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Session NP15.03/ Scaling, multifractals and nonlinearity in geophysics: Stochastic rainfall modelling: scaling and other approaches; downscaling

Stochastic disaggregation of spatial-temporal rainfall with limited data

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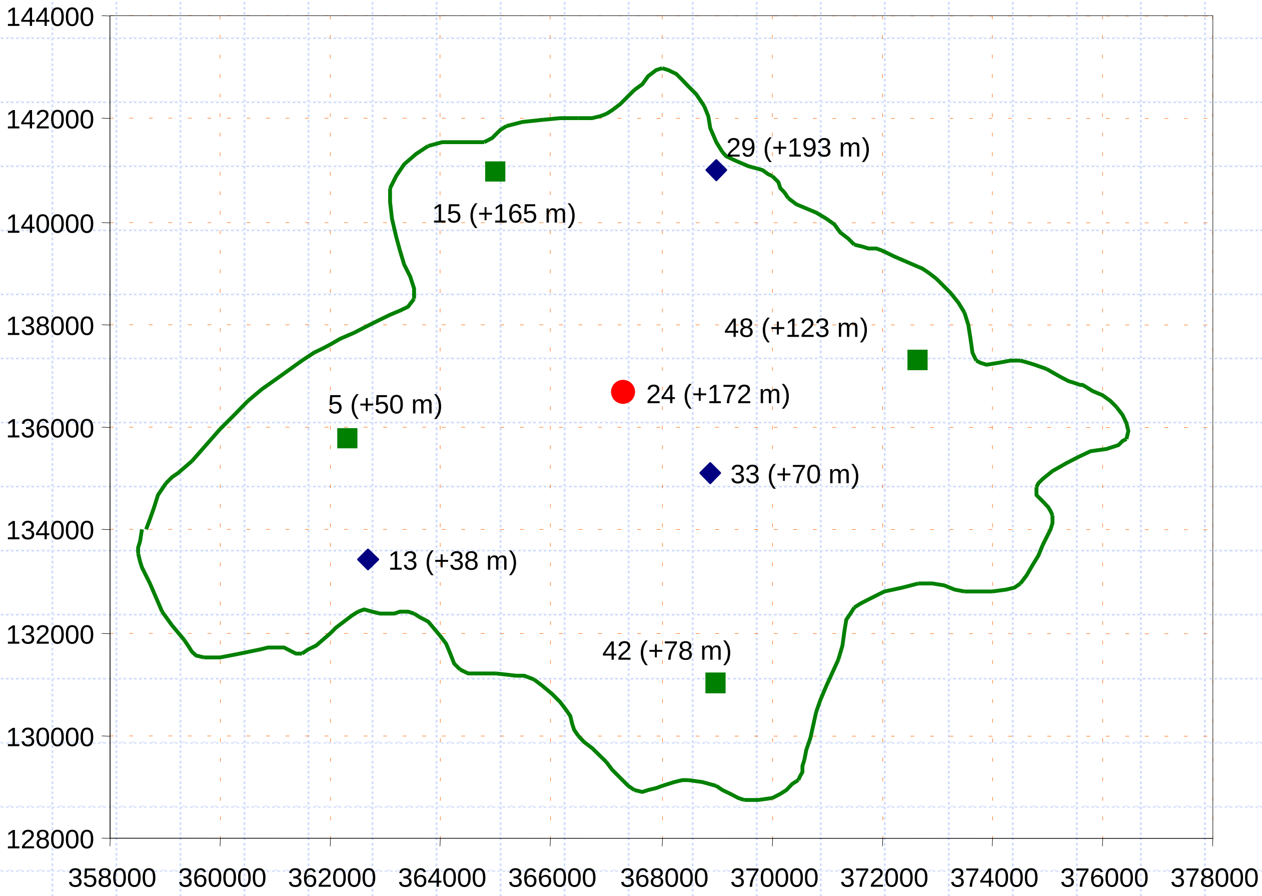
Questions studied

1. Can a spatial-temporal stochastic rainfall model be fitted using limited raingauge data?
   - Case (a): A limited number of points with hourly rainfall measurements
   - Case (b): Only one point with hourly rainfall measurements plus a limited number of points with daily rainfall measurements
2. If the fit of the spatial-temporal stochastic rainfall model is feasible in case (b), can it be utilised to enhance the available rainfall information?
   - Disaggregation of the daily data into hourly series.

Study area and raingauges



Case (a)
- ✓ 8 raingauges with hourly data (● ■ ◆)

Case (b)
- ✓ 1 raingauge with hourly data (●)
- ✓ 4 raingauges with daily data (■)

- ✓Brue catchment (South-Western England)
- ✓Equipped with 49 raingauges (8 were used in case (a) and 5 in case (b))
- ✓Five years of data available (September 1994 to August 1998)

Investigation of statistical properties of the rainfall field using raingauge data

- The rainfall process can be regarded as spatially stationary
- No effect of storm orientation is apparent
- The spatial correlation of rainfall is impressively high at the daily and hourly scales
- The lag-zero spatial correlations are by far larger than the lagged correlations, which indicates insignificance of storm movement in the case study and the time scale examined
- The lag-zero spatial correlation reduces with increasing distance whereas lagged correlation exhibit no variation with distance within the catchment

The Gaussian displacement spatial-temporal rainfall model – Principles

Storms

- Storm centers arrive in a homogeneous Poisson process of rate λ in two-dimensional space (x, y) and time (t).
- Each storm has a finite duration L, an infinite areal extent, represented by an elliptical geometry with eccentricity ε and orientation θ. It moves with a velocity V and incorporates a number of rainfall cells.

Cells

- The time origin of each rainfall cell follows a Poisson process starting at the time ordinate of the storm origin.
- The spatial displacements from storm center are random variables jointly normally distributed.
- Each cell has a finite duration D, exponentially distributed.
- All cells have elliptical shape with randomly varying areas and same orientation θ.
- Each cell has a uniform intensity exponentially distributed.

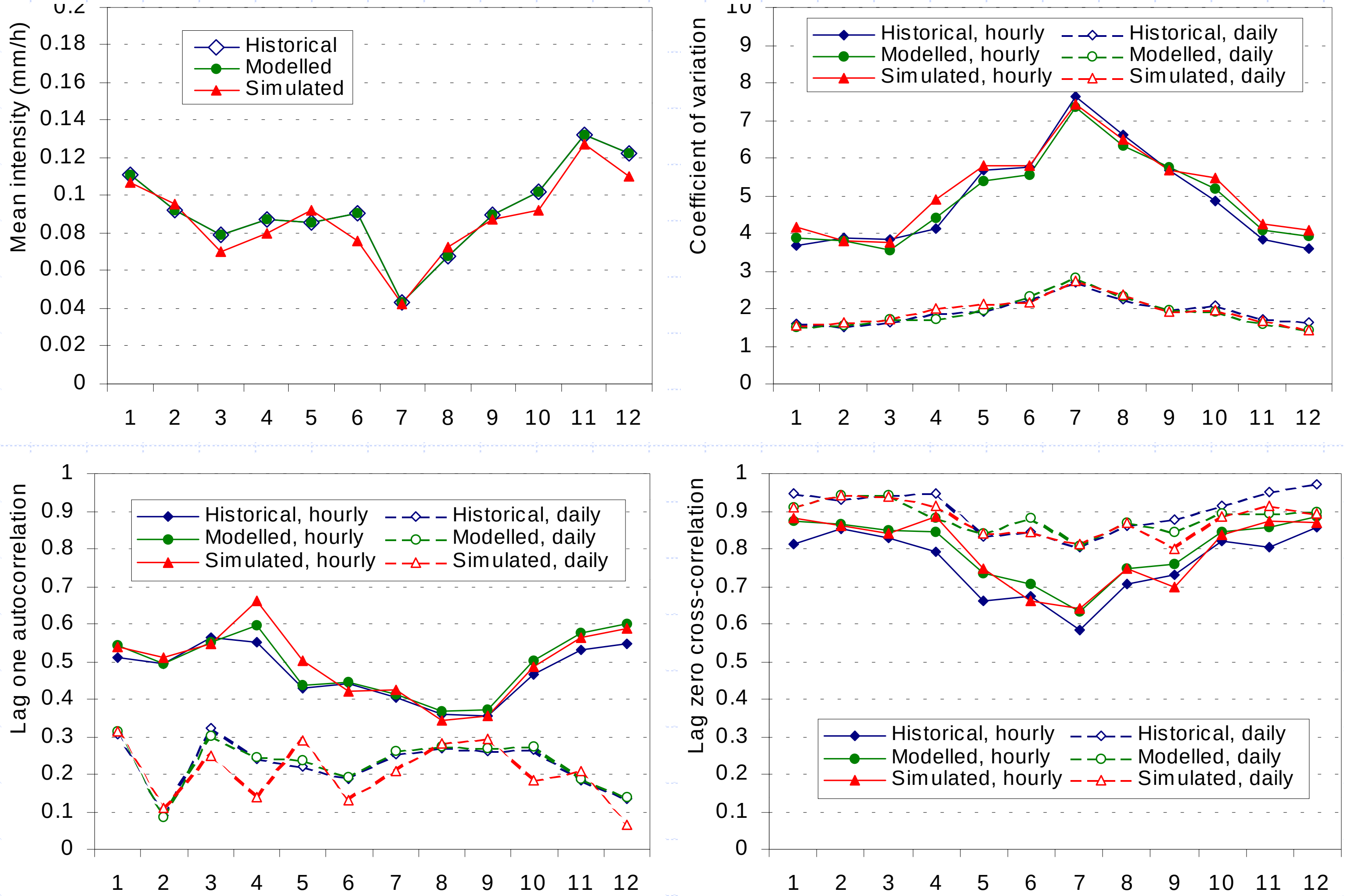
The Gaussian displacement spatial-temporal rainfall model – Parameters

- Parameters that can be estimated from raingauge data
  - Rate of storm arrivals, λ (number of storms per km^2 per hour)
  - Mean cell duration, μ_D (h)
  - Mean storm duration, μ_L (h)
  - Mean cell area, μ_A (km^2)
  - Mean characteristic storm area, μ_s (km^2)
  - Mean number of cells per storm, μ_c
  - Mean cell intensity, μ_X (mm/h)
- Parameters that cannot be estimated from raingauge data
  - Component of cell/storm velocity in the x direction, V_x (km/h)
  - Component of cell/storm velocity in the y direction, V_y (km/h)
  - Cell/storm eccentricity, ε
  - Cell/storm orientation (in radians from east), θ

Parameter estimation

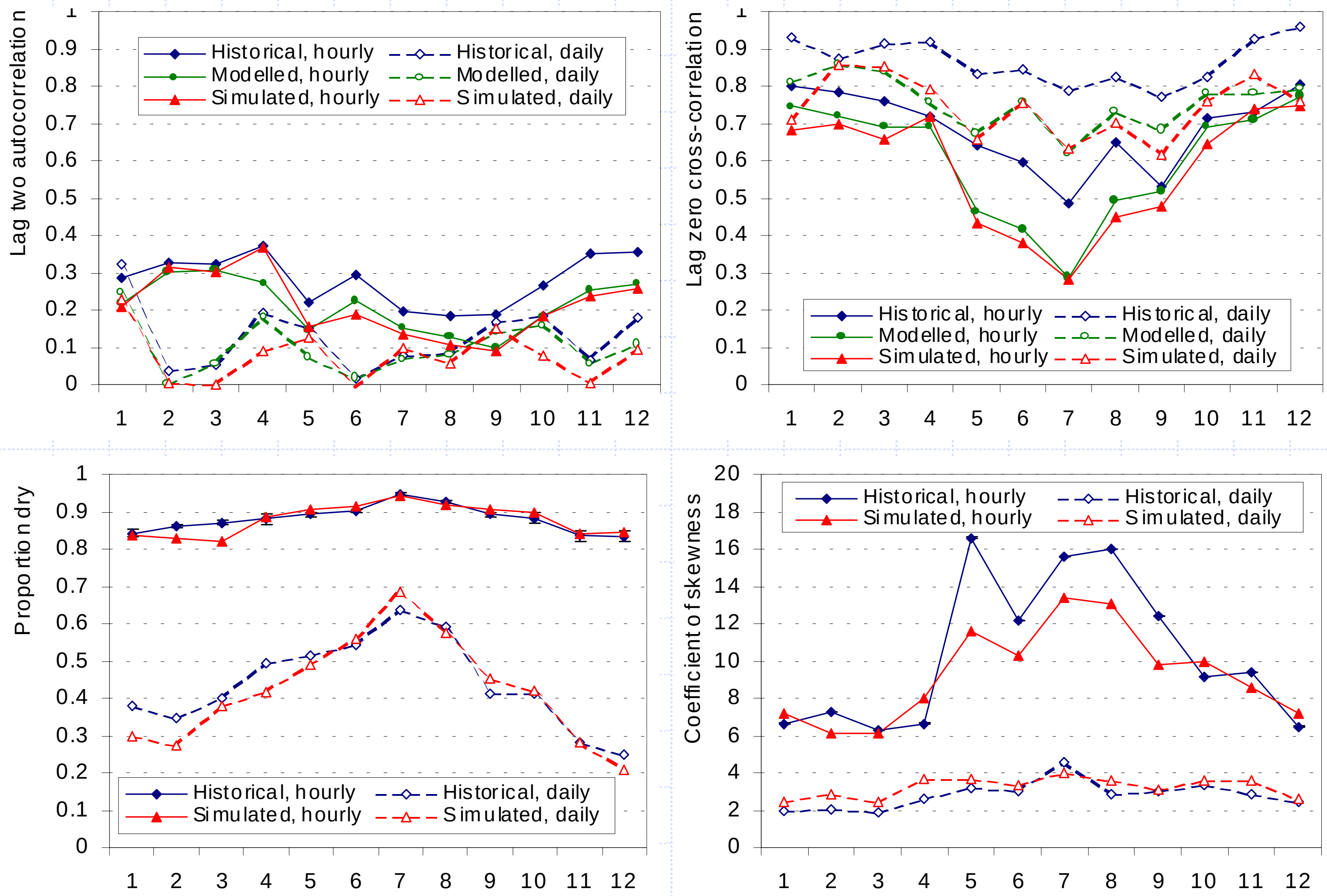
- Statistical properties of historical records used and preserved (one set for each month)
  - Mean rainfall intensity
  - Variance in hourly and daily scale
  - Lag-one autocorrelation coefficients at hourly and daily scale
  - Lag-zero cross-correlation coefficients for a characteristic spatial distance at daily scale
  - Lag-zero cross-correlation coefficients for the same distance at hourly scale (case a) or lag-two autocorrelation coefficients at hourly and daily scale (case b)
- Theoretical expressions of the same properties in terms of the model parameters
  - Obtained by numerical integration and/or Taylor expansion
- Parameter estimation
  - Optimisation framework using gradient and evolutionary methods
  - Objective function expressed as a weighted sum of square errors in matching theoretical and historical statistics

Comparison of simulated, modelled and historical statistics for each month – Case (a)



1. Statistics used in model fitting

Comparison of simulated, modelled and historical statistics for each month – Case (a)



2. Statistics not used in model fitting

Answer of initial question 1

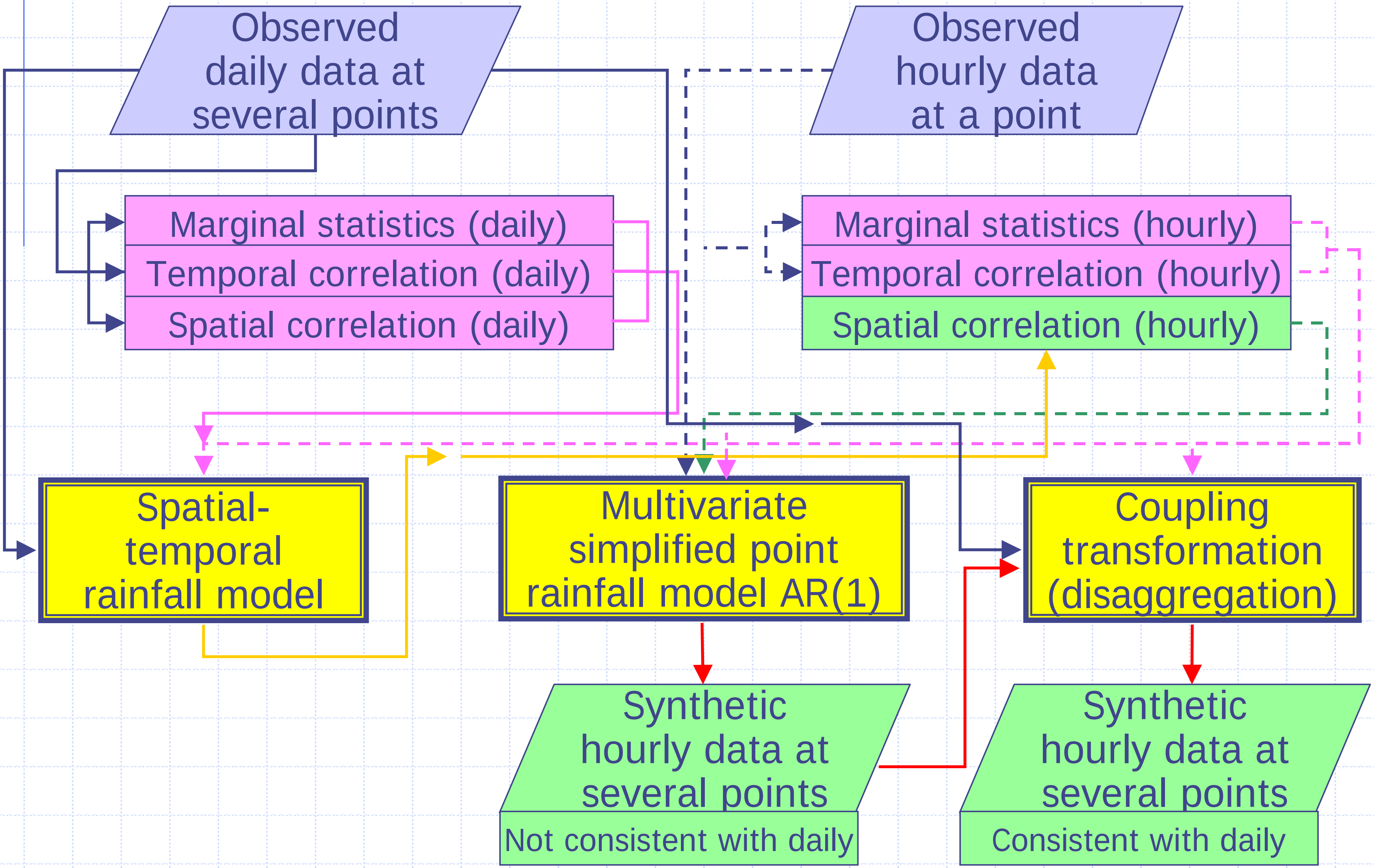
a) The Gaussian displacement spatial-temporal rainfall model can be calibrated using a number of raingauges with hourly data with the exception of parameters related to storm movement.

b) Provided that the rainfall field is stationary and isotropic in space, one raingauge with hourly data plus some raingauges with daily data may suffice for an approximate fitting of the model.

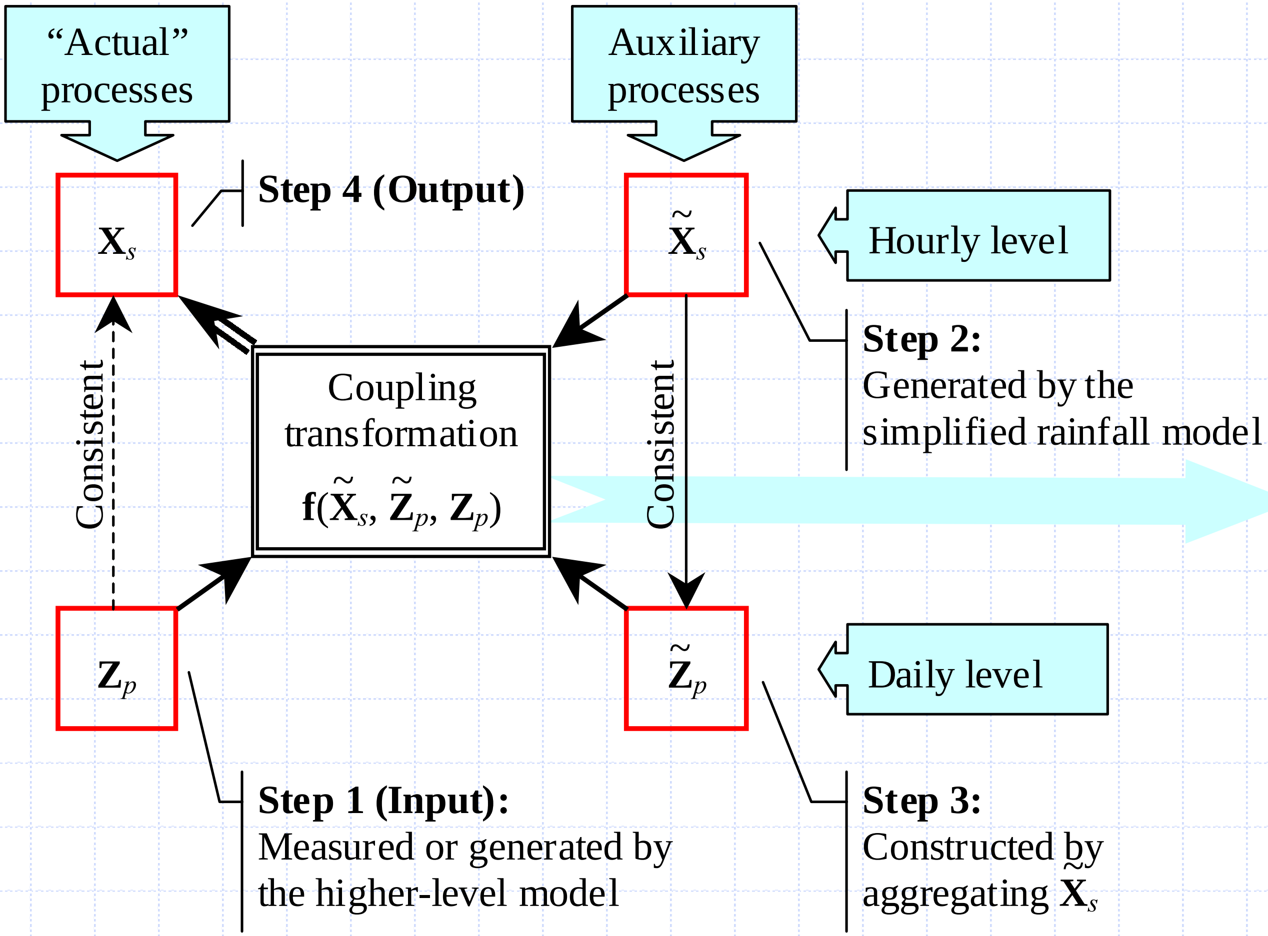
Revisit of question 2

- Can the fitted rainfall model be utilised to enhance the available rainfall information?
- How can measured daily values be disaggregated into hourly values, guided by the measured at a single raingauge hourly rainfall data?
- Can the rainfall model be adapted so as to be conditioned by:
  - the known hourly values at a single point?
  - the known daily values at several points?

A hybrid modelling approach



The coupling transformation



The linear transformation

$$\mathbf{X}_s = \tilde{\mathbf{X}}_s + \mathbf{h}\,(\mathbf{Z}_p - \tilde{\mathbf{Z}}_p)$$

where

$$\mathbf{h} = \mathrm{Cov}[\mathbf{X}_s, \mathbf{Z}_p] \cdot \{\mathrm{Cov}[\mathbf{Z}_p, \mathbf{Z}_p]\}^{-1}$$

preserves the vectors of means, the variance-covariance matrix and any linear relationship that holds among $\mathbf{X}_s$ and $\mathbf{Z}_p$.

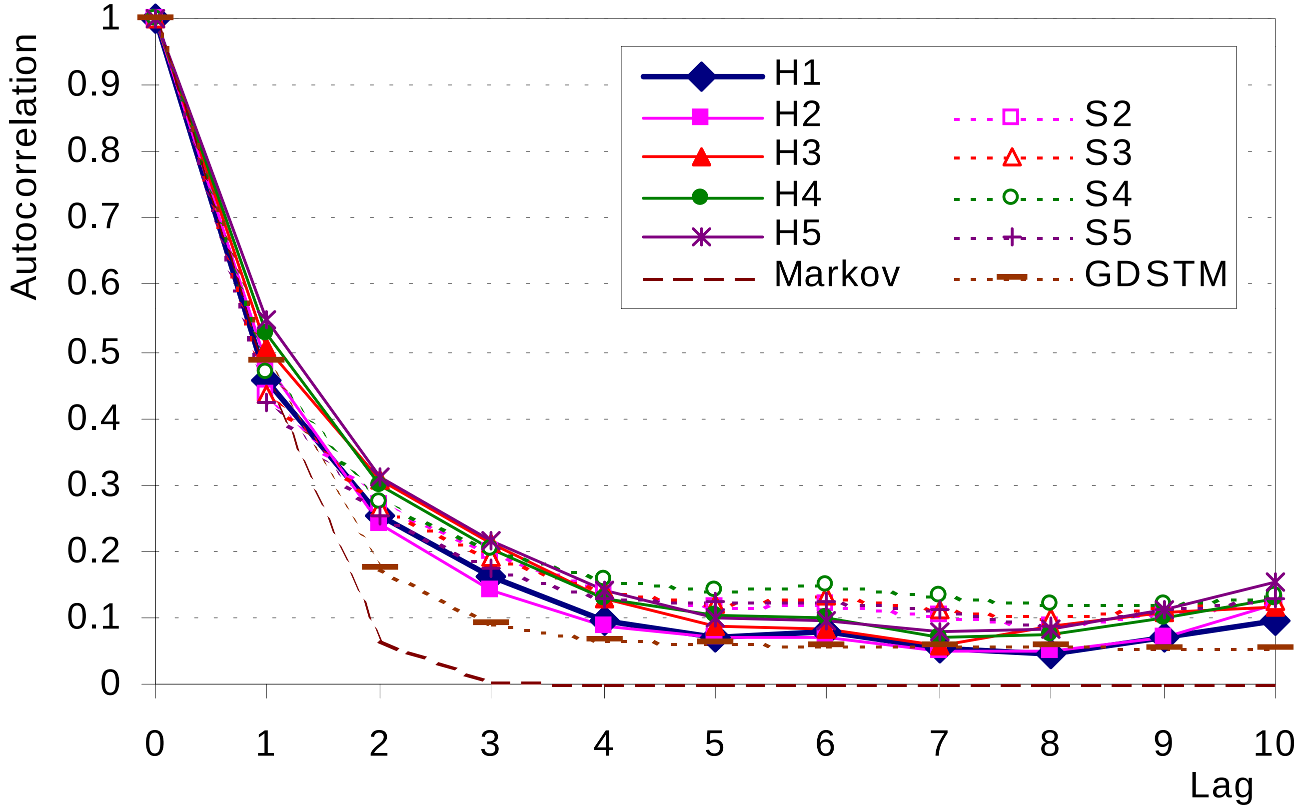
Disaggregation results 1: Statistics of hourly rainfall depths at each gauge – January

	Gage	24	5	15	42	48
Proportion dry	Hist.	0.84	0.85	0.84	0.85	0.84
	Model	0.84	0.84	0.84	0.84	0.84
	Synth.	0.84	0.82	0.80	0.82	0.80
Mean	Hist.	0.10	0.10	0.12	0.11	0.12
	Model	0.10	0.10	0.10	0.10	0.10
	Synth.	0.10	0.10	0.12	0.11	0.12
Maximum value	Hist.	8.0	7.6	5.8	4.8	4.8
	Model	8.0	8.0	8.0	8.0	8.0
	Synth.	8.0	6.7	7.1	6.9	7.2
Standard deviation	Hist.	0.39	0.40	0.44	0.40	0.41
	Model	0.39	0.39	0.39	0.39	0.39
	Synth.	0.39	0.37	0.41	0.39	0.42
Skewness	Hist.	7.6	7.2	5.8	6.0	5.3
	Model	7.6	7.6	7.6	7.6	7.6
	Synth.	7.6	6.9	6.9	7.9	7.2
Lag 1 autocorrelation	Hist.	0.46	0.48	0.50	0.53	0.55
	Model	0.46	0.46	0.46	0.46	0.46
	Synth.	0.46	0.44	0.44	0.47	0.43

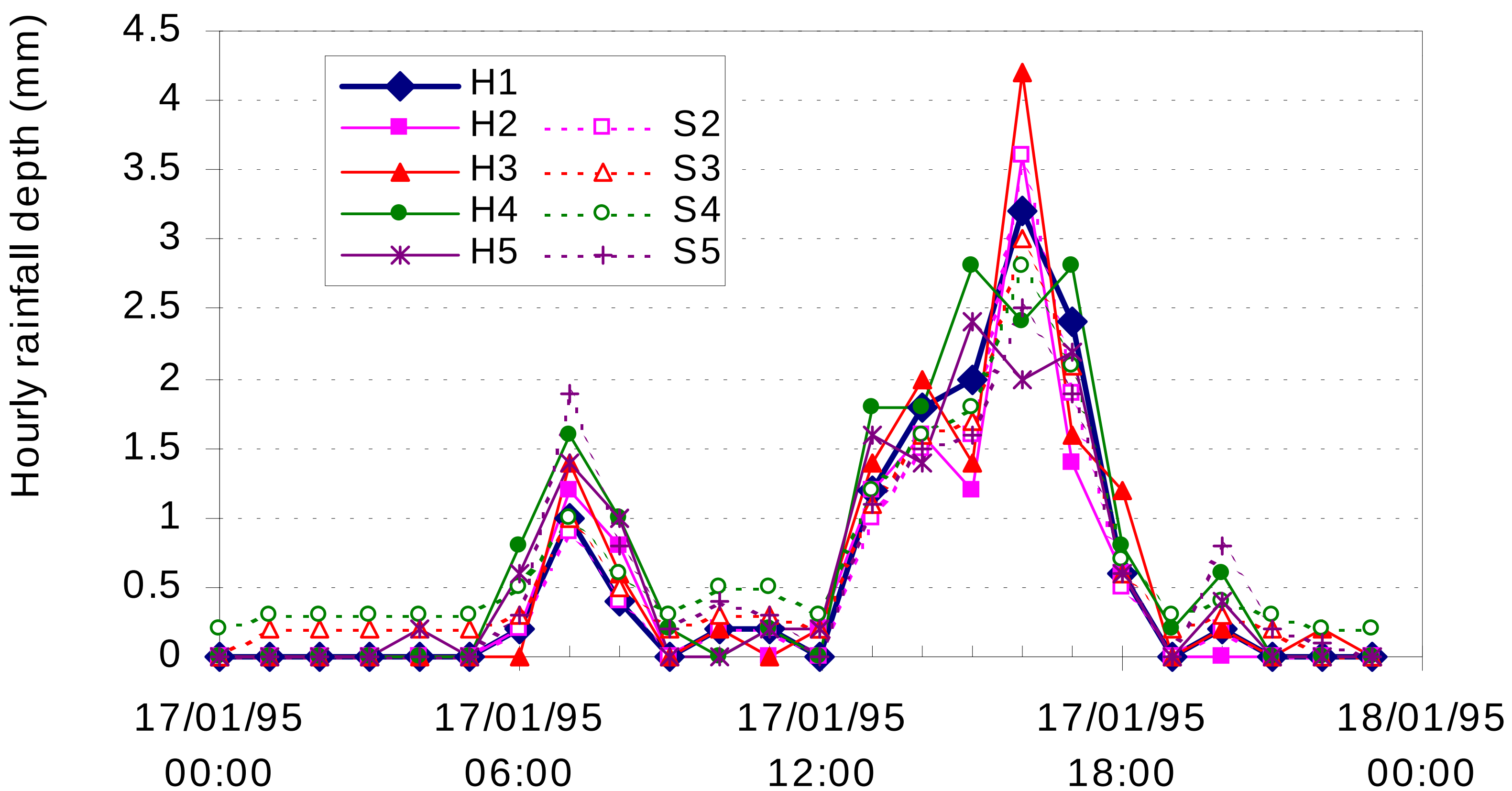
Disaggregation results 2: Cross-correlation coefficients for the five gauges at hourly level – January

Gage		1	2	3	4	5
1	Hist.	1.00	0.84	0.80	0.72	0.83
	Model	1.00	0.89	0.89	0.87	0.88
	Synth.	1.00	0.93	0.92	0.84	0.86
2	Hist.	0.84	1.00	0.82	0.80	0.83
	Model	0.89	1.00	0.87	0.82	0.77
	Synth.	0.93	1.00	0.90	0.84	0.83
3	Hist.	0.80	0.82	1.00	0.79	0.86
	Model	0.89	0.87	1.00	0.77	0.82
	Synth.	0.92	0.90	1.00	0.81	0.83
4	Hist.	0.72	0.80	0.79	1.00	0.87
	Model	0.87	0.82	0.77	1.00	0.84
	Synth.	0.84	0.84	0.81	1.00	0.79
5	Hist.	0.83	0.83	0.86	0.87	1.00
	Model	0.88	0.77	0.82	0.84	1.00
	Synth.	0.86	0.83	0.83	0.79	1.00

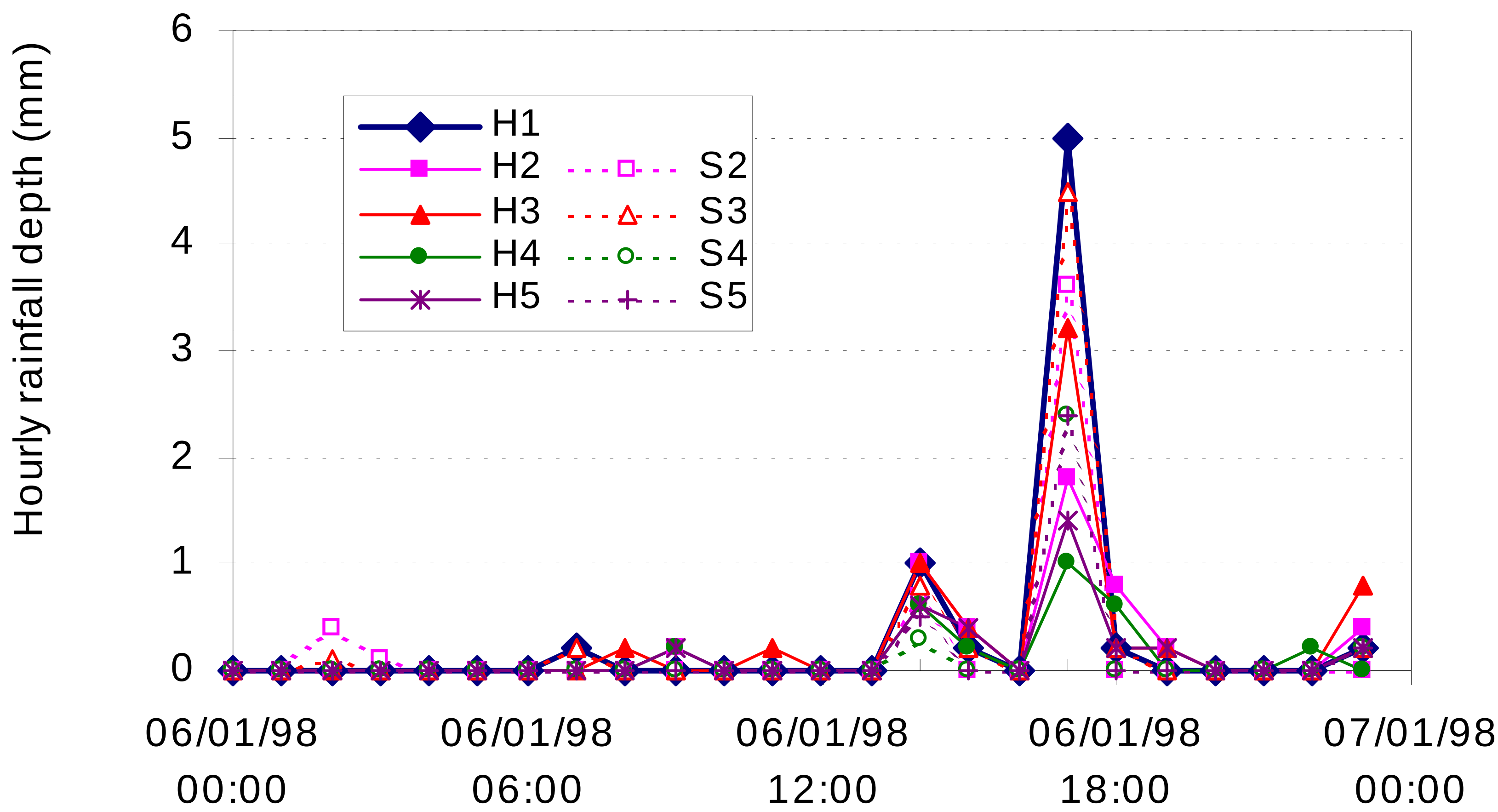
Disaggregation results 3: Comparison of autocorrelation functions of hourly rainfall in January



Disaggregation results 4: Comparison of historical and simulated hyetographs (event 1)



Disaggregation results 5: Comparison of historical and simulated hyetographs (event 2)



Concluding remarks

- A stochastic modelling framework for spatial-temporal rainfall, appropriate for catchments with limited data availability, has been developed.
- The methodology involves the combination of full spatial-temporal rainfall models and simplified multivariate models operating in a disaggregation framework.
- Potential hydrologic applications include enhancement of historical data series and generation of simulated data series.