A stochastic hydrology framework for the management of multiple reservoir systems

Motivation: The management of the hydrosystem for the water supply of Athens
Requirements for stochastic simulation

1. Multivariate model
2. Time scales from annual to monthly or sub-monthly
3. Preservation of essential marginal statistics up to third order (skewness)
4. Preservation of joint second order statistics (auto- and cross-correlations)
5. Capturing/reproduction of “patterns” observed in the last severe drought – Preservation of long-term persistence

Climatic persistence versus climatic variability

Annual minimum water level of the Nile river for the years 622 to 1284 A.D. (663 years)
Hurst exponent = 0.85

Standardised tree ring widths from a paleoclimatological study at Mammoth Creek, Utah, for the years 0-1989 (1990 years)
Hurst exponent = 0.75
Methodology 1: The generalised autocovariance function (GAS)

General expression
\[ Y_j = Y_0 (1 + \kappa \beta j)^{-1/\beta} \]
where
- \( Y_j \): autocovariance for lag \( j \)
- \( Y_0 \): variance
- \( \kappa, \beta \): parameters
(The two parameters allow for preservation of \( y_1 \) and Hurst exponent)
For \( \beta = 0 \Rightarrow \text{ARMA} \)
\[ Y_j = Y_0 \exp(-\kappa j) \]
For \( \kappa = (1/\beta) (1 - 1/\beta)^{-\beta} \)
\( (1 - 1/2\beta)^{-\beta} \Rightarrow \text{FGN} \)


Methodology 2: Generalised generating scheme for any covariance structure

Typical (backward) moving average (BMA) scheme
\[ X_i = \ldots + a_1 V_{i-1} + a_0 V_i \]
where \( V_i \) innovations and \( a_i \) parameters.
Symmetric moving average (SMA) scheme
\[ X_i = \ldots + a_1 V_{i-1} + a_0 V_i + a_1 V_{i+1} + \ldots \]
SMA has several advantages over BMA. Among them, it allows a closed solution for \( a_i \):
\[ s_a(\omega) = [2 s_y(\omega)]^{1/2} \]
where \( s_a(\omega) \) and \( s_y(\omega) \) the DFTs of the series \( a_j \) and \( y_j \) respectively.
Both schemes are applicable for multivariate problems.

Methodology 3: Stochastic simulation in forecast mode

- In terminating simulations of a hydrosystem the present and past states must be considered.
- The observed values of the present and past must condition the hydrologic time series of the future.
- This is attainable using a two-step algorithm
  1. Generate future time series without reference to the known present and past values.
  2. Adjust future time series using the known present and past values and a linear adjusting algorithm.

The linear adjusting algorithm:
1. is expressed in terms of covariances among variables;
2. preserves exactly means, variances and covariances;
3. is easily implemented.


Methodology 4: Coupling stochastic models of different time scales

The linear transformation
\[ X_s = \tilde{X}_s + h (Z_p - \tilde{Z}_p) \]
where
\[ h = \text{Cov}[X_s, Z_p] \cdot \{\text{Cov}[Z_p, Z_p]\}^{-1} \]
preserves the vectors of means, the variance-covariance matrix and any linear relationship that holds among \( X_s \) and \( Z_p \).

Methodology 5: Preservation of skewness in multivariate problems via appropriate decomposition of covariance matrices

Consider any linear multivariate stochastic model of the form

\[ Y = aZ + bV \]

where \( Y \): vector of variables to be generated, \( Z \): vector of variables with known values, \( V \): vector of innovations, and \( a \) and \( b \): matrices of parameters.

The parameter matrix \( b \) is related to a covariance matrix \( c \) by

\[ bb^T = c \]

This equation may have infinite solutions or no solution.

The skewness coefficients \( \xi \) of innovations \( V \) depend on \( b \).

The smaller the values of \( \xi \), the more attainable the preservation of the skewness coefficients of the actual variables \( Y \).

Therefore, the problem of determination of \( b \) can be solved in an optimisation framework, that combines

- minimisation of skewness \( \xi \), and
- minimisation of the error \( \| bb^T - c \| \).

A fast optimisation algorithm has been developed for this problem.


Implementation of the methodology:
The Castalia software

- Designed as part of a decision support system for the water resource system of Athens
- Linked to a simulation-optimisation model of a hydrosystem
- Can also perform as a stand-alone software
- Written in Delphi; utilises Oracle.
- Simulates several hydrological variables at multiple sites
- Uses annual and monthly time scales
- Preserves:
  - essential marginal statistics up to third order (skewness)
  - joint second order statistics (auto- and cross-correlations)
  - long-term persistence
### Parameter estimation - Parameters of autocorrelation and persistence

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Castalia: Stochastic simulation without long term persistence

Castalia: Stochastic simulation with long term persistence

Koutsoyiannis & Efstratiadis, A stochastic hydrology framework for the management of multiple reservoir systems 13

Koutsoyiannis & Efstratiadis, A stochastic hydrology framework for the management of multiple reservoir systems 14
Castalia:
Stochastic forecasting with long term persistence

Castalia:
Preservation of marginal statistics – Skewness
Utilisation of Castalia’s results in the hydrosystem of the Athens water supply: System’s firm yield

Results of steady-state simulations for 2000 years with and without long-term persistence

Utilisation of Castalia’s results in the hydrosystem of the Athens water supply: Stochastic forecast of system storage

Evolution of quantiles of system storage (for several levels of probability of exceedance) for the next 10 years as a result of 200 terminating simulations with long-term persistence
Summary

- A generalised stochastic modelling framework for hydrological variables has been developed.
- The methodology involves the combination of novel stochastic techniques, and preserves long-term persistence and asymmetric distributions in multivariate, sequential or disaggregation, problems.
- The methodology has been implemented in the **Castalia** program.
- The methodology and the program have been tested in a large hydrosystem involving 4 hydrologic catchments with 4 reservoirs.