A parametric rule for planning and management of multiple-reservoir systems

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Abstract. A parametric rule for multireservoir system operation is formulated and tested. It is a generalization of the well-known space rule of simultaneously accounting for various system operating goals, in addition to the standard goal of avoiding unnecessary spills, including avoiding leakage losses, avoiding conveyance problems, taking into account the impacts of the reservoir system topology, and assuring satisfaction of secondary uses. Theoretical values of the rule’s parameters for each one of these isolated goals are derived. In practice, parameters are evaluated to optimize one or more objective functions selected by the user. The rule is embedded in a simulation model so that optimization requires repeated simulations of the system operation with specific values of the parameters each time. The rule is tested on the case of the multireservoir water supply system of the city of Athens, Greece, which is driven by all of the operating goals listed above. Two problems at the system design level are tackled. First, the total release from the system is maximized for a selected level of failure probability. Second, the annual operating cost is minimized for given levels of water demand and failure probability. A detailed simulation model is used in the case study. Sensitivity analysis of the rule’s parameters revealed a subset of insensitive parameters that allowed for rule simplification. Finally, the rule is validated through comparison with a number of heuristic rules also applied to the test case.

1. Introduction

Planning and management of multiple reservoir systems have been and continue to be the subject of numerous research works. This attention is due to the benefits arising from reservoir system operation (e.g., hydropower) in combination with the reduction of natural risks (e.g., flood control). The problem of reservoir planning and/or management is most often stated as an optimal control problem. Its solution is not an easy task because of the large number of variables involved, the nonlinearity of the system dynamics, the stochastic nature of future inflows, and other uncertainties of the system (e.g., leakage from reservoirs).

Stochastic dynamic programming (SDP) has been repeatedly used by many researchers to study the problem [e.g., Su and Deininger, 1972, 1974; Askew, 1974a, b; Sniedovich, 1979; 1980a, b; Bras et al., 1983; Stedinger et al., 1984]. SDP could be satisfactory if it did not require excessive amounts of computer time and storage. To increase the efficiency of the solution algorithm, some researchers have treated the inflows’ stochasticity in an analytic way without state-space discretization and then applied efficient deterministic optimization methods [Wasiimi and Kitanidis, 1983; Loaiciga and Mariño, 1985; Georgakakos and Marks, 1987]. For example, Georgakakos and Marks [1987] represented the reservoir system dynamics in a state-space form and proposed an extension of stochastic control theory, which they termed extended linear quadratic Gaussian (ELQG). In this way these authors obtained a very efficient algorithm at the expense of an accurate representation of the stochastic structure of inflows (i.e., only Gaussian independent inflows were considered). In later studies [Georgakakos, 1989] the problem of the representation of the stochastic structure of inflows was effectively tackled. Other researchers continued their studies in the direction of stochastic dynamic programming with the purpose of remediating its deficiencies. Efficient interpolation schemes for dynamic programming (DP) algorithms are discussed by Johnson et al. [1993]. The problem of errors resulting from the state-space discretization in discrete dynamic programming was tackled [Kitanidis and Foufoula-Georgiou, 1987; Foufoula-Georgiou and Kitanidis, 1988; Foufoula-Georgiou, 1991]. These authors proposed gradient dynamic programming, which is based on an interpolation scheme of the cost-to-go function at each stage and reduces significantly the error due to discretization.

In spite of the large number of optimization techniques available in the literature, simulation models still remain the primary tool for reservoir planning and management studies in practice. The reason is that simulation models allow a more detailed and faithful representation of the system studied than optimization techniques do [Loucks and Sigvaldason, 1982]. Moreover, they can be easily combined with synthetically generated streamflow sequences [Young, 1967; Loucks et al., 1981, p. 277]. The main drawback of simulation is that unlike optimization, it requires prior specification of the system operating policy. To remedy this problem, Young [1967] combined the use of synthetically generated annual inflows into a single reservoir with deterministic dynamic programming and inferred simple parametric rules for the operating policy using regression techniques. Other researches have employed optimization methods within simulation models [Evenson and Moseley, 1970; Sigvaldason, 1976; Gunn and Houck, 1989; Johnson et al., 1991; Tejada-Guibert et al., 1993]. Tejada-Guibert et al. [1993] compared two alternative approaches for defining the operation
A system of $N$ reservoirs is assumed for which an operating policy is sought. The policy is focused on consumptive water uses such as water supply for domestic and industrial use and irrigation. Other uses such as hydropower generation, recreation, or navigation are assumed absent or of secondary importance in this study. Our approach, however, can easily accommodate such nonconsumptive uses. The reservoirs are connected in series or in parallel to form a network with any topology. Water is withdrawn from all of them to meet a common downstream target release $D$ (equal to the water demand). The continuity equation for each reservoir $i$ is given for a certain time period by

$$ S_i = BS_i + Q_i - R_i - L_i - SP_i $$

where $BS_i$ is the beginning-of-period storage for reservoir $i$ (known); $S_i$ is the end-of-period storage, which is unknown; $Q_i$ is the inflow; $R_i$ is the total release from the reservoir; $L_i$ is the total loss due to evaporation and leakages, and $SP_i$ is the reservoir spill. Reference to time interval is omitted for convenience.

Let $V$ denote the total storage in the system at the end of the time period of interest. In the simple case of one reservoir, $V$ is completely determined by (1), in which case we omit the subscript $i$ and replace $S$ with $V$. The operation of a system of $N$ reservoirs is much more complicated as, this time, the state of the system is described by $N$ variables $S_i$, satisfying

$$ \sum_{i=1}^{N} S_i = V $$

Assuming that the target release is fulfilled and the inflows, losses, and spills from all reservoirs are estimated in some manner, the total end-of-period storage of the system is given by

$$ V = \sum_{i=1}^{N} (BS_i + Q_i - L_i - SP_i) - D $$

Thereafter the problem is to determine the releases from all reservoirs such that their sum equals $D$. Equivalently, the problem is to distribute the total volume $V$ into the $N$ reservoirs such that (3) is satisfied. This can be done in numerous ways, as the problem has several degrees of freedom. We call a specific way to perform this distribution an operating rule. To avoid ambiguity, we express the operating rules by means of some quantities $S_i^*$, which stand for the target storage for the reservoir $i$ at the end of the period. The real storage $S_i$ is generally different from the target storage $S_i^*$ because of the physical constraints that were not considered in the determination of $S_i^*$. We propose to distribute $V$ according to the following rule:

$$ S_i^* = a_i + b_i V $$

where $a_i$ and $b_i$, $i \in \{1, \cdots, N\}$, are unknown parameters.

There exist $2N$ parameters for a system of $N$ reservoirs. We note that because of (2) we have two constraints on the parameters, i.e.,

$$ \sum_{i=1}^{N} a_i = 0 \quad \sum_{i=1}^{N} b_i = 1 $$

and thus the number of unknown parameters is finally $2(N - 1)$. It will be shown in the next subsection that the rule specified by (4) is a generalization of the well-known space rule.

Having defined the operating rule in the linear form of (4) with parameters $a_i$ and $b_i$ obeying (5), we have introduced a convenient parameterization of the problem. This raises important issues regarding the validity of the rule proposed. These are related to (1) the ability of equation (4) to take into account various policies that result from different concerns.
about the system, (2) the need for further mathematical development of the rule to take into account physical constraints of the system, and (3) parameter issues such as whether the linearity of (4) is appropriate and whether the number of parameters is sufficient. These issues are discussed in the following subsections.

2.2. Justification of the Rule’s Form

In this subsection we study five particular operating policies, which result from different concerns about the system properties and objectives. In each case we deal with one isolated objective of the system such as the minimization of spills or losses. To be able to obtain the operating rule for each case as an analytical solution, based on a theoretical objective function, we do not consider all physical constraints of the system at this stage. At a later stage we will incorporate the physical constraints in the rule. The five cases examined do not exhaust all possible concerns about the reservoir system operation, but they are indicative of the form such policies can take. As we will see, in all cases the result is the linear rule (4) with the particular values of coefficients \(a_i\) and \(b_i\) dependent on the main concern chosen. This justifies the linear form of (4) as a generalization of various operating rules.

2.2.1. Restricting spills. Assume that the primary concern is to avoid unnecessary spills from one reservoir while others still have empty space. This rule is appropriate for the refill cycle of the reservoirs or, equivalently, for the wet season. Spills are more likely to be avoided when more empty space is left for the reservoirs with larger expected cumulated inflows up to the end of their refill cycle. It has been shown [Sands, 1984; Johnson et al., 1991] that the minimum expected value of the total spills of the system corresponds to the case in which the probability of spill is the same for each reservoir, i.e.,

\[
\text{prob}(CQ_i \geq K_i - S_i) = \text{const} \quad \forall \ i \tag{6}
\]

where \(CQ_i\) is the cumulative inflow to reservoir \(i\) from the end of the current period to the end of the refill cycle, \(K_i\) is the storage capacity of reservoir \(i\), and \(\text{prob}(\cdot)\) denotes probability. Johnson et al. [1991] showed that under the assumption that the distribution of \(CQ_i/E[CQ_i]\) (with \(E[\cdot]\) denoting expectation) is the same for each reservoir \(i\), (6) results in

\[
K_i - S_i^* = \frac{\left( \sum_{j=1}^{N} K_j - V \right)}{\left( \sum_{j=1}^{N} E[CQ_j] \right)} \tag{7}
\]

This is the well-known space rule, which consists in keeping equal for all reservoirs the ratio of the empty space to the expected cumulative inflow for the rest of the refill cycle. Equation (7) can be rewritten in the form of (4) for each \(i\) with values of parameters

\[
a_i = K_i - b_i \sum_{j=1}^{N} K_j, \quad b_i = \frac{E[CQ_i]}{\sum_{j=1}^{N} E[CQ_j]} \tag{8}
\]

If the reservoirs are all located in a region with the same climatic regime, the ratios \(b_i\) of (8) do not vary significantly from one month to another as demonstrated in Figure 5 for the case study. Thus quantities \(a_i\) and \(b_i\) of (8) can be considered as time invariant.

Furthermore, the assumption that the distribution of \(CQ_i/E[CQ_i]\) is the same for all reservoirs is not obligatory to get the linear rule (4), as different assumptions can result in the same equation. For example, if all \(CQ_i\) have Gaussian distributions, one can easily obtain that (6) results again in (4) with \(a_i\) and \(b_i\) given by equations slightly different from (8).

2.2.2. Restricting losses. Very often the leakages from reservoirs are not negligible, especially if the reservoirs are natural lakes on a karstic background. It is also likely that evaporation losses are of main concern, especially if we consider natural shallow lakes. Thus let us assume that the losses due to leakage and evaporation are of much more importance when compared with spills. The losses due to leakage are commonly a function of water surface elevation, and those due to evaporation are a function of the surface area of the reservoir. Given the reservoir storage-elevation and area-elevation relationships, we can express the total losses of this kind as a function of storage, i.e.,

\[
L_i = l_i(S_i) \tag{9}
\]

If our concern is to minimize losses, using algebra and some rather general assumptions (functions \(l_i(S_i)\) increasing and concave, which holds for almost any reservoir; see Appendix A1), we find that the most efficient rule is the one that stores all water \(V\) at the reservoir \(m\) whose value \(l_m(V)\) is the minimum among those of other reservoirs \(l_i(V)\). Mathematically, this is expressed again by the linear equation (4) but with coefficients \(a_i = 0\) for all \(i\), \(b_i = 1\) for the specific reservoir \(m\) whose value \(l_m(V)\) is the minimum among all other \(l_i(V)\), and \(b_i = 0\) for all other \(i\) (except for \(i = m\)).

2.2.3. Ensuring conveyance. A third rule will be considered for periods with low system storage. In such periods the main concern is not avoiding reservoir spill but making withdrawals so as not to drive one or more reservoirs empty while demand cannot be satisfied from the remaining reservoirs because of limited conveyance capacity. In such a case it is straightforward that the optimal distribution is such that the storage in each reservoir is proportional to the conveyance capacity of the relative aqueduct. This rule is expressed by the same linear rule (4) but with coefficients

\[
a_i = 0, \quad b_i = \frac{C_i}{\sum_{j=1}^{N} C_j} \tag{10}
\]

for all \(i\), where \(C_i\) is the conveyance capacity of the aqueduct through which the release from reservoir \(i\) is made.

2.2.4. Taking into account the impacts of topology. In the above cases all reservoirs were assumed implicitly to be topologically equivalent; that is, each of them is located at a different river or branch of river, and they are all connected by separate aqueducts with the consumption location. However, in many cases there appear to be differences in the topology of the reservoir system that may affect greatly the operating rule. Let us consider, for example, the case where the reservoirs

\[\text{Appendices are available on microfiche. Order from American Geophysical Union, 2000 Florida Avenue, N.W., Washington, DC 20009. Document 97WR01034M; $2.50. Payment must accompany order.}\]
form a cascade along the same river. In such a case the spills of all reservoirs but the most downstream one are not a loss for the system. Moreover, for energy-saving reasons (e.g., minimization of pumping) it may be a gain for the system to store the water as far upstream as possible. In addition it is always possible to move the water from upstream to downstream if necessary, while the opposite needs pumping. Thus a good operating rule for such a case would be to keep the water at the most upstream reservoir (if feasible), leaving the downstream reservoirs empty. Mathematically, this is expressed by the same linear rule (4) with coefficients $a_i = 0$ for all $i$, $b_m = 1$ for the most upstream reservoir $m$, and $b_i = 0$ for all other $i$ (except for $i = m$).

2.2.5. Assuring satisfaction of secondary water uses. In many cases, apart from the main water use, there are some secondary water uses in the neighborhood of each reservoir (e.g., irrigation, satisfaction of environmental demands, etc.). In such cases we want to avoid situations where some reservoirs are almost empty, while others are almost full. Thus we can set a rule that stores the water proportionally to cumulative local water demand for consumptive use CLD in order to balance the satisfaction of all local uses. This leads again to the linear rule (4) with

$$a_i = 0, \quad b_i = \frac{E[CLD_i]}{\sum_{j=1}^{N} E[CLD_j] \forall i}$$

(11)

We have seen that in each of the above simple situations the operation rule has always the linear form (4) with parameters $a_i$ and $b_i$ given by different simple equations for each case. In real-world situations we have to deal with more than one such concern (or goal) simultaneously. In these situations we can keep the formalism and parameterization of the linear rule, but the parameters $a_i$ and $b_i$ are no longer determined by simple equations such as the above because the objective function is not simple enough to be treated analytically. The parameterization of the rule allows for estimation of parameters using simulation via sampling and search procedures [Loucks et al., 1981, p. 65]. Before we proceed to the description of the models for simulation and optimization it is necessary to incorporate physical constraints into the linear rule in order for it to be operational for real-world situations.

2.3. Further Development of the Rule and Parameter Issues

In introducing (4) we have ignored the physical constraints, which demand that the storage cannot be negative nor can it exceed the reservoir capacity. To correct this inconsistency, we modify (4) so that

$$S_i^* = \begin{cases} 0 & a_i + b_i V < 0 \\ a_i + b_i V & 0 \leq a_i + b_i V \leq K_i \\ K_i & a_i + b_i V > K_i \end{cases}$$

(12)

However, this creates another inconsistency as the quantities $S_i^*$ defined by (12) may no longer add up to $V$. Several adjustment procedures can be used, the most refined being the transformation of straight lines of (4) into broken lines. Here we adopt another procedure that is computationally simpler. We distribute the departure $V - \sum_{j=1}^{N} S_j^*$ proportionally to the quantity $S_i^*$ ($1 - S_j^*/K_j$) so that $(S_i^* = 0)$ maps to $(S_i^* = K_i)$ and $(S_i^* = 0)$ maps to $(S_i^* = K_i)$. In this way the adjustment procedure does not affect the cases where the reservoir $i$ was found by (12) to be either empty or full. Thus we get the final target storage $S_i^*$ by

$$S_i^* = S_i^* + \frac{S_i^* (1 - S_i^*/K_i)}{\sum_{j=1}^{N} S_j^* (1 - S_j^*/K_j)} \left( V - \sum_{j=1}^{N} S_j^* \right)$$

$$= S_i^* \left[ 1 + \phi(1 - S_i^*/K_i) \right]$$

(13)

with

$$\phi := \frac{V - \sum_{j=1}^{N} S_j^*}{\sum_{j=1}^{N} S_j^* (1 - S_j^*/K_j)}$$

(14)

We note that under certain circumstances (e.g., for $\phi$ lying outside of the interval $[-1, 1]$), (13) may lead to values of $S_i^*$ that still violate the physical constraints. These circumstances are described in detail in the Appendix A2 along with an iterative algorithm to obtain $S_i^*$ such that $0 \leq S_i^* \leq K_i$ in all cases. We emphasize that the final operating rule, expressed by means of $S_i^*$, is completely determined from the initial parameters $a_i$ and $b_i$. An example of an initial rule expressed in terms of $S_i^*$ along with its corresponding final rule expressed in terms of $S_i^*$ are given in Figure 6 for the case study described in section 3.

Having introduced the full mathematical description of the rule proposed, several issues concerning the rule’s parameters are raised: (1) Is the linear form (4) of the rule adequate, or do we need a more complicated nonlinear form? (2) Is the number of parameters in the rule (two parameters per reservoir) adequate, or do we need more or fewer parameters? (3) Do we need to introduce a seasonal variation of the parameters?

It is difficult to answer these questions in a strict mathematical sense. However, we will attempt to give some detailed but rather intuitive answers. The answer to question 1 is threefold. As we have shown in section 2.2, the linear form is justified for several simple cases. Second, the operational form of the rule is not strictly linear since the corrections (12) and (13) introduce strong nonlinearity as demonstrated in the example of Figure 6, where the final target storages and their initial values are compared. The initial linear form is, in fact, used as an efficient way to parameterize the problem using two parameters for each reservoir. Third, the physical constraints of a reservoir system strongly modify the form of any initial rule no matter which this specific form is. Different initial rules thus have very similar final operational forms. To demonstrate that numerically, we can experiment using a quadratic rule instead of the linear, i.e.,

$$S_i^* = a_i^* + b_i^* V + c_i^* V^2$$

(15)

where $a_i^*$, $b_i^*$, and $c_i^*$ are parameters for each reservoir $i$. Experimenting with different sets of parameters $a_i$ and $b_i$ of (4), we can find a parameter set of this linear rule such that the final rules (after introducing corrections for constraints) of both the linear and quadratic form are very close to each other. A comparison of the two rules (linear and quadratic) is illus-
trated in Appendix A3 for the quadratic rule with the highest possible curvature, where the final forms are almost indistinguishable (the overall root-mean-square error, normalized by the respective reservoir capacity, is less than 0.1%).

The above discussion already gives some indication of the adequacy of the number of parameters (question 2): the use of three parameters per reservoir instead of two essentially makes no difference. We could also consider reducing the number of parameters to one parameter per reservoir, thus formulating the rule as a homogeneous line of the form \( S_i^* = b_i V \). To test this, we approximated a quadratic and a linear nonhomogeneous rule with a linear homogeneous rule (see Appendix A3). In both cases we obtained approximations of the final operational rules with overall root-mean-square error less than 10%, although the initial rules differed by as much as 100%. This suggests that the rule may be satisfactory for practical applications even in its reduced homogeneous form. However, to develop a clearer idea of the adequacy of the number of parameters, we must assess the sensitivity of the objective function to some parameters. As it will be shown in the section 3, in our test case we started by using two parameters per reservoir \((a_i, b_i)\) and found that the optimum of the objective function was practically insensitive to \(a_i\), which indicates that one parameter per reservoir suffices. This, however, cannot be transferred to any reservoir system without prior investigation.

Question 3 concerns another form of nonlinearity that can be introduced through seasonal variation of the parameters. First, we note that in systems consisting of reservoirs with very high capacities that perform overyear regulation, there is no reason to consider target storages dependent on the season, as the overyear variation of storage is more important than the within-the-year variation. In systems with smaller capacities it seems reasonable to have the target storages dependent on the season. However, the parametric rule implicitly contains such a dependence of the target storages on \(V\). This is particularly true for reservoirs with considerable drawdown in the dry season. In such cases \(V\) takes large values only in the wet season. We note, though, that intermediate values of \(V\) are normally attained twice a year: once during the refill period and once during the drawdown period. It may be beneficial to distribute among the reservoirs the same total volume \(V\) in a different way in each of the two periods. This means that the use of two parameter sets for the rule, one for the refill and one for the drawdown period, may be advantageous. For simplicity the parameters \(a_i\) and \(b_i\) are considered in this study as time invariant and constant for each reservoir. However, the approach proposed can be directly modified to include two parameter sets, but this will require more computations because of the doubling of the number of parameters.

2.4. Optimization Model

As described above our proposal in this paper is to consider the coefficients \(a_i\) and \(b_i\) of the operating rule as unknown parameters and to determine them by optimization. Their values are optimized in the following way:

1. A simulation model of the reservoir system operation is built together with a multivariate stochastic model of the system’s inflows. A long series of synthetic inflows is generated and is passed into the simulation model to evaluate the objective function of the optimization model described in point (2), below.

2. An optimization method is used to determine \(a_i\) and \(b_i\). At each evaluation of the objective function one or more simulations of the system operation (depending on the constraints of the optimization) for the whole operation period are performed. Thus, in our case, simulation is embedded in the optimization algorithm.

To formulate the objective function of the optimization model, we consider two typical problems. In the first problem the objective is to maximize the target release of the system for a given reliability level. For example, this is the objective in the first three simple cases examined in section 2.2, which can be represented by a common objective function. Mathematically, this is expressed by

\[
\max D = f_1(a, b) \tag{16}
\]

where \(a = (a_1, \ldots, a_N)^T\) and \(b = (b_1, \ldots, b_N)^T\). The constraint for this optimization is related to a total reliability measure that the system should have, i.e.,

\[
\text{prob} \left( \sum_{i=1}^N R_i = D \right) = \alpha \tag{17}
\]

where \(\alpha\) is the reliability level. For example, if \(\alpha = 0.95\), the above equation means that in a simulated period of 2000 years the total release equals the target release \(D\) during 1900 years (95%), whereas we allow 100 years (5%) where the target release is not completely satisfied. The failure probability \(\alpha'\) corresponds to the case of partial satisfaction of the demand and \(\alpha' = 1 - \alpha\). Apparently, failure occurs in cases where release targets are not physically achievable.

In the second problem our concern is the cost of conveyance (or the profit, in cases of energy production). This is, for example, our concern for the fourth case examined in section 2.2. In this problem we can formulate the objective function as

\[
\min E \left[ \sum_{i=1}^N c_i(R_i) \right] = f_2(a, b) \tag{18}
\]

where \(c_i(R_i)\) is the cost paid for conveying the quantity \(R_i\) to the consumption location (negative in cases of energy production) and expectation is taken over the releases. Equation (17) still remains a constraint for (18).

Other concerns of the system may lead to different objective functions (single or multivariate) or to different constraints. In this paper we consider only the above two problems with single-objective optimizations having the form of equations (16) and (18).

2.5. Simulation of the System Operation

As we have seen previously, the optimization process involves a certain number of simulations of the system operation. In each simulation, trial values of the parameters \(a_i\) and \(b_i\) are used. At each time period of simulation the following computations are performed:

1. The end-of-period storage in the system \(V\) is estimated from (3).

2. The target storages \(S_i^*\) are obtained from (4). Then, these are corrected according to (12) and (13) to give the final values of the target storages \(S_i^{\ast\ast}\).

3. The releases from each reservoir are determined so as to meet the target storages \(S_i^{\ast\ast}\) while also satisfying

\[
0 \leq R_i \leq C_i \tag{19}
\]
In case the releases \( R_i \) are outside the limits set by (19) they are set equal to these limits, and the remainder from the total target release is redistributed among the remaining reservoirs.

4. The spill from each reservoir \( i \) is given by

\[
SP_i = \max \{0, (BS + Qi - R_i - L_i - K_i)\} \quad (20)
\]

In some cases this procedure may require an iteration. Initially, to estimate \( V \) in step (1), spills are assumed zero. If nonzero spills are derived from (20), \( V \) is reevaluated on the basis of those spills, and the whole procedure is repeated again. Finally, the simulation model may include other equations that determine leakages and safety storages. Examples are discussed in the next section in the presentation of the case study.

3. Case Study

3.1. The Reservoir System for Water Supply of the Greater Athens Area and Its Simulation

The reservoir system of greater Athens is used to supply water mainly for domestic and industrial use to the metropolitan area of Athens. It comprises two main reservoirs (Figure 1): (1) the Mornos Reservoir with an active storage capacity of 643 hm\(^3\) and (2) the natural Lake Iliki with a storage capacity of 587 hm\(^3\). A small reservoir near Athens, the Marathon Reservoir, with a storage capacity of 41 hm\(^3\), is also part of the system. This reservoir is considered full all the time for emergency situations. Major water transfer works are (1) the Mornos Aqueduct, some 200 km long, which carries water from the Mornos Reservoir to Athens and comprises a number of different hydraulic works, for example, 70 km of tunnels, and (2) the Iliki Aqueduct from Iliki to the Marathon Reservoir, with a storage capacity of 41 hm\(^3\), is also part of the system. This reservoir is considered full all the time for emergency situations. Major water transfer works are (1) the Mornos Aqueduct, some 200 km long, which carries water from the Mornos Reservoir to Athens and comprises a number of different hydraulic works, for example, 70 km of tunnels, and (2) the Iliki Aqueduct from Iliki to the Marathon Reservoir, which is some 60 km long. The growing water demand and the system’s vulnerability to drought during the severe drought of 1989–1990, which was followed by 6 years with low flows except for 1990–1991 [Nalbantis et al., 1994], led public authorities to decide to construct a new reservoir (Evinos) with a dam at the site of Aghios Dimitrios on the Evinos River just west of the Mornos River Basin. Water from the new reservoir will be diverted to the neighboring Mornos Reservoir and from there to Athens via the Mornos Aqueduct. The storage capacity of the reservoir is small (104 hm\(^3\)) as compared to that of the Mornos Reservoir. On the other hand, inflows to the new reservoir are of a magnitude comparable to that of the inflows to the Mornos Reservoir. As a result, the Mornos Reservoir will be the main storage work for the Evinos River flows as well. A map with the reservoir system is given in Figure 1, while a schematic layout is sketched in Figure 2, where, also, the technical characteristics of the system are annotated. Mean values, standard deviations, and lag-one autocorrelation coefficients for monthly inflows to the three main reservoirs (Evinos, Mornos, and Iliki) of the system are given in Table 1.

Water from the western part of the system (Evinos and Mornos reservoirs) flows to Athens via gravity. Contrary to this, water from Lake Iliki has to be pumped. Another important feature of the system is that Lake Iliki lies on a karstic geologic formation that causes significant leakages. These depend strongly on the water surface elevation of the lake and may equal half of the annual inflow for high elevations. Analysis of historical data established two distinct leakage-elevation relationships: a first one for the dry period (April through September) and a second one for the wet period (October through March). The relationship for the dry period is given by

\[
L_L = 0.01242Z^2 - 0.999Z + 17.46 + e \quad (21)
\]

where \( L_L \) is the monthly leakage in cubic hectometers and \( Z \) is the water elevation of the lake in meters. For the wet period the following relationship was found

\[
L_L = 0.01242Z^2 - 0.999Z + 22.16 + e \quad (22)
\]

In both (21) and (22) a random term \( e \) is added to account for discrepancies from the deterministic \( L_L = Z \) relationship. For this term, \( E[e] = 0 \), while its standard deviation is \( \sigma_e = 2.64 \) hm\(^3\) for the dry period and \( \sigma_e = 5.96 \) hm\(^3\) for the wet period. These two statistics are used to produce simulated values of leakages through random generation of \( e \) that are added to the deterministic part in (21) and (22) during simulation.

The Mornos Reservoir leakages are concentrated in a limited area of the reservoir and are rather small compared to...
those of Lake Iliki. They are effectively modeled via the following linear relationship:

\[ L_L = 22.865 \times 10^{-3} (Z - 384.2) \quad Z \geq 384.2 \text{ m} \quad (23) \]

Apart from water supply to the greater Athens area, the system provides water for irrigation of the Kopais Plain in the Boeotia district. This secondary water use is fixed by decree at 50 hm\(^3\)/yr but may be reduced in case of water shortages in the water supply of Athens.

In the simulation model of the system operation an arrangement has been made for keeping safety storages in case of possible damages to the system aqueducts. For the case of

![Figure 2. Schematic representation of the Athens water supply system. Characteristic data of the system are annotated: for rivers, the watershed area and the mean annual reservoir inflow; for reservoirs, the minimum and maximum water level and the active storage capacity; for aqueducts, the length and conveyance capacity; and for other components, the characteristic water levels.](image)

### Table 1. Mean Values, Standard Deviations, and Lag-One Autocorrelation Coefficients of Monthly Inflows to the Reservoirs of the Athens Water Supply System

<table>
<thead>
<tr>
<th></th>
<th>Evinos</th>
<th>Mornos</th>
<th>Iliki§</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(m)</td>
<td>(s)</td>
<td>(r)</td>
</tr>
<tr>
<td>October</td>
<td>7.2</td>
<td>6.5</td>
<td>0.32</td>
</tr>
<tr>
<td>November</td>
<td>30.4</td>
<td>23.5</td>
<td>0.17</td>
</tr>
<tr>
<td>December</td>
<td>60.0</td>
<td>47.1</td>
<td>0.49</td>
</tr>
<tr>
<td>January</td>
<td>48.3</td>
<td>34.6</td>
<td>0.19</td>
</tr>
<tr>
<td>February</td>
<td>56.4</td>
<td>32.0</td>
<td>0.75</td>
</tr>
<tr>
<td>March</td>
<td>47.8</td>
<td>27.1</td>
<td>0.80</td>
</tr>
<tr>
<td>April</td>
<td>34.0</td>
<td>12.2</td>
<td>0.29</td>
</tr>
<tr>
<td>May</td>
<td>18.5</td>
<td>7.1</td>
<td>0.60</td>
</tr>
<tr>
<td>June</td>
<td>8.2</td>
<td>3.1</td>
<td>0.73</td>
</tr>
<tr>
<td>July</td>
<td>4.7</td>
<td>1.5</td>
<td>0.81</td>
</tr>
<tr>
<td>August</td>
<td>3.1</td>
<td>0.8</td>
<td>0.68</td>
</tr>
<tr>
<td>September</td>
<td>2.9</td>
<td>0.9</td>
<td>0.11</td>
</tr>
<tr>
<td>Year</td>
<td>321.5</td>
<td>111.2</td>
<td>0.17</td>
</tr>
</tbody>
</table>


*Mean value, in cubic hectometers.
†Standard deviation, in cubic hectometers.
‡Lag-one autocorrelation coefficient.
§Inflow from B. Kifissos River (not including inflow from Iliki’s own basin)
The monthly water demand distribution coefficients are derived using the Dynamic Disaggregation Model (DDM), which is sufficient for the system simulation. The disaggregation into monthly depths as the monthly step was proven using a multivariate AR(1) model. Then these quantities are disaggregated into annual quantities, which is performed by a system simulation, given the concurrent rainfall and runoff. These generations result in the variation of the reservoir areas. For each of the two cases, the model generates the runoff of the three basins and the target release for month $j$ ($j = 1, 2, \cdots, 12$), which, in turn, is distributed throughout the months of the year via the water demand distribution coefficients

$$d_j = \frac{D_j}{D_{zn}}$$

where $d_j$ and $D_j$ are the water demand distribution coefficient and the target release for month $j$ ($j = 1, 2, 3, \cdots, 12$), respectively. Water demand distribution coefficients for both the water supply of the greater Athens area and the irrigation of Kopais Plain are given in Table 2.

### Table 2. Monthly Water Demand Distribution Coefficients $d_j$ ($j = 1, 2, \cdots, 12$) for the Athens Water Supply System

<table>
<thead>
<tr>
<th></th>
<th>Water Supply of Athens, %</th>
<th>Irrigation of Kopais Plain, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>October</td>
<td>8.75</td>
<td>0.00</td>
</tr>
<tr>
<td>November</td>
<td>7.75</td>
<td>0.00</td>
</tr>
<tr>
<td>December</td>
<td>7.75</td>
<td>0.00</td>
</tr>
<tr>
<td>January</td>
<td>7.17</td>
<td>0.00</td>
</tr>
<tr>
<td>February</td>
<td>6.58</td>
<td>0.00</td>
</tr>
<tr>
<td>March</td>
<td>7.42</td>
<td>0.00</td>
</tr>
<tr>
<td>April</td>
<td>7.58</td>
<td>2.58</td>
</tr>
<tr>
<td>May</td>
<td>8.67</td>
<td>7.17</td>
</tr>
<tr>
<td>June</td>
<td>9.33</td>
<td>17.58</td>
</tr>
<tr>
<td>July</td>
<td>10.08</td>
<td>39.84</td>
</tr>
<tr>
<td>August</td>
<td>9.75</td>
<td>32.83</td>
</tr>
<tr>
<td>September</td>
<td>9.17</td>
<td>0.00</td>
</tr>
<tr>
<td>Annual sum</td>
<td>100.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>

damage to the Mornos Aqueduct a sufficient volume of water is always kept in Lake Iliki to satisfy water demand of Athens and irrigation of the Kopais Plain for six months to come. Minimum inflow to Lake Iliki as well as the storage in the Marathon Reservoir are considered to contribute to safety storage. Owing to the large dead volume of the Mornos Reservoir (119 hm$^3$), which can be pumped in emergency situations, and to the absence of local water uses from that reservoir, no such safety concern was necessary for the case of damage to the Iliki aqueduct.

The annual target release $D$ is expressed in cubic hectometers per year. In the calculations this is first transformed into a monthly mean value $D_m(= D/12)$, which, in turn, is distributed throughout the months of the year via the water demand distribution coefficients.

$$d_j = \frac{D_j}{D_{zn}}$$

where $d_j$ and $D_j$ are the water demand distribution coefficient and the target release for month $j$ ($j = 1, 2, 3, \cdots, 12$), respectively. Water demand distribution coefficients for both the water supply of the greater Athens area and the irrigation of Kopais Plain are given in Table 2.

### 3.3. Results

The proposed method was applied in two real-world problems related to the water supply system of greater Athens. In the first problem (problem 1) the ultimate development of the system is studied. Specifically, the maximum possible system release is sought by taking no account of the operating cost (i.e., for pumping). In the second problem (problem 2) the system operation is studied for a level of development lower than the ultimate, considering this time the related economic aspects. Specifically, a target release level is assumed to be less than the maximum that is estimated in problem 1, and the minimization of the operating cost is sought.

In problem 1 the total target release from the system, $D_j$, is maximized for a selected level of failure probability. The objective function to be maximized is given by (16), while constraint (17) must also be satisfied. The adopted level of the probability of failure for the water supply system of greater Athens is $\alpha' = 1\%$ [Koutsoyiannis and Xanthopoulos, 1990], a value that provides a high level of security. So during the optimization process, the point $(a, b)$ in the parameter space, which yields the maximum target release for $\alpha' = 1\%$ is sought. However, the simulation of the system operation for a specific set of parameter values yields $\alpha'$ for a given water demand $D_j$.

To avoid an excessive number of simulations with large computing times, we followed a procedure with two steps. In step 1 a level of target release $D$ is selected and parameters are estimated that minimize the probability of failure or, otherwise, maximize the system reliability

$$\max \text{prob} \left( \sum_{i=1}^{N} R_i = D \right) = f_j(a, b)$$

with the constraint

$$D = \text{const}$$

Step 2 simply involves finding a target release that gives the desired level of reliability with the parameter values already estimated in step 1. The basic hypothesis behind this two-step optimization lies in the fact that (16) and (17) can be interchanged as far as their role as objective function and constraint is concerned. This is reasonable when the assumed level of target release in step 1 does not differ significantly from the target release estimated in step 2, a condition that must be checked a posteriori.

All simulations are based on a synthetic data set for a period of 2000 hydrological years. Nine hydrological variables are simulated, i.e., three reservoirs (Evinos, Mornos, and Iliki) × three variables (runoff, precipitation, and evaporation).

The main focus of this work is to explore the features of the approach associated with the parameterization of the proposed reservoir system operating rule and not to establish an efficient optimization algorithm. Our purpose is served better by using the uniform grid method of parameter optimization already described in classical texts [Loucks et al., 1981, pp. 65–68]. In this study the method is applied in the form of successive steps or iterations with grids that are nested to each other and
become progressively finer. In this method the objective function is evaluated via simulation at all the grid points of the parameter space that satisfy constraints (5). The algorithm starts by dividing the interval of variation \( P_i(p) \) of each parameter \( p \) with an interval divider \( \delta_i \) to obtain the initial (coarsest) grid. Then, we construct a second grid with finer resolution by taking a smaller interval \( P_j(p) \) of each parameter \( p \) only in the vicinity of the optimum and dividing it by a divider \( \delta_j \). This is considered as the first iteration. The algorithm proceeds in this way for a number of iterations \( M \) until convergence to one or more optima. Note that simulation runs are performed for \( M + 1 \) grids. The convergence criterion depends on the objective function to be optimized. For (25), of step 1, iterations are stopped when maximum difference between failure probability values within a grid drops below the critical value \( e_1 = 0.002 \). This is chosen as a small multiple of 0.0005, which is the minimum probability level that can be calculated for a period of simulation of 2000 years.

In our study the parameter set is six-dimensional, i.e., \( (a_1, a_2, a_3, b_1, b_2, b_3) \), where indexes 1, 2, and 3 correspond to Evinos, Mornos, and Iliki, respectively, but owing to (5), this is reduced to a four-dimensional problem. Preliminary tests showed little sensitivity to parameters \( a_i \) (\( i = 1, 2, 3 \)). One example is given in Table 3 for a particular set of parameters \( b = (0.20, 0.80, 0.00) \) and \( D = 700 \text{ hm}^3 \). This table shows that results are insensitive to parameters \( a \), and the rule proposed was initially overparameterized, at least for our case study.

Given the results of the sensitivity analysis and the discussion of the number of parameters presented in section 2.3, we opted to proceed to the optimization of parameters \( b_i \) (\( i = 1, 2, 3 \)) by selecting constant values for \( a_i \), i.e., \( a_i = 0 \) (\( i = 1, 2, 3 \)), or equivalently, to use the homogeneous instead of the complete linear rule. In this case the parameter space is initially three-dimensional with \( 0 \leq b_i \leq 1 \) (\( i = 1, 2, 3 \)) and is restricted to a two-dimensional parameter space given that (5) holds. The results are presented in Table 4. In Figure 3 we depict the results of the initial (coarsest) grid in contours with equal probability of failure for the space of parameters \( b_i \) that is mapped to an equilateral triangle. We observe that (1) the probability of failure follows a rather smooth and continuously curved surface; (2) this surface is not symmetrical with respect to the sides of the triangle, which is explained by the different conditions of the three reservoirs; (3) the lowest values of the surface correspond to \( b_3 = 0 \), which is explained by the high leakages of Lake Iliki (the zero value means that we withdraw water from Iliki as much as possible); (4) there is a flat area with minimum probability (equal to 1.4%) rather than a single point; and (5) further investigation of this area is needed for the selection of the final parameter set.

After three iterations we obtained the final grid. The flat area already detected in the initial grid was proved larger, and no probability less than 1.4% appears. The flat area is advantageous as it provides flexibility: any point with \( \alpha' = 1.4\% \) could be chosen. The selection of the final parameter set was based on engineering criteria. We have chosen the point with the lowest value of \( b_1 \), which corresponds to conveying as much water as possible from the Evinos to the Mornos Reservoir. The idea behind this is to store water as close to Athens as possible for safety reasons. Thus the final parameters set is \( (a, b) = [(0, 0, 0)^T, (0.08, 0.88, 0.04)^T] \). In Table 4 we depict the main characteristics of the optimization process. The optimization process for problem 1 is completed with

---

**Table 3.** Sensitivity Analysis of the Failure Probability \( \alpha' \) of the Athens Water Supply System to Parameters \( a_i \) (\( i = 1, 2, 3 \)) for Step 1 of the Optimization Process

<table>
<thead>
<tr>
<th>Test</th>
<th>Evinos</th>
<th>Mornos</th>
<th>Iliki</th>
<th>( \alpha' ), %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-500</td>
<td>500</td>
<td>1.40</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-400</td>
<td>400</td>
<td>1.40</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-300</td>
<td>300</td>
<td>1.40</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>-200</td>
<td>200</td>
<td>1.40</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>-100</td>
<td>100</td>
<td>1.40</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.40</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>-400</td>
<td>300</td>
<td>1.40</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>-300</td>
<td>200</td>
<td>1.40</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>-200</td>
<td>100</td>
<td>1.40</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>-100</td>
<td>0</td>
<td>1.40</td>
</tr>
</tbody>
</table>

In the case of maximization of the expected annual total release from the system (Problem 1). The annual target release is 700 \( \text{hm}^3 \). Parameters \( b_i \) are held constant: \( b_1 = 0.20 \) for Evinos, \( b_2 = 0.80 \) for Mornos and \( b_3 = 0.00 \) for Iliki.

---

**Table 4.** Summary of Results of the Optimization Process for Problem 1 (Step 1) and Problem 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Problem 1 (Step 1)</th>
<th>Problem 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean annual target release, ( \text{hm}^3 )</td>
<td>700</td>
<td>600</td>
</tr>
<tr>
<td>Number of iterations, ( M )</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Initial interval for ( b_i )</td>
<td>[0, 1]</td>
<td>[0, 1]</td>
</tr>
<tr>
<td>Interval divider ( b_i ) (( i = 1, \cdots, M ))</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Critical value for stopping</td>
<td>0.002</td>
<td>0.005</td>
</tr>
<tr>
<td>Final failure probability, %</td>
<td>1.40</td>
<td>1.00</td>
</tr>
<tr>
<td>Mean annual abstraction from Lake Iliki ( E[R_i] ), ( \text{hm}^3 )</td>
<td>182</td>
<td>104</td>
</tr>
</tbody>
</table>

*For all iterations except the first, where \( \delta_1 = 5 \).
As in problem 1, the uniform grid method is applied with parameters $a_i$ (5). The procedure here tries, for a given target release $R$, more, the cost of pumping is a linear function of withdrawals can be neglected if compared to the cost from Iliki. Furthermore, the operating cost of the Evinos and Mornos works, part of the system (the Evinos-Mornos subsystem) flows to Athens via gravity, while water from Iliki is pumped. Consequently, the operating cost of the Evinos and Mornos works can be neglected if compared to the cost from Iliki. Furthermore, the cost of pumping is a linear function of withdrawals $R_3$ from Iliki. So the objective function (18) becomes

$$\min E[R_3] = f_2(a, b)$$  \hspace{1cm} (27)

As in problem 1, the uniform grid method is applied with parameters $a_i = 0$ ($i = 1, 2, 3$) and parameters $b_j$ satisfying (5). The procedure here tries, for a given target release $D$, to get a solution that is closer to satisfying constraint (17) while at the same time optimizing $f_2$ in (27).

The results are presented in Table 4. The values of the objective function for the initial (coarsest) grid are also shown in Figure 4, where we have drawn contours of equal probability of failure and of equal mean annual abstraction from Lake Iliki. We observe that the general shape of the surface of probability is quite similar to that of Figure 3 and has its minimum values in the same region (although the absolute values of probability are different in the two figures). Figure 4 allows us to localize the area where the contour with probability of failure 1% passes, i.e., where the constraint (17) is valid. Guided by this we constructed a finer grid and so on. The criterion to stop the iteration was to obtain improvements of the objective function that are less than a certain critical value $\varepsilon_2$ in relative terms. In our case $\varepsilon_2 = 0.005$. Table 4 summarizes the results. The final set of optimal parameters is $(a, b) = [(0, 0, 0, 0)^T, (0.106, 0.291, 0.603)^T]$. The value of the objective function is $E[R_3] = 104$ m$^3$. Note that this value is 78 m$^3$ lower than the corresponding value for problem 1 (182 m$^3$). We can also easily notice that the optimal parameter set of problem 2 is clearly different from that of problem 1.

To validate the rule proposed, we compared the above results with those obtained by heuristic rules with no parameters to be optimized. These are (1) the well-known space rule expressed by (8), (2) the leakage rule as described in section 2.2, and (3) the conveyance rule given by (10). We have tested all three rules applied throughout the year as well as combinations of them applied separately for the dry and wet season, as shown in Table 6 (except for three combinations that had no meaning).

The comparison is performed only for Problem 1 since in this problem we can find the maximum target release from the system that corresponds to a failure probability equal to 1%. The application of the above heuristic rules to Problem 2 is not possible because, in that case, there is no degree of freedom: once the target release is fixed the failure probability is also fixed and cannot be made equal to its desired level (1% in our case).

For each one of the three basic heuristic rules we estimated the values of the parameters in equation (4). First, the parameter values for the space rule are estimated. From Figure 5 we conclude that ratios $E[(COQ)]/\sum_{i=1}^{N} E[(COQ)]$ are nearly constant for all months with mean values 0.313, 0.297, and 0.390 for Evinos, Mornos, and Iliki, respectively. With these values we obtain from (8) the values of $(a, b)$ shown in Table 5. The graphical representation of the space rule is given in Figure 6, in comparison with the optimized rules of problems 1 and 2. The parameter sets for all other heuristic rules, determined from the corresponding equations of section 2.2, as well as those obtained by optimizing the parametric rule for problems 1 and 2, are shown also in Table 5. We observe that (1) in all rules the parameters $a_i$ are zero except for the space rule, (2) the parameter $b_3$ for Lake Iliki optimized for problem 1 (parametric rule) takes a value similar to that of the leakage rule, and (3) the parameters $b_i$ for the Evinos and Mornos reservoirs.
voirs optimized for problem 1 are not well approximated by any one of the heuristic constant-parameter rules.

In Table 6 we depict the annual target release corresponding to the 1% failure probability for each one of the rules tested. These results allow us to make the following observations and interpretations. First, the space rule, applied throughout the year (case 1), gives a total annual release of 620 hm$^3$, which is 70 hm$^3$ less than that obtained by our method. This is expected since the avoidance of spills results in storing water mainly in the Mornos and Iliki reservoirs, thus leading to high leakage losses especially from Iliki. Second, the introduction of the

<table>
<thead>
<tr>
<th>Rule</th>
<th>Evinos</th>
<th>Mornos</th>
<th>Iliki</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_1$, hm$^3$</td>
<td>$b_1$</td>
<td>$a_2$, hm$^3$</td>
</tr>
<tr>
<td>Space</td>
<td>-315</td>
<td>0.313</td>
<td>247</td>
</tr>
<tr>
<td>Leakage</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Conveyance</td>
<td>0</td>
<td>0.377</td>
<td>0</td>
</tr>
<tr>
<td>Parametric, problem 1</td>
<td>0</td>
<td>0.080</td>
<td>0</td>
</tr>
<tr>
<td>Parametric, problem 2</td>
<td>0</td>
<td>0.106</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 6. Graphical representation of operating rules for (a) the final parameter set of problem 1, (b) the final parameter set of problem 2, and (c) the parameter set of the space rule. Solid lines with rhombi, squares, and circles correspond to reservoirs 1, 2, and 3 (Evinos, Mornos, and Iliki), respectively, and represent the adjusted rule (equation (13)). Dashed lines represent the initial linear rule (equation (4)).
leakage rule in the dry season while the space rule is still used in the wet season (case 2) does not improve the results. In this case the leakage rule tries to store all water of the dry season in the Evinos Reservoir, while in the previous wet period this was almost emptied by the space rule to keep empty space for the significant inflows from the Evinos basin. Because of the very low inflows in the dry season, no sensitivity to the introduction of the leakage rule is revealed. Third, the introduction of the conveyance rule in the dry season while the space rule is still used in the wet season (case 3) gives a small improvement of 8 hm³ with regard to the previous case. We note that the conveyance rule tries to store more water in the Evinos-Mornos subsystem, which has also lower leakage losses. As a result, the combination of the latter two rules was revealed in the case study, which allowed further simplification of the rule and restriction of the dimension of the parameter space to half the initial value.

The proposed parametric rule was superior in its performance. Since many real-world problems involve more than one of these goals, parameters are evaluated numerically to optimize one or more objective functions that are selected by the user. The rule drives a simulation model of the reservoir system, which is embedded in a scheme that optimizes the rule’s parameters.

The parametric rule proposed is tested on the case of the water supply system of the city of Athens, Greece, comprising three main reservoirs on three separate water basins. Its complexity and idiosyncrasies make the system ideal as a test system, since many of the operating goals examined theoretically appear in this case study. Two problems are tackled in this case study. First, the ultimate development of the system is considered, and the total release from the system is maximized for a selected level of system reliability. Second, an intermediate development of the system is sought, and the pumping cost is minimized for a given reliability and a given level of target release less than that obtained in the first problem. A detailed simulation model on a monthly timescale has been used in the analyses. This included a generation model of synthetic annual hydrological data and a model for disaggregation of annual values into monthly values. Also, it included models describing system losses such as leakages and evaporation. The system’s operating details such as the maintenance of safety storages were also taken into consideration. It appears that the parametric rule proposed has proven satisfactory in tackling the problem of finding the capabilities of a reservoir system on a long-term basis. Through its parameterization it effectively accommodates various system operating goals into a single-objective function. Insensitivity to a subset of the parameters was revealed in the case study, which allowed further simplification of the rule and restriction of the dimension of the parameter space to half the initial value.

Finally, the rule proposed is validated through comparison with other heuristic rules that satisfy specific goals (avoidance of spills, leakage losses, and conveyance problems). In all cases the proposed parametric rule was superior in its performance. Of course, storage and release trajectories obtained are not “optimal” in the absolute mathematical sense as the trajecto-

### Table 6. Annual Target Release Satisfied With 1% Failure Probability for Various Heuristic Operating Rules and the Optimized Proposed Rule (problem 1)

<table>
<thead>
<tr>
<th>Case</th>
<th>Rule throughout the Year</th>
<th>Wet Season</th>
<th>Dry Season</th>
<th>Annual Target Release, hm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>S</td>
<td></td>
<td></td>
<td>620</td>
</tr>
<tr>
<td>2</td>
<td>S</td>
<td>L</td>
<td></td>
<td>620</td>
</tr>
<tr>
<td>3</td>
<td>S</td>
<td>C</td>
<td></td>
<td>628</td>
</tr>
<tr>
<td>4</td>
<td>L</td>
<td></td>
<td></td>
<td>633</td>
</tr>
<tr>
<td>5</td>
<td>L</td>
<td></td>
<td></td>
<td>635</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td></td>
<td></td>
<td>652</td>
</tr>
<tr>
<td>7</td>
<td>P</td>
<td></td>
<td></td>
<td>690</td>
</tr>
</tbody>
</table>

The rules are applied throughout the year or by season. S is the space rule; L is the leakage rule; C is the conveyance rule; and P is the parametric rule proposed.
ries must comply with a simple parametric relation. Nevertheless, once optimized, the proposed rule is very simple, mathematically, to apply even for a nonexpert user and is therefore recommended for situations with long-term studies of reservoir systems.

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References


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