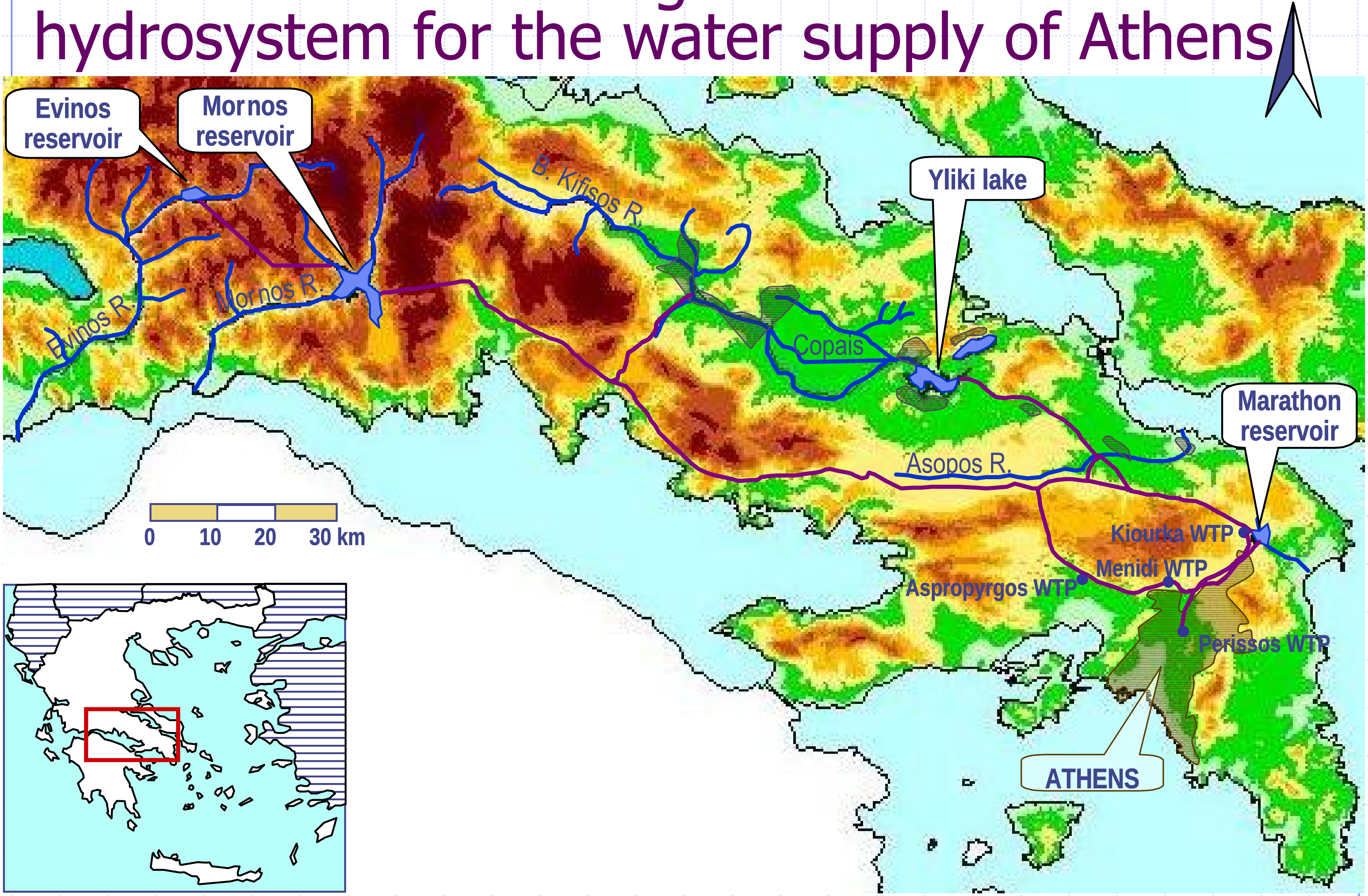
XXVI General Assembly of the European Geophysical Society
Nice, France, 25 - 30 March 2001

Session HSC3/ Concepts of risk and uncertainty
in reservoir design and control

A stochastic hydrology framework for the management of multiple reservoir systems

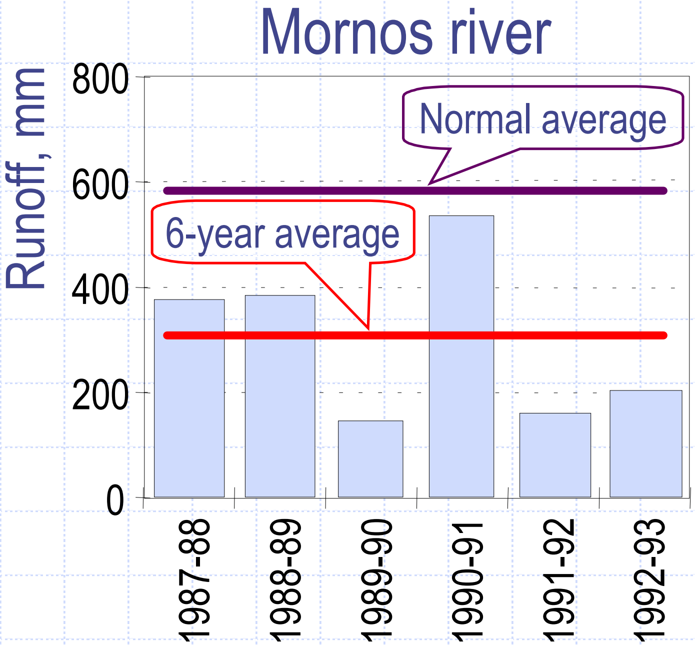
Demetris Koutsoyiannis & Andreas Efstratiadis
Department of Civil Engineering, National Technical University, Athens

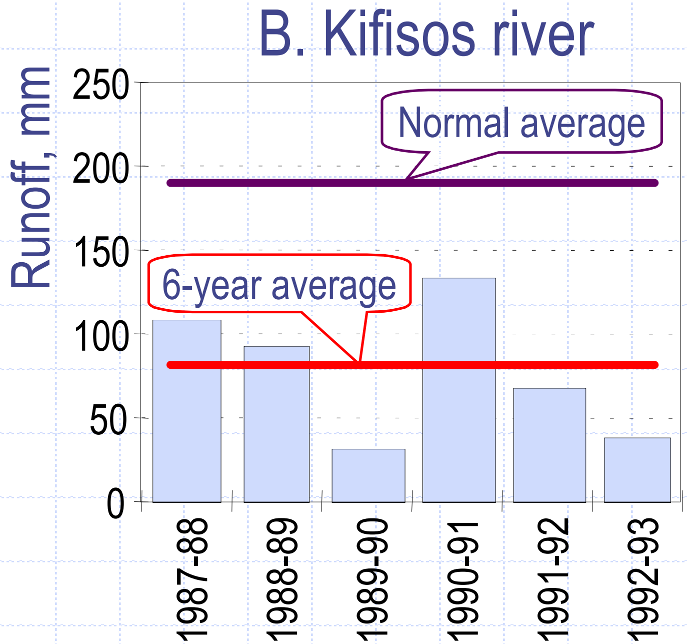
Motivation: The management of the hydrosystem for the water supply of Athens



Requirements for stochastic simulation

1. Multivariate model
2. Time scales from annual to monthly or sub-monthly
3. Preservation of essential marginal statistics up to third order (skewness)
4. Preservation of joint second order statistics (auto- and cross-correlations)
5. Capturing/reproduction of “patterns” observed in the last severe drought – Preservation of long-term persistence

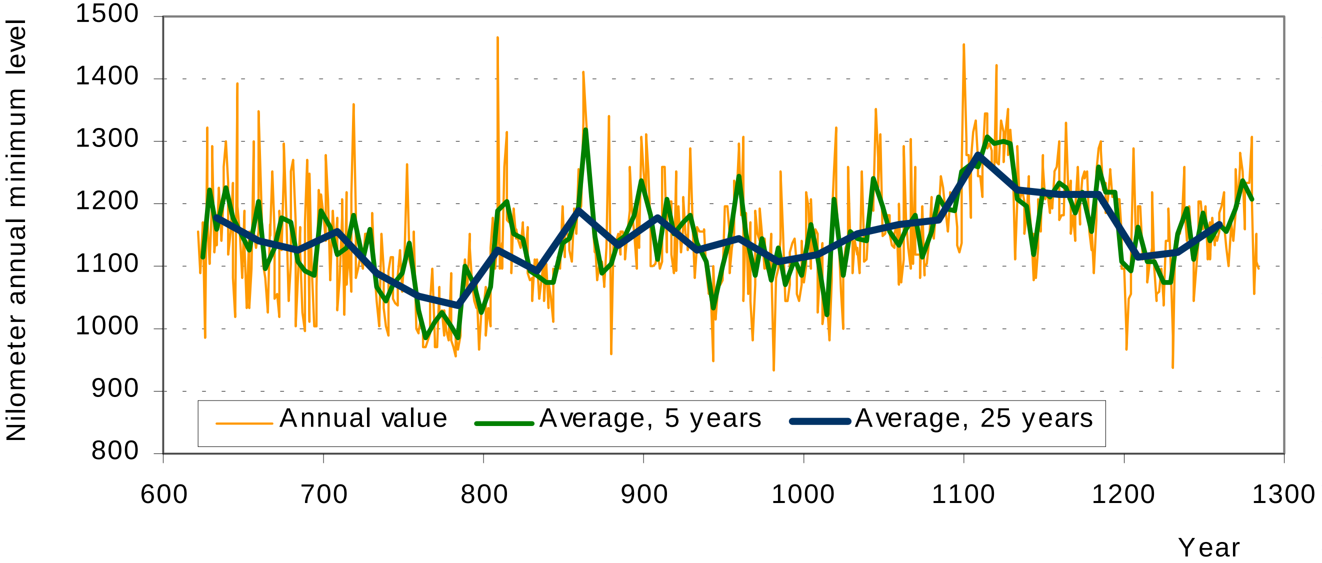




Climatic persistence versus climatic variability

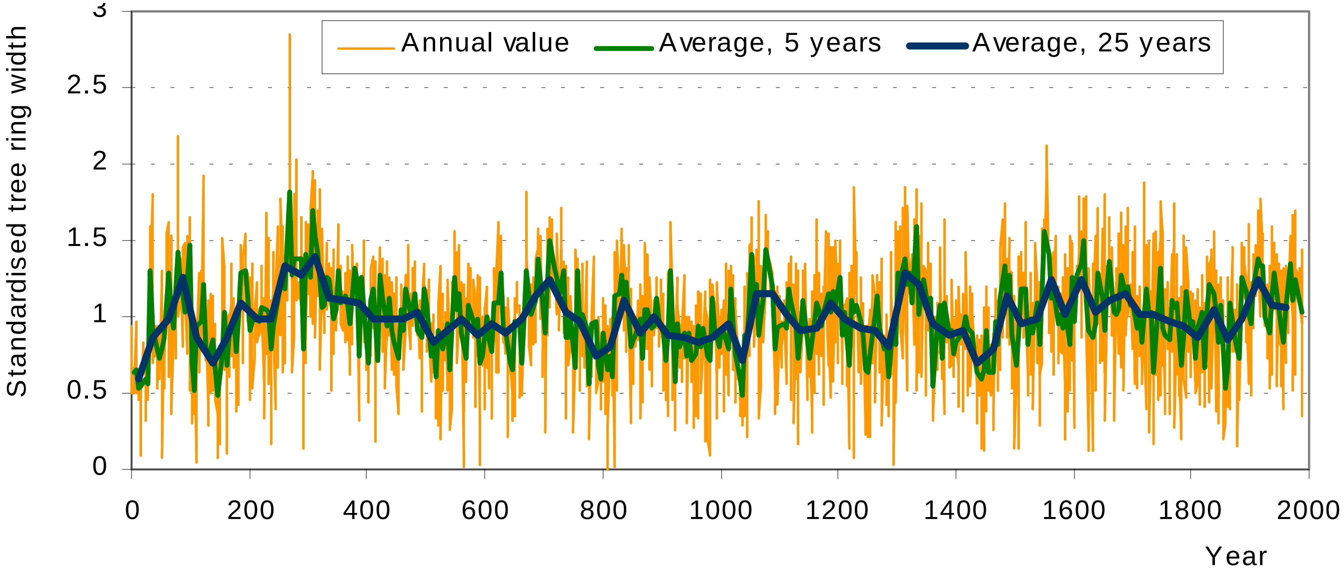
Annual minimum water level of the Nile river for the years 622 to 1284 A.D. (663 years)

Hurst exponent = 0.85



Standardised tree ring widths from a paleoclimatological study at Mammoth Creek, Utah, for the years 0-1989 (1990 years)

Hurst exponent = 0.75



Methodology 1: The generalised autocovariance function (GAS)

General expression

$$\gamma_j = \gamma_0 \, (1 + \kappa \, \beta \, j)^{-1/\beta}$$

where

γ_j: autocovariance for lag j

γ_0 : variance

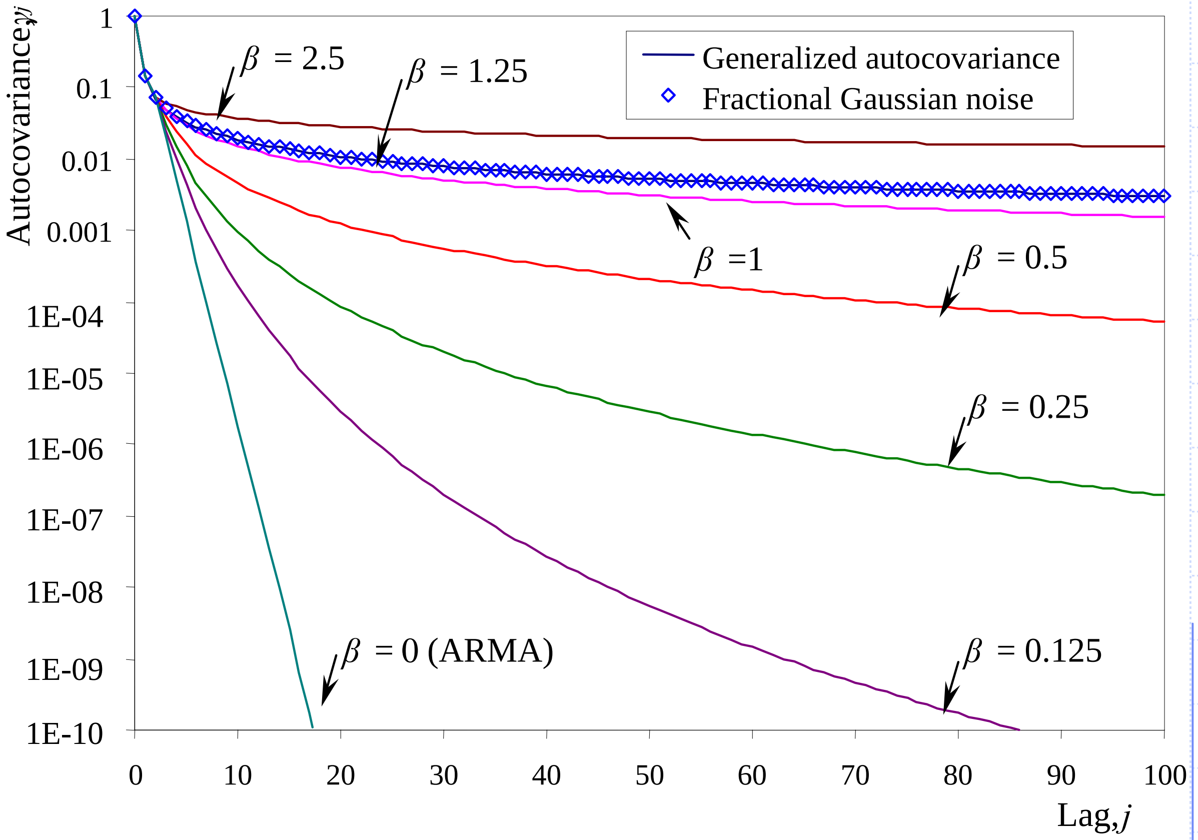
κ, β: parameters

(The two parameters allow for preservation of γ_1 and Hurst exponent)

For $\beta = 0 \Rightarrow$ ARMA

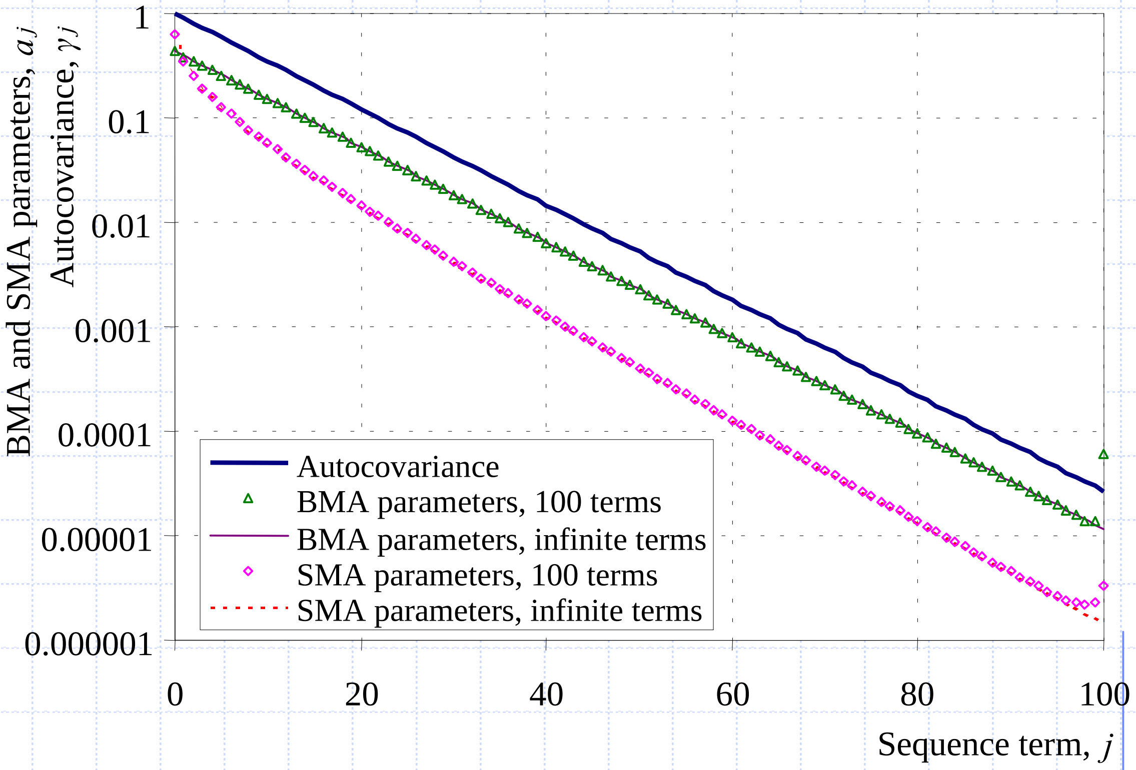
$$\gamma_j = \gamma_0 \exp(-\kappa \, j)$$

For $\kappa = (1/\beta)\,(1 - 1/\beta)^{-\beta} \, (1 - 1/2\beta)^{-\beta} \Rightarrow$ FGN



See details in: Koutsoyiannis, D., A generalized mathematical framework for stochastic simulation and forecast of hydrologic time series *Water Resources Research*, 36(6), 1519-1534, 2000.

Methodology 2: Generalised generating scheme for any covariance structure



Typical (backward) moving average (BMA) scheme

$$X_i = \ldots + a_1 V_{i-1} + a_0 V_i$$

where V_i innovations and a_i parameters.

Symmetric moving average (SMA) scheme

$$X_i = \ldots + a_1 V_{i-1} + a_0 V_i + a_1 V_{i+1} + \ldots$$

SMA has several advantages over BMA. Among them, it allows a closed solution for a_i:

$$s_a(\omega) = [2\, s_\gamma(\omega)]^{1/2}$$

where $s_a(\omega)$ and $s_\gamma(\omega)$ the DFTs of the series a_j and γ_j, respectively.

Both schemes are applicable for multivariate problems.

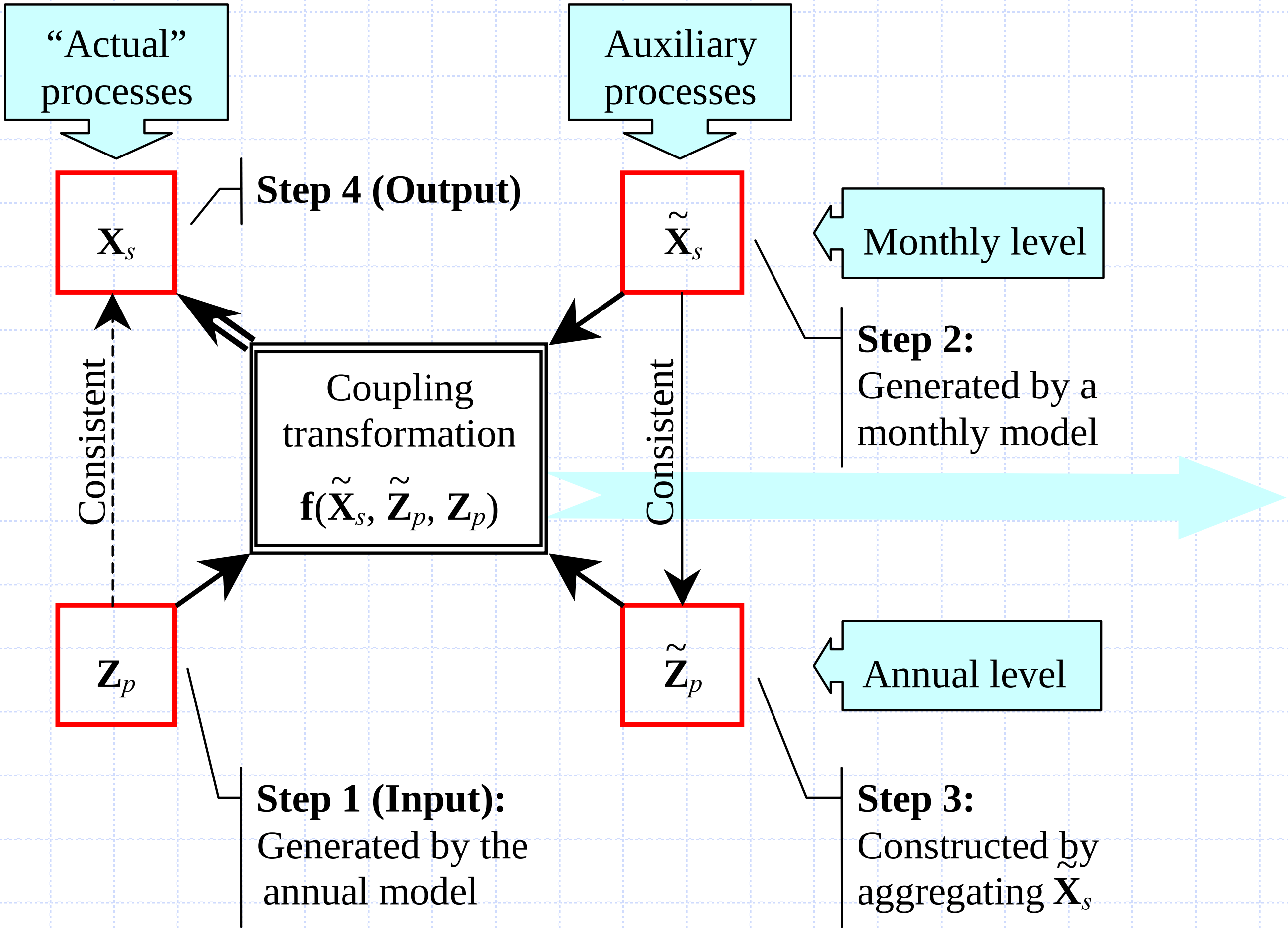
See details in: Koutsoyiannis, D., A generalized mathematical framework for stochastic simulation and forecast of hydrologic time series *Water Resources Research*, 36(6), 1519-1534, 2000.

Methodology 3: Stochastic simulation in forecast mode

- In terminating simulations of a hydrosystem the present and past states must be considered.
- The observed values of the present and past must condition the hydrologic time series of the future.
- This is attainable using a two-step algorithm
  1. Generate future time series without reference to the known present and past values.
  2. Adjust future time series using the known present and past values and a linear adjusting algorithm.
- The linear adjusting algorithm:
  1. is expressed in terms of covariances among variables;
  2. preserves exactly means, variances and covariances;
  3. is easily implemented.

See details in: Koutsoyiannis, D., A generalized mathematical framework for stochastic simulation and forecast of hydrologic time series *Water Resources Research*, 36(6), 1519-1534, 2000.

Methodology 4: Coupling stochastic models of different time scales



"Actual" processes

Auxiliary processes

$\mathbf{X}_s$

$\tilde{\mathbf{X}}_s$

Step 4 (Output)

Monthly level

Step 2: Generated by a monthly model

Consistent

Coupling transformation $\mathbf{f}(\tilde{\mathbf{X}}_s, \tilde{\mathbf{Z}}_p, \mathbf{Z}_p)$

Consistent

$\mathbf{Z}_p$

$\tilde{\mathbf{Z}}_p$

Annual level

Step 1 (Input): Generated by the annual model

Step 3: Constructed by aggregating $\tilde{\mathbf{X}}_s$

The linear transformation

$$\mathbf{X}_s = \tilde{\mathbf{X}}_s + \mathbf{h}\,(\mathbf{Z}_p - \tilde{\mathbf{Z}}_p)$$

where

$$\mathbf{h} = \mathrm{Cov}[\mathbf{X}_s, \mathbf{Z}_p] \cdot \{\mathrm{Cov}[\mathbf{Z}_p, \mathbf{Z}_p]\}^{-1}$$

preserves the vectors of means, the variance-covariance matrix and any linear relationship that holds among $\mathbf{X}_s$ and $\mathbf{Z}_p$.

See details in: Koutsoyiannis, D., Coupling stochastic models of different time scales, *Water Resources Research*, 37(2), 379-392, 2001.

Methodology 5: Preservation of skewness in multivariate problems via appropriate decomposition of covariance matrices

- Consider any linear multivariate stochastic model of the form

$$\mathbf{Y} = \mathbf{a}\,\mathbf{Z} + \mathbf{b}\,\mathbf{V}$$

where **Y**: vector of variables to be generated, **Z**: vector of variables with known values, **V**: vector of innovations, and **a** and **b**: matrices of parameters.

- The parameter matrix **b** is related to a covariance matrix **c** by

$$\mathbf{b}\,\mathbf{b}^T = \mathbf{c}$$

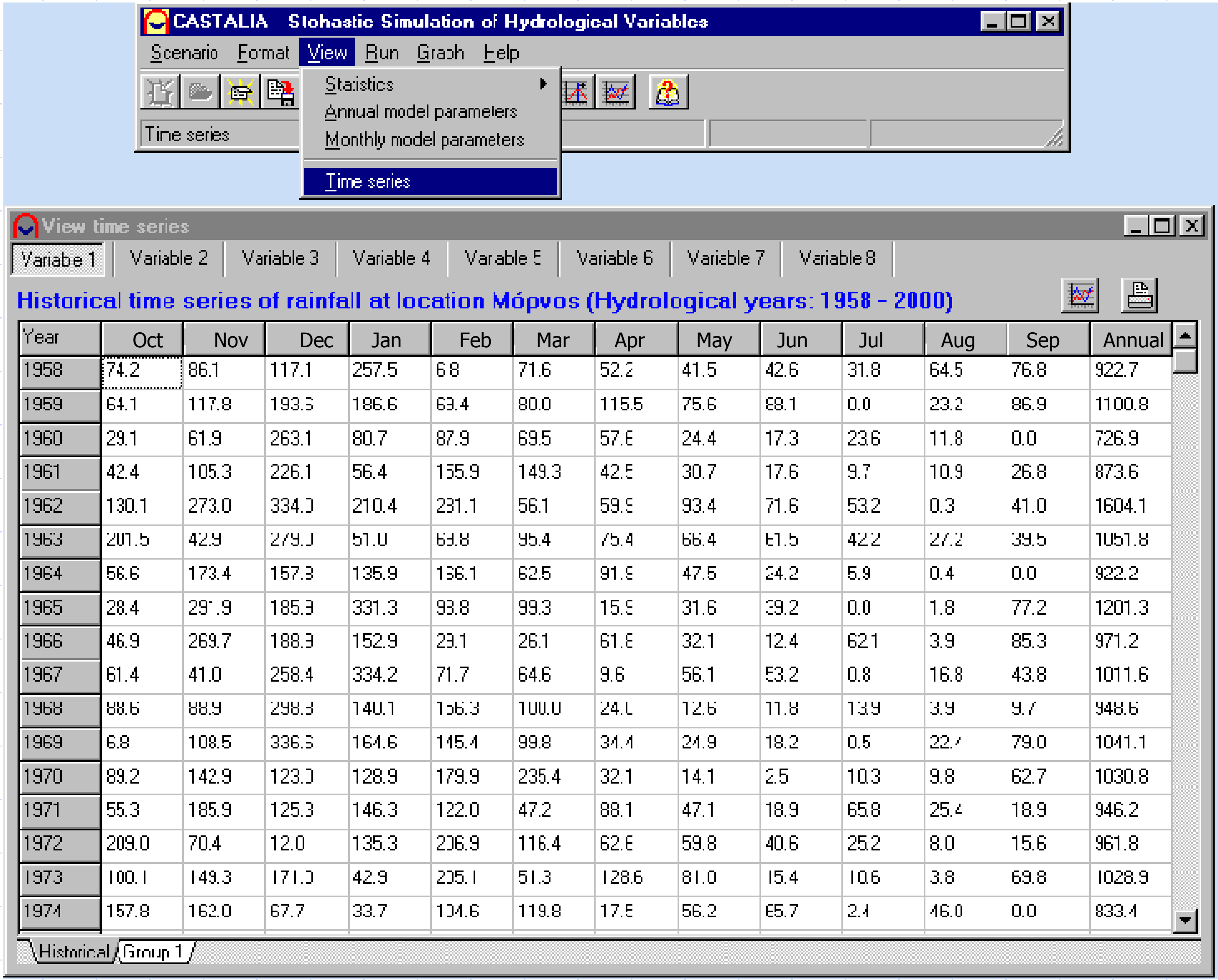
- This equation may have infinite solutions or no solution.
- The skewness coefficients **ξ** of innovations **V** depend on **b**.
- The smaller the values of **ξ**, the more attainable the preservation of the skewness coefficients of the actual variables **Y**.
- Therefore, the problem of determination of **b** can be solved in an optimisation framework, that combines
  - minimisation of skewness **ξ**, and
  - minimisation of the error $||\mathbf{b}\,\mathbf{b}^T - \mathbf{c}||$.
- A fast optimisation algorithm has been developed for this problem.

See details in: Koutsoyiannis, D., Optimal decomposition of covariance matrices for multivariate stochastic models in hydrology, *Water Resources Research* 35(4), 1219-1229, 1999.

Implementation of the methodology: The **Castalia** software

- Designed as part of a decision support system for the water resource system of Athens
- Linked to a simulation-optimisation model of a hydrosystem
- Can also perform as a stand-alone software
- Written in **Delphi**; utilises **Oracle**.
- Simulates several hydrological variables at multiple sites
- Uses annual and monthly time scales
- Preserves:
  - essential marginal statistics up to third order (skewness)
  - joint second order statistics (auto- and cross-correlations)
  - long-term persistence

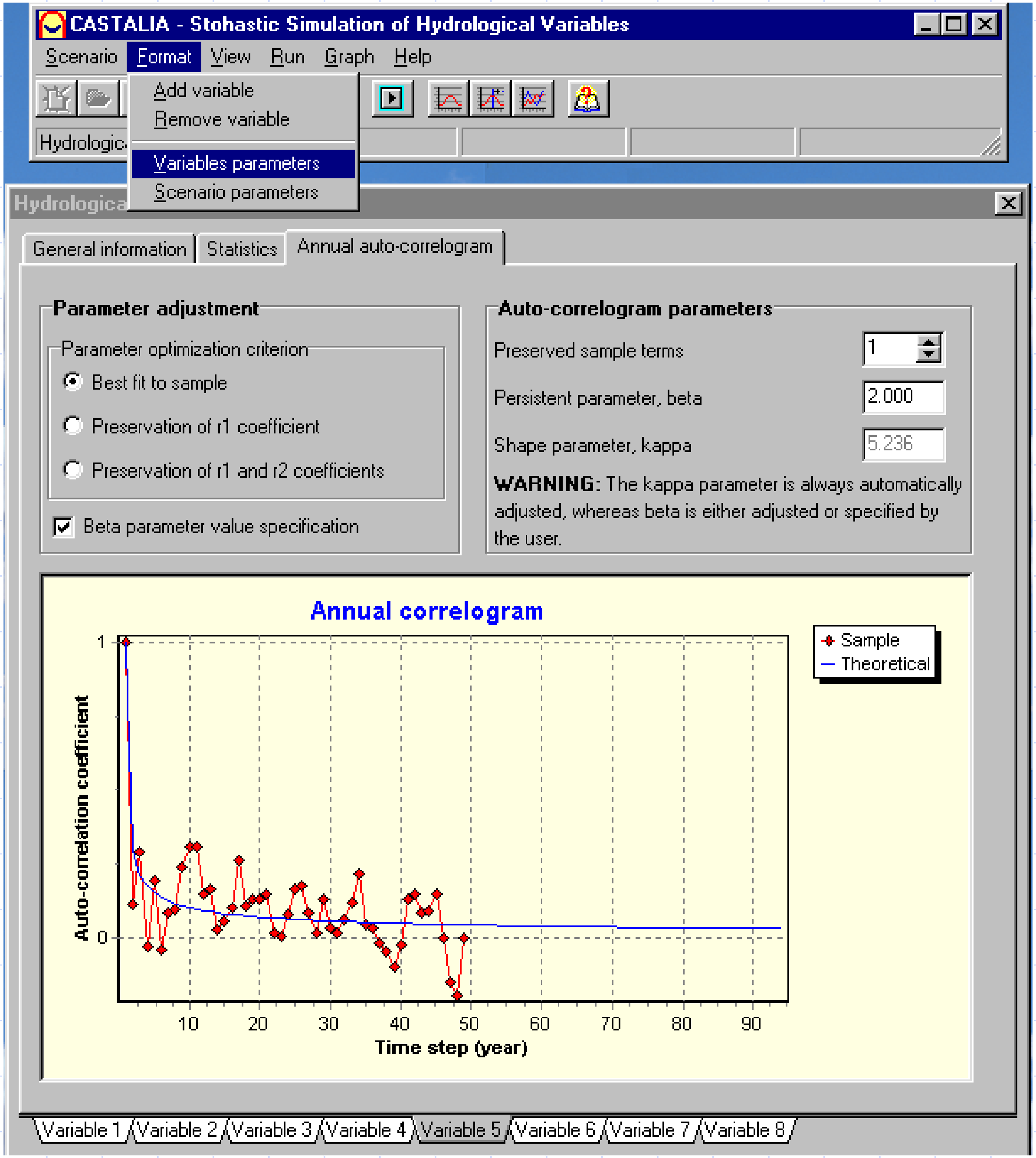
Castalia:Data base operations for time series



Historical time series of rainfall at location Mópvos (Hydrological years: 1958 - 2000)

Year	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Annual
1958	74.2	86.1	117.1	257.5	6.8	71.6	52.2	41.5	42.6	31.8	64.5	76.8	922.7
1959	64.1	117.8	193.3	186.6	63.4	80.0	115.5	75.6	88.1	0.0	23.2	86.9	1100.8
1960	29.1	61.9	263.1	80.7	87.9	69.5	57.6	24.4	17.3	23.6	11.8	0.0	726.9
1961	42.4	105.3	226.1	56.4	155.9	149.3	42.5	30.7	17.6	9.7	10.9	26.8	873.6
1962	130.1	273.0	334.0	210.4	231.1	56.1	59.9	93.4	71.6	53.2	0.3	41.0	1604.1
1963	201.5	42.9	279.0	51.0	63.8	95.4	75.4	66.4	61.5	42.2	27.2	39.5	1051.8
1964	56.6	173.4	157.3	135.9	136.1	62.5	91.9	47.5	24.2	5.9	0.4	0.0	922.2
1965	28.4	297.9	185.3	331.3	93.8	99.3	15.9	31.6	39.2	0.0	1.8	77.2	1201.3
1966	46.9	269.7	188.3	152.9	23.1	26.1	61.8	32.1	12.4	62.1	3.9	85.3	971.2
1967	61.4	41.0	258.4	334.2	71.7	64.6	9.6	56.1	53.2	0.8	16.8	43.8	1011.6
1968	88.6	88.9	298.3	140.1	156.3	100.0	24.0	12.6	11.8	13.9	3.9	9.7	948.6
1969	6.8	108.5	336.3	164.6	145.4	99.8	34.4	24.9	18.2	0.5	22.4	79.0	1041.1
1970	89.2	142.9	123.0	128.9	179.9	235.4	32.1	14.1	2.5	10.3	9.8	62.7	1030.8
1971	55.3	185.9	125.3	146.3	122.0	47.2	88.1	47.1	18.9	65.8	25.4	18.9	946.2
1972	209.0	70.4	12.0	135.3	206.9	116.4	62.6	59.8	40.6	25.2	8.0	15.6	961.8
1973	100.1	149.3	171.0	42.9	205.1	51.3	128.6	81.0	15.4	10.6	3.8	69.8	1028.9
1974	157.8	162.0	67.7	33.7	104.6	119.8	17.5	56.2	65.7	2.4	46.0	0.0	833.4

Castalia: Parameter estimation-Parameters of autocorrelation and persistence



Parameter adjustment

Parameter optimization criterion

● Best fit to sample

○ Preservation of r1 coefficient

○ Preservation of r1 and r2 coefficients

☑ Beta parameter value specification

Auto-correlogram parameters

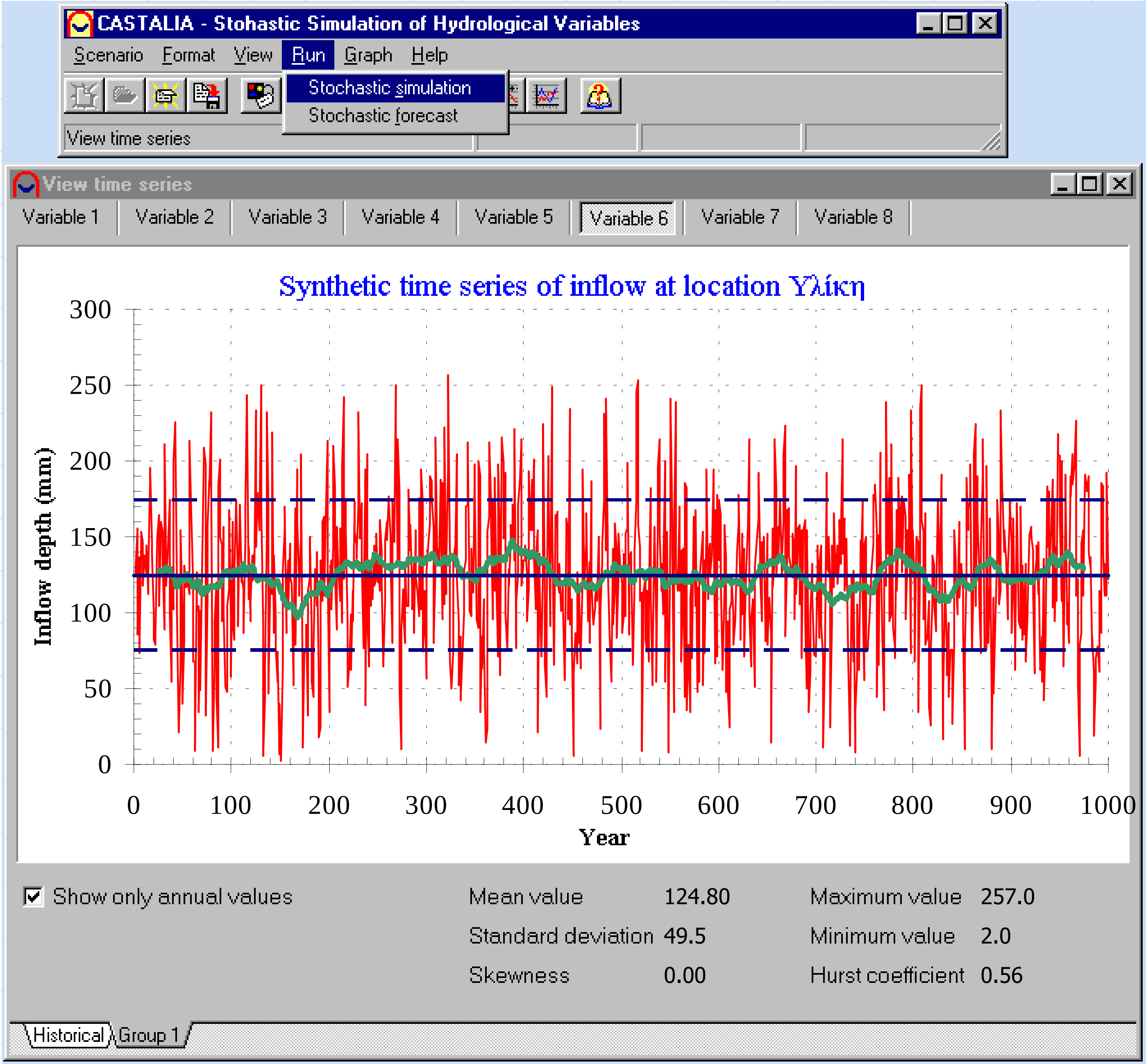
Preserved sample terms: 1

Persistent parameter, beta: 2.000

Shape parameter, kappa: 5.236

WARNING: The kappa parameter is always automatically adjusted, whereas beta is either adjusted or specified by the user.

Castalia: Stochastic simulation without long term persistence



☑ Show only annual values

Mean value 124.80

Standard deviation 49.5

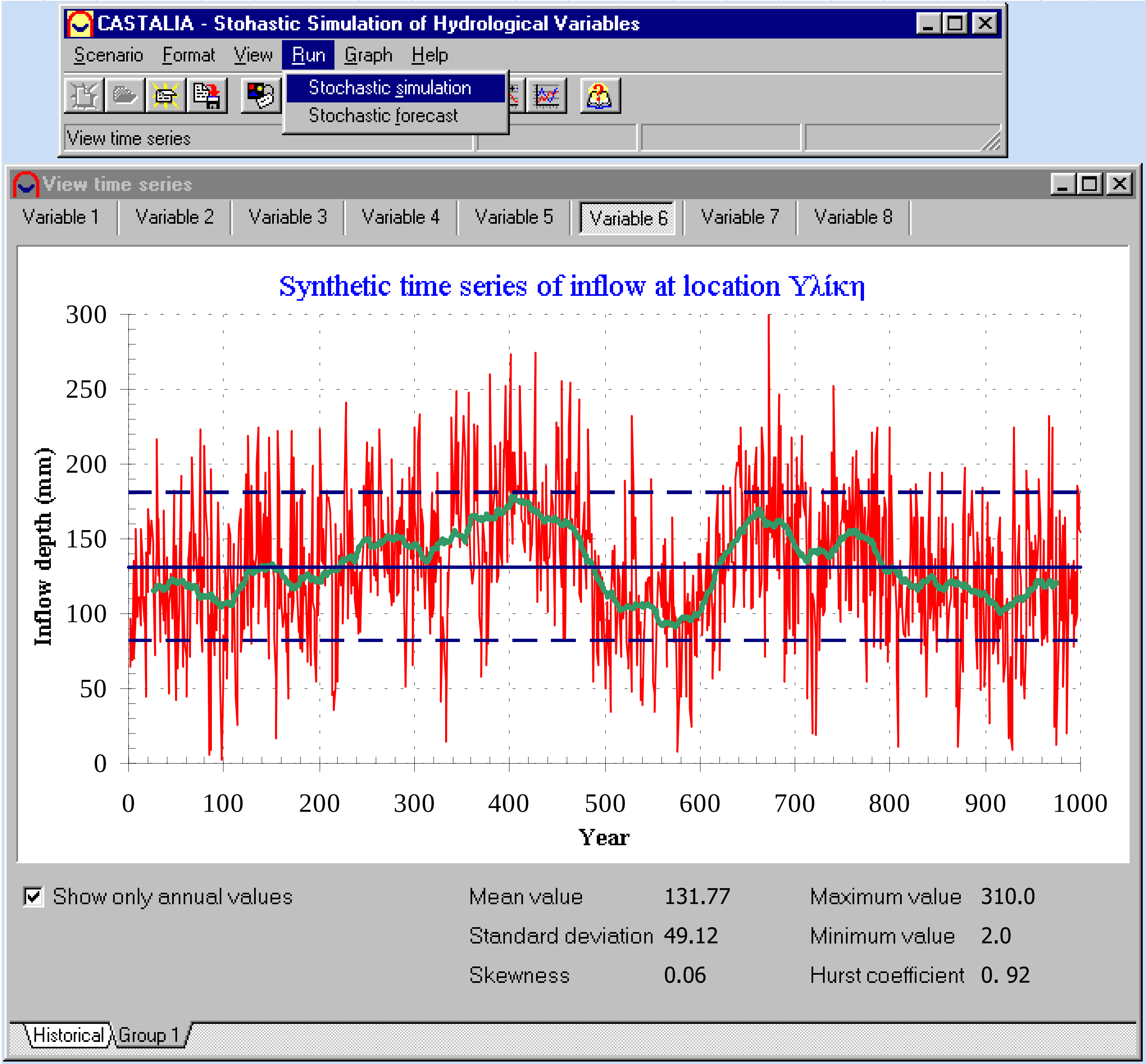
Skewness 0.00

Maximum value 257.0

Minimum value 2.0

Hurst coefficient 0.56

Castalia: Stochastic simulation with long term persistence



☑ Show only annual values

Mean value 131.77

Standard deviation 49.12

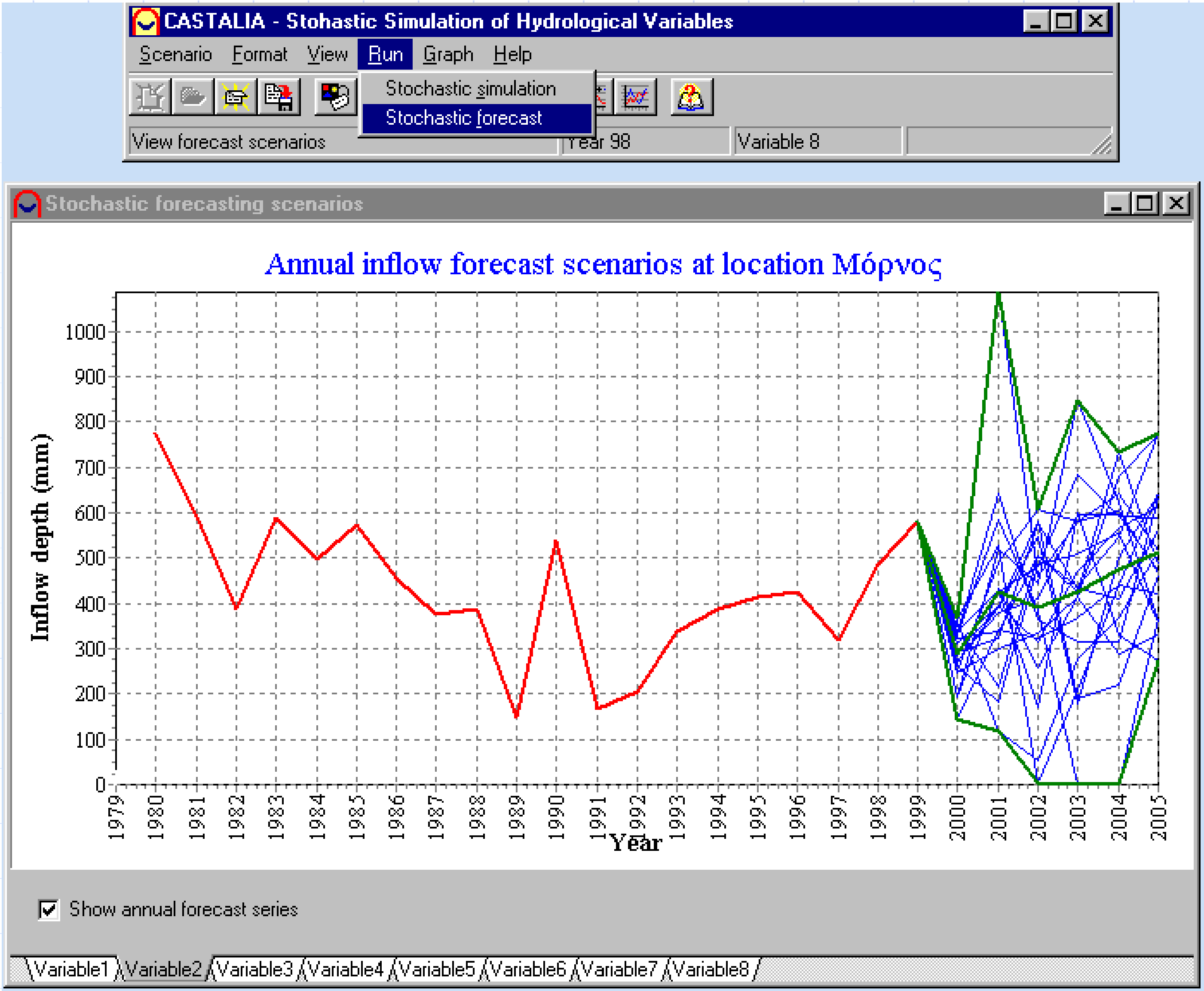
Skewness 0.06

Maximum value 310.0

Minimum value 2.0

Hurst coefficient 0. 92

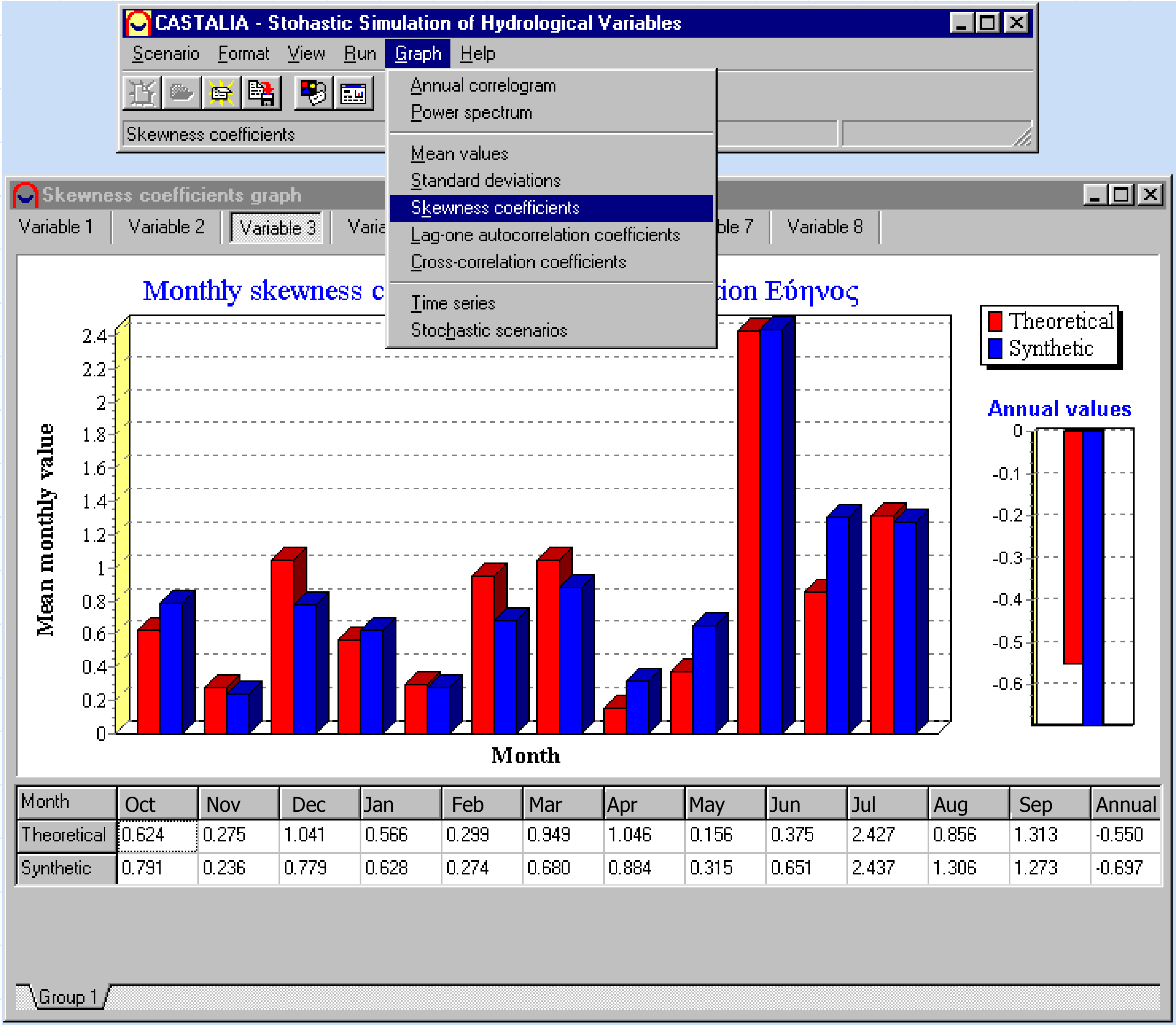
Castalia: Stochastic forecasting with long term persistence



View forecast scenarios | Year 98 | Variable 8

☑ Show annual forecast series

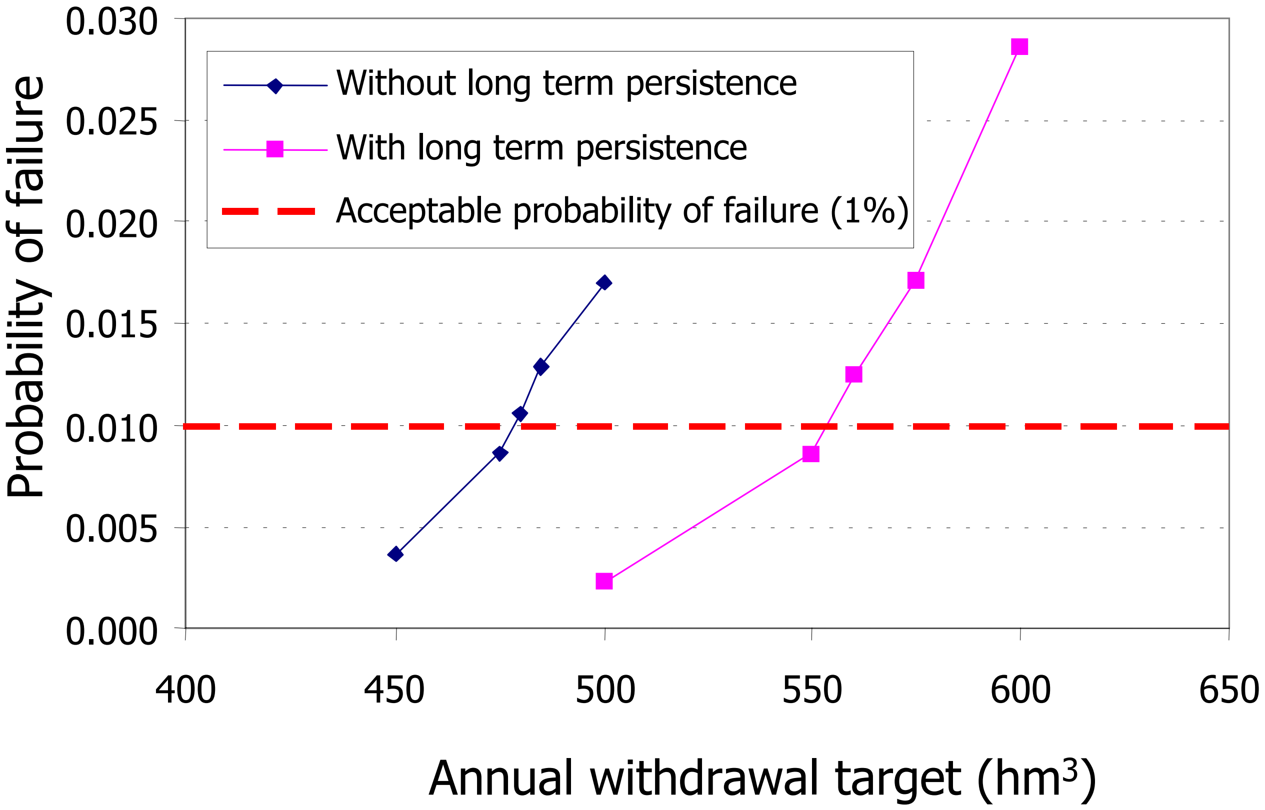
Castalia: Preservation of marginal statistics – Skewness



Month	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Annual
Theoretical	0.624	0.275	1.041	0.566	0.299	0.949	1.046	0.156	0.375	2.427	0.856	1.313	-0.550
Synthetic	0.791	0.236	0.779	0.628	0.274	0.680	0.884	0.315	0.651	2.437	1.306	1.273	-0.697

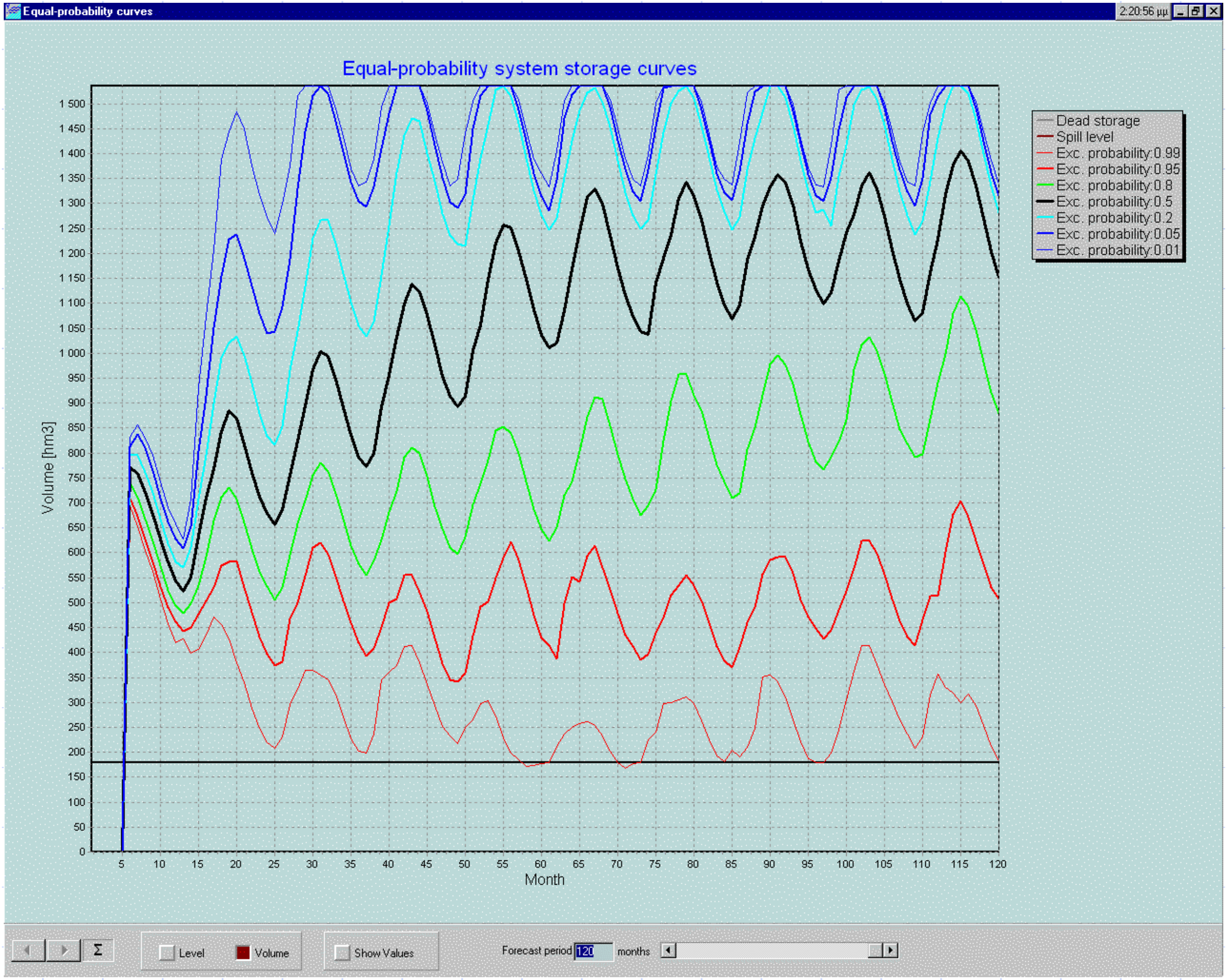
Utilisation of Castalia’s results in the hydrosystem of the Athens water supply: System’s firm yield

Results of steady-state simulations for 2000 years with and without long-term persistence



Utilisation of Castalia's results in the hydrosystem of the Athens water supply: Stochastic forecast of system storage

Evolution of quantiles of system storage (for several levels of probability of exceedance) for the next 10 years as a result of 200 terminating simulations with long-term persistence



Summary

- A generalised stochastic modelling framework for hydrological variables has been developed.
- The methodology involves the combination of novel stochastic techniques, and preserves long-term persistence and asymmetric distributions in multivariate, sequential or disaggregation, problems.
- The methodology has been implemented in the **Castalia** program.
- The methodology and the program have been tested in a large hydrosystem involving 4 hydrologic catchments with 4 reservoirs.