hydrodynamic module, based on the Saint Venant equations, capable to calculate critical hydraulic processes such as overloading, backflow, etc., and predicting flows, stages, and pollutant concentrations. As it has been designed to simulate water quality over long time periods it is able to perform single and multiple event simulation and includes additional modules for the simplification of large sewer systems.

APPLICATION

The method is applied on a small urban catchment of the Greater Athens region (in the suburb of Argyroupoli) with a relative simple sewer network. The area of the catchment is 14 ha and its average slope is 4.4%. The sewer network is composed of 40 branches with diameters 0.5-0.8 m and average slope 4.3%. The total length of the sewer system is about 2 km with a maximum tributary length 0.76 km. The total storage volume of the sewer pipes is 888 m^3.

The IDF curves have been estimated for this region using data from two stations of the Greater Athens (Helliniko and Nymphon Hill) and have the following expression (Koutsoyiannis and Baloutsos, 1998)

$$i(d, T) = \frac{40.6\,(T^{0.185} - 0.45)}{(d + 0.189)^{0.796}} \tag{1}$$

where i is the rainfall intensity (in mm/h) for duration d (in h) and return period T corresponding to partial duration series. In can be shown that (1) implies that the rainfall intensity i has a Pareto distribution with parameters that can be inferred directly from (1). Assuming a total duration $D = d = 2$ h (about 10 times the basin's response time) and using (1), it can be found (from the equations of the Pareto distribution) that the mean and standard deviation of the total depth H are $\mu_{H|D} = 33.83$ mm and $\sigma_{H|D} = 12.45$ mm, respectively. Using a time step of 5 min (= D / 24), we find the mean of the incremental rainfall depth is $\mu = 33.83/24 = 1.41$ mm. The remaining parameters of the surrogate process i.e., σ and ρ, can be found with the already mentioned trial-and error procedure, whose details are given by Koutsoyiannis (1994). Their values are $\sigma = 1.99$ mm and $\rho = 0.25$.

Using the method described above, a series of 1000 rainfall events has been generated; all events have total duration $D = 2$ h and total depth $H = 39.06$ mm, which according to (1) corresponds to return period $T = 5$ years (a value suggested by the literature for small urban catchments). All the hyetographs have been then entered into the HAuSS model and the flows in the network were calculated. The calculations were made with a time step of 5-minutes assuming an mean imperviousness of the study area 72.3%. The flood peaks at the outlet of the network together with the maximum 10-minute depths are plotted on a Gumbel paper as it is shown in Figure 1. The adoption of a 10-minute rainfall peak for Figure 1 was based on an estimation of a representative time lag of the system, which is roughly that value. Note that non-exceedance probability in Figure 1 is conditional, the conditions being $D = 2$ h and $H = 39.06$ mm. Also, note that the values of rainfall and flow appearing in Figure 1 are standardised by dividing each value with the corresponding mean. Maximum 10-minute rainfall depths roughly follow a Gumbel distribution of maxima. The distribution of the peak flows is very similar to that of rainfall peaks for conditional probability of exceedance less than 90%, but it deviates largely for greater probability levels. The reason is that for high rainfall the produced runoff exceeds the maximum capacity of the sewer network and an overflow occurs. This is better shown in Figure 2, where the flow peaks are plotted against the rainfall peaks using different symbols for the cases of overflowing or not. Interestingly, Figure 2 demonstrates that there is no clear separation limit (either in rainfall or discharge at the outlet) of the states of overflowing or not; instead, there exists a region where two events with the same rainfall and flood peak may produce overflow or not. Another observation from Figure

2 is the fact that there is no one-to-one correspondence between the rainfall and flow peak but rather large deviations exist, although the examined sewer network due its small extent and simple structure could be otherwise considered as a linear one. The wide spread of the rainfall-discharge relationship is better demonstrated in Figure 3 in terms of the exceedance probabilities of the corresponding variables.

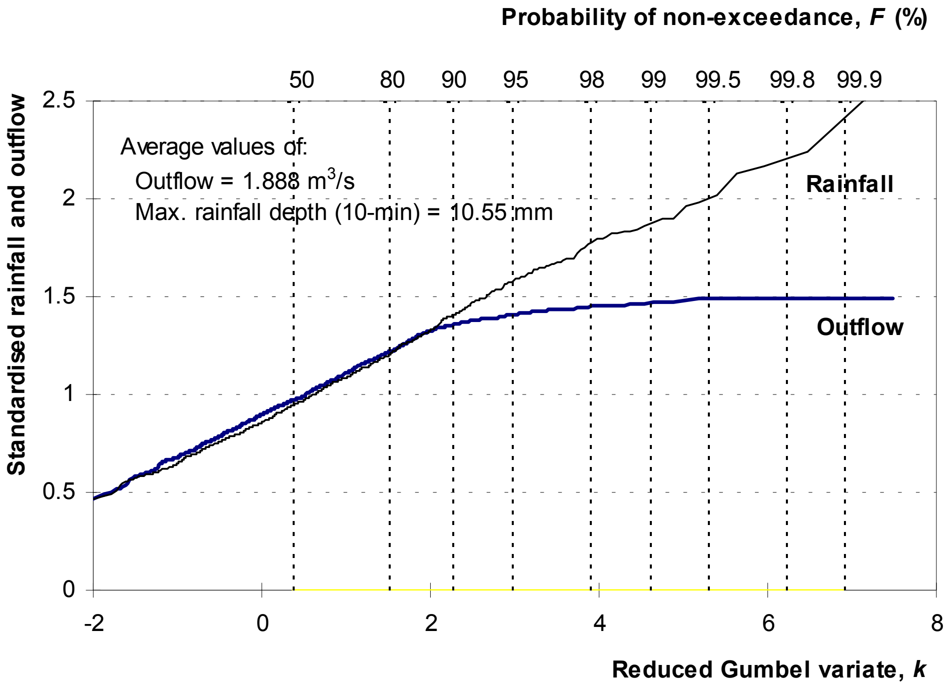

Figure 1. Conditional probability distribution functions of the maximum 10-minute rainfall depths ($T = 5$ years) and peaks of outflow hydrographs at the outlet of the sewer system.

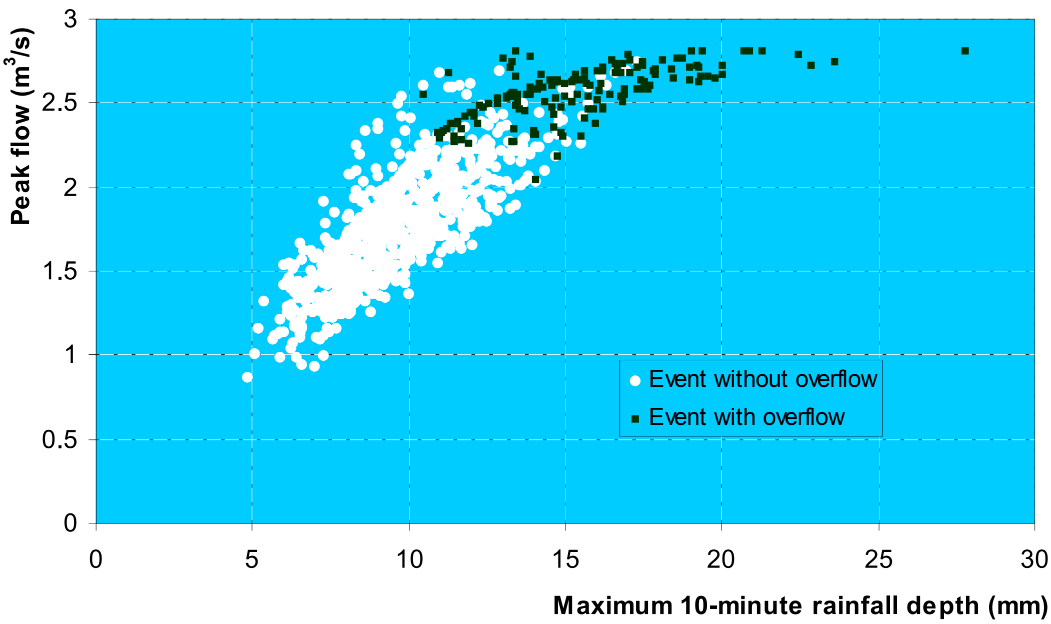

Figure 2. Plot of peak flow at the sewer outlet versus the maximum 10-minutes rainfall depth for the 1000 generated rainfall and flood events.

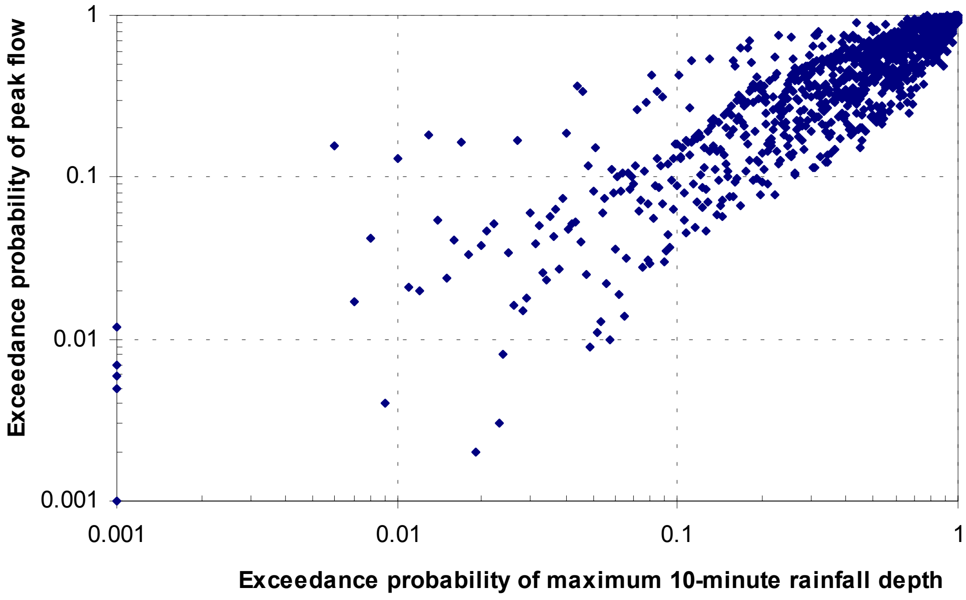

Figure 3. Plot of conditional exceedance probability of peak flow at the sewer outlet versus the conditional exceedance probability of maximum 10-minutes rainfall depth for the 1000 generated rainfall and flood events.

Table 1. Comparison of rainfall and flow characteristics estimated using the proposed method and other empirical design storm methods.

Method	**Case**	**Rainfall Peak (5-minute increment) (mm)**	**Rainfall Peak (10-minute increment) (mm)**	**Outflow Peak (m^3/s)**
Proposed method	Mean	8.12	10.55	1.89
D = 2 h, *H* = 39.06 mm, *T* = 5	50% probability level	7.68	9.96	1.83
years,	80% probability level	9.77	12.61	2.29
Imperviousness 72.3%	90% probability level	11.41	14.77	2.55
	95% probability level	12.77	16.65	2.65
Huff curve, 50% first quartile storm, *D* = 2 h, *H* = 39.06 mm, Imperviousness 72.3%	-	6.64	12.89	2.03
Alternating block method *D* = 2 h, *H* = 39.06 mm, Imperviousness 72.3%	-	8.55	13.83	2.40
Rational method *D* = 10 min, *H* = 13.83 mm, Imperviousness 72.3%	-	6.915	13.83	2.21
Proposed method	Mean	13.76	18.08	2.67
D = 2 h, *H* = 70.2 mm, *T* = 50	50% probability level	13.05	17.01	2.75
years,	80% probability level	16.69	21.79	2.81
Imperviousness 72.3%	90% probability level	19.05	25.19	2.81
	95% probability level	21.55	27.56	2.81
Proposed method	Mean	8.12	10.55	1.66
D = 2 h, H = 39.06 mm, T = 5	50% probability level	7.68	9.96	1.60
years,	80% probability level	9.77	12.61	1.99
Imperviousness 62.8%	90% probability level	11.41	14.77	2.32
	95% probability level	12.77	16.65	2.47

Additionally to the above main case, other scenarios were also examined. These include: (a) the adoption of a smaller imperviousness of the catchment, equal to 62.8%; (b) the use of another series of rainfall events generated with $T = 50$ years; (c) the application of three well-known empirical design storms, i.e., using the 50% first-quartile Huff time profile (with $D = 2$ h), the alternating block method (with $D = 2$ h) and the uniform time distribution (with $D = 10$ min). The results are summarised in Table 1. We observe that the effect of the imperviousness and the return period is as expected; the results indicate a rather linear behaviour of the catchment with regard to changes of these characteristics. An exception is observed for the flow peaks for large probabilities of non-exceedance, which are rather insensitive to changes of the imperviousness or the return period of total rainfall depth. Apparently, this is explained by the limited discharge capacity of the network. Interestingly, all empirical design storms exhibit a good compliance with the proposed method, as their resulting peak flows lie between the 50% and 90% percentiles estimated by the proposed probabilistic method. However, all these methods assume arbitrary temporal distribution of rainfall, to which no risk level can be assigned. On the contrary, the proposed method can specify a probability level to each value of flow peak, based on the simulated distribution function. The expected value of this distribution may be accepted as a design value; alternatively the design value may be based on a certain conditional probability level (e.g. 90% to assure higher safety). Another advantage of the proposed method is the fact that the resulting hyetographs and hydrographs are more realistic (i.e., with random shape with one or more peaks; see Figure 4) than those of empirical methods, which have too smooth and orderly shapes. The obvious drawback of the proposed method is that it does not combine the probability of non-exceedance of the total rainfall depth with the selected conditional probability level to obtain a total measure of the design safety.

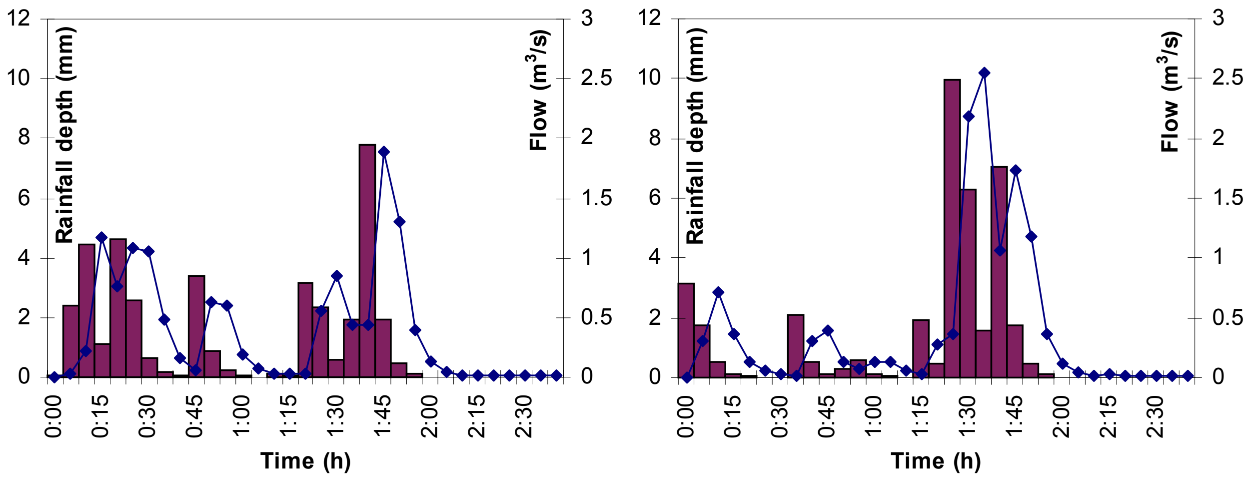

Figure 4 Examples of 'design' storms and floods generated by the proposed method: event corresponding to the average peak flow (left) and event corresponding to 90% non-exceedance probability for the peak flow (right).

SUMMARY AND CONCLUSIONS

A method for studying storm sewer systems, alternative to both the design storm and the continuous simulation approaches, has been proposed and applied. This method is actually an implementation to urban sewer networks of a recent generalised framework proposed by Koutsoyiannis (1994). The method generates an ensemble of storms via a simple stochastic rainfall model, and routes these storms through the sewer network implementing a flow model that incorporates rainfall-runoff and flood routing components. The stochastic rainfall model has a simple GAR structure with only three parameters that are estimated so that the IDF curves of the area of interest are preserved. In its application phase, the rainfall model, like most empirical design storm methods, assumes a specified total duration and storm depth corresponding to an appropriately selected return period. However, unlike empirical methods that adopt a unique (and arbitrary) time profile, the proposed model generates an ensemble of hyetographs by stochastically disaggregating the total depth to incremental depths. An equal number of hydrographs and flood peaks are accordingly produced using the flow model. Therefore, instead of using a single design hydrograph, the behaviour and capacity of the sewer network is explored in a probabilistic basis.

Overall, the rainfall model is characterised by its simple stochastic structure and parsimony of parameters. These parameters are estimated from the IDF curves only, which are usually the only available rainfall information even in developed countries. Apparently however, the simplicity must have a cost and this is paid in terms of not knowing the actual rainfall process, but rather a simplified surrogate rainfall process, which takes into account only the flood producing characteristics of the actual process. The proposed method may also act as a tool for intercomparison of empirical methods implying a single design event. In the sewer system used for the application of the method, it was found that three such methods, i.e. those using the Huff standardised curves, the alternating blocks, and the uniform time distribution perform relatively well. However, the proposed method has some advantages over empirical methods, and mainly it avoids arbitrary time profiles, it allows a thorough exploration of the system behaviour through an ensemble of events, which, notably, can be used simultaneously for the entire sewer system (as opposed to the empirical methods that need to synthesise different design events for different parts of the system), and it can assign a conditional probability level to each event based on the simulated probability distribution function of the flow peak.

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