A new stochastic hydrologic framework inspired by the Athens water resource system

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A seminar given at the …

School of Civil and Environmental Engineering, Georgia Institute of Technology

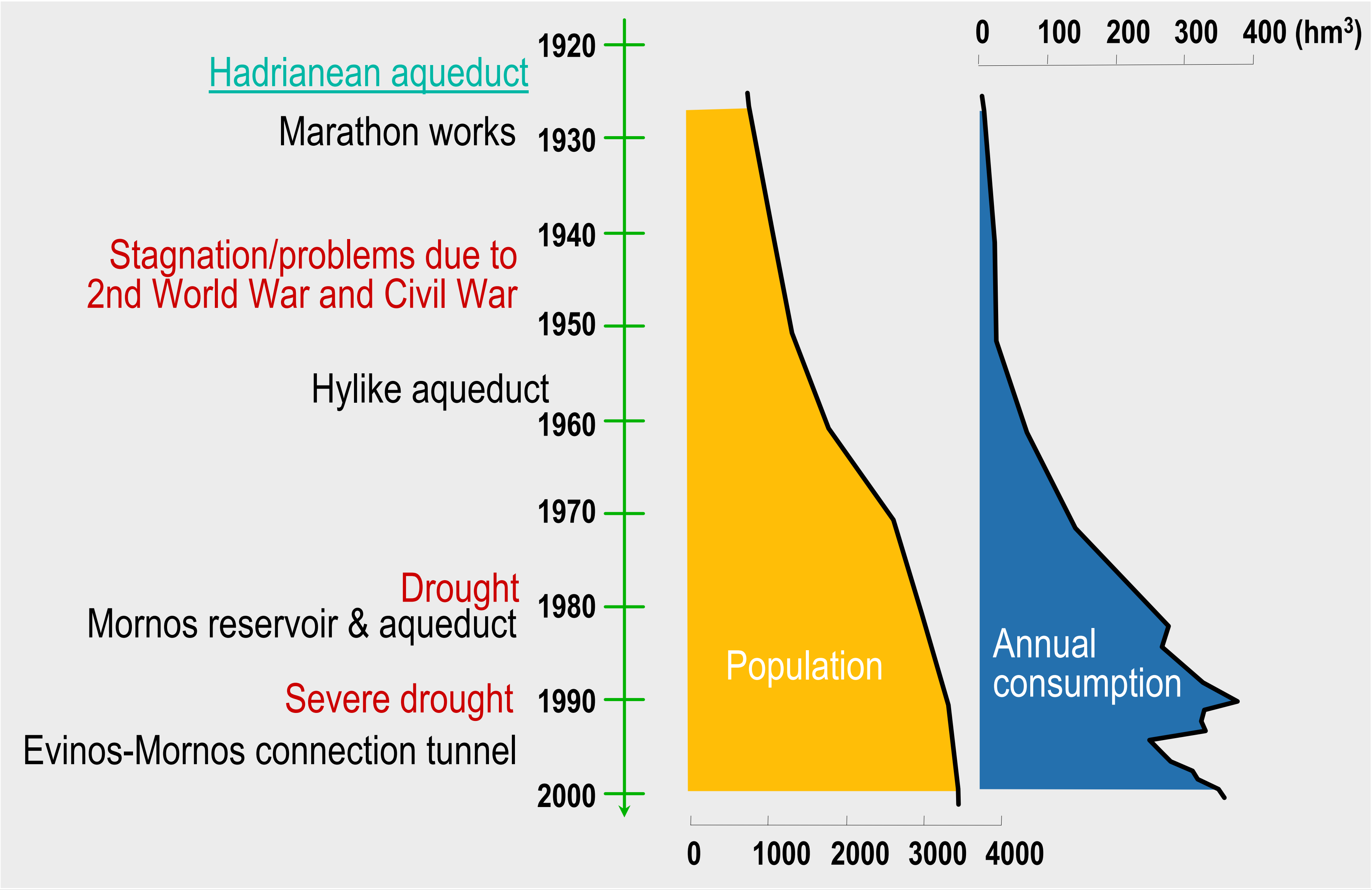
Atlanta, 13 January 2006

… and at the …

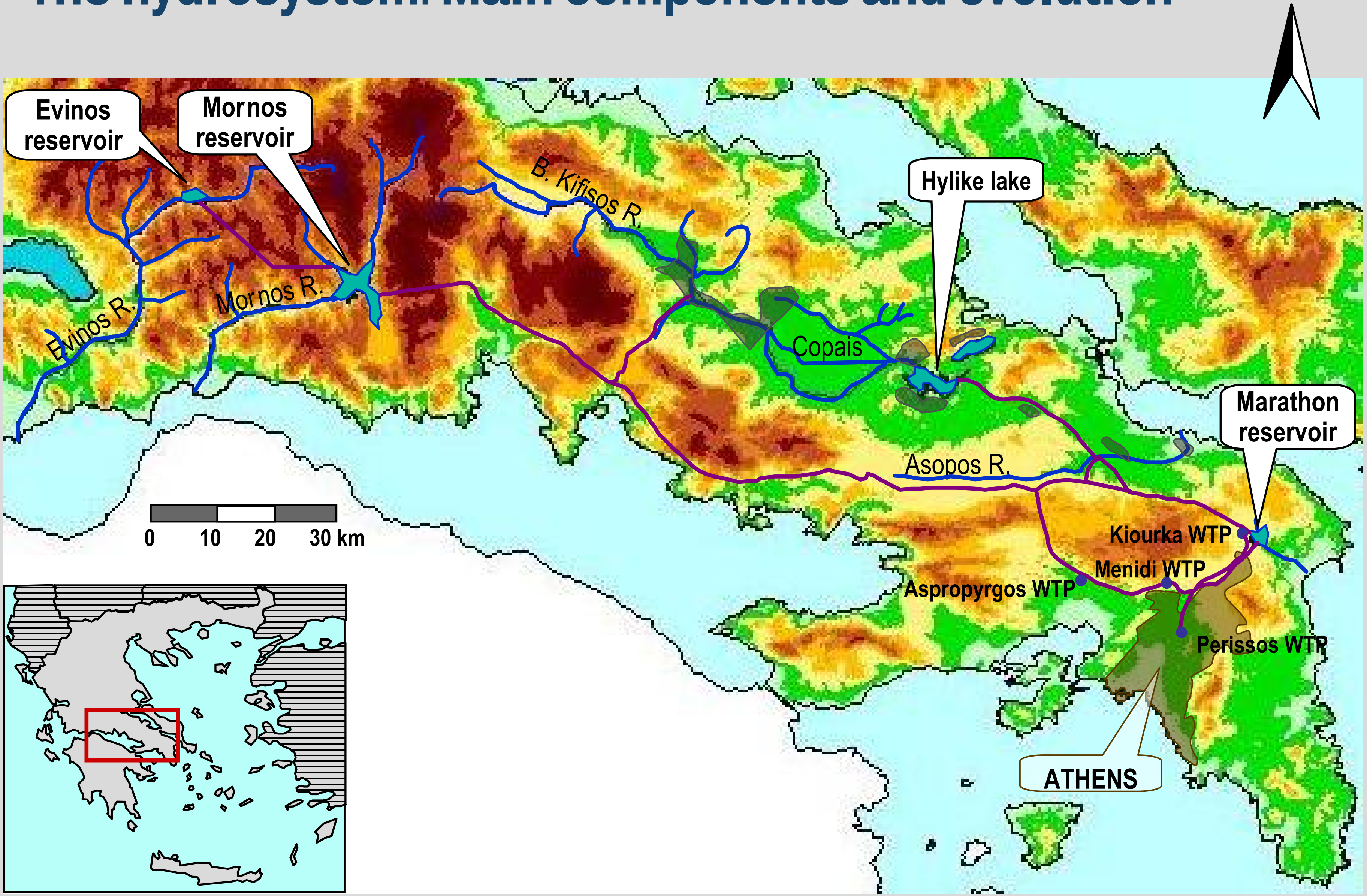
School of Engineering, Duke University

Durham, 16 January 2006

Evolution of water consumption – Milestones



The hydrosystem: Main components and evolution



Parts of the presentation

1. Diagnosis

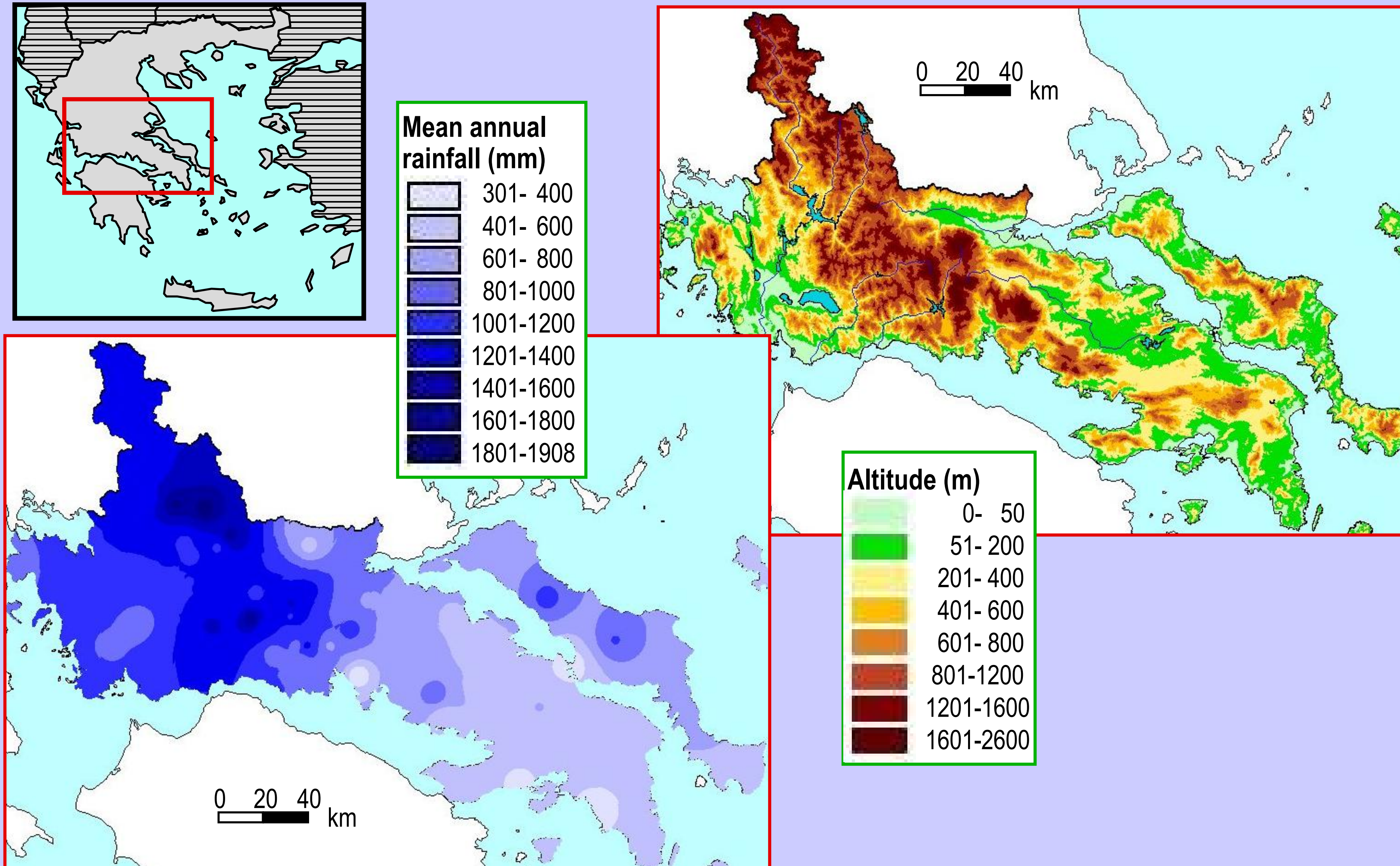
2. Explanation

3. Operational synthesis

1. Diagnosis

Ὕσον, ὕσον Ζεῦ κατὰ τῆς ἀρούρης τῶν Ἀθηναίων

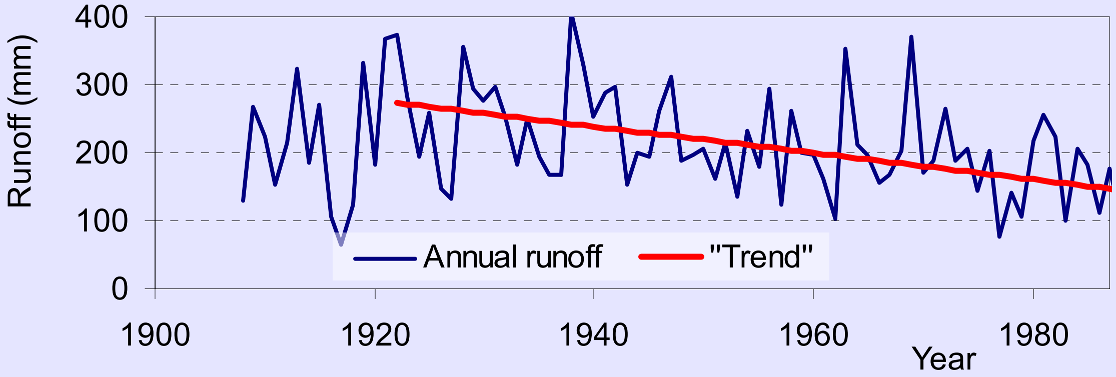
Do rain, do rain Zeus against the earth of Athenians (Ancient Greek prayer)



Back in 1990s – Initial empirical observations

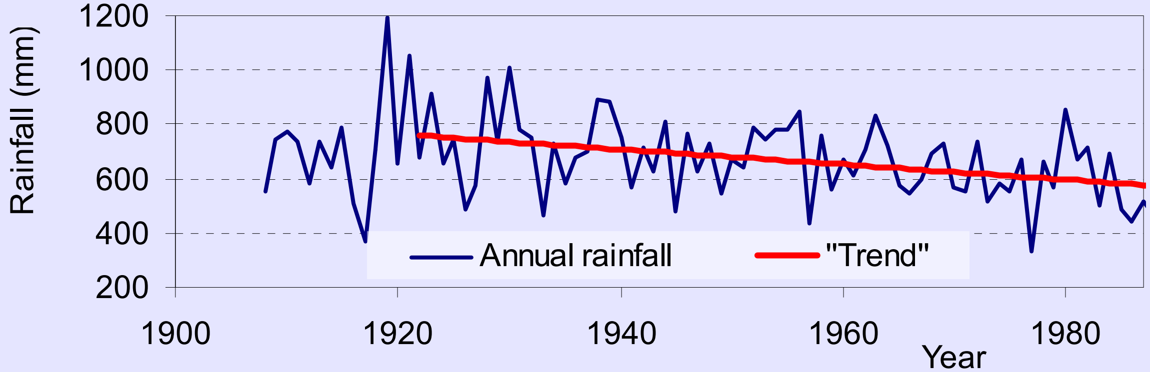
The historic time series of Boeoticos Kephisos runoff (Hydrologic years 1907/08-1986/87)

A multi-year «trend» is observed



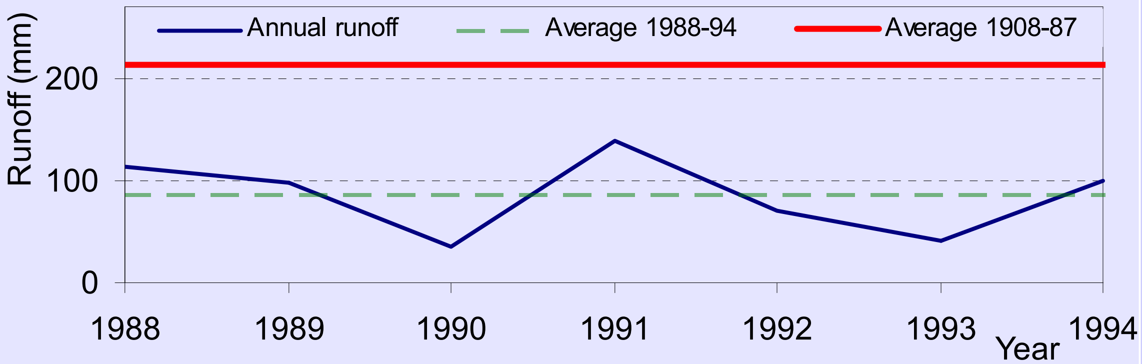
A similar «trend» in the rainfall time series

Explains the «trend» in runoff



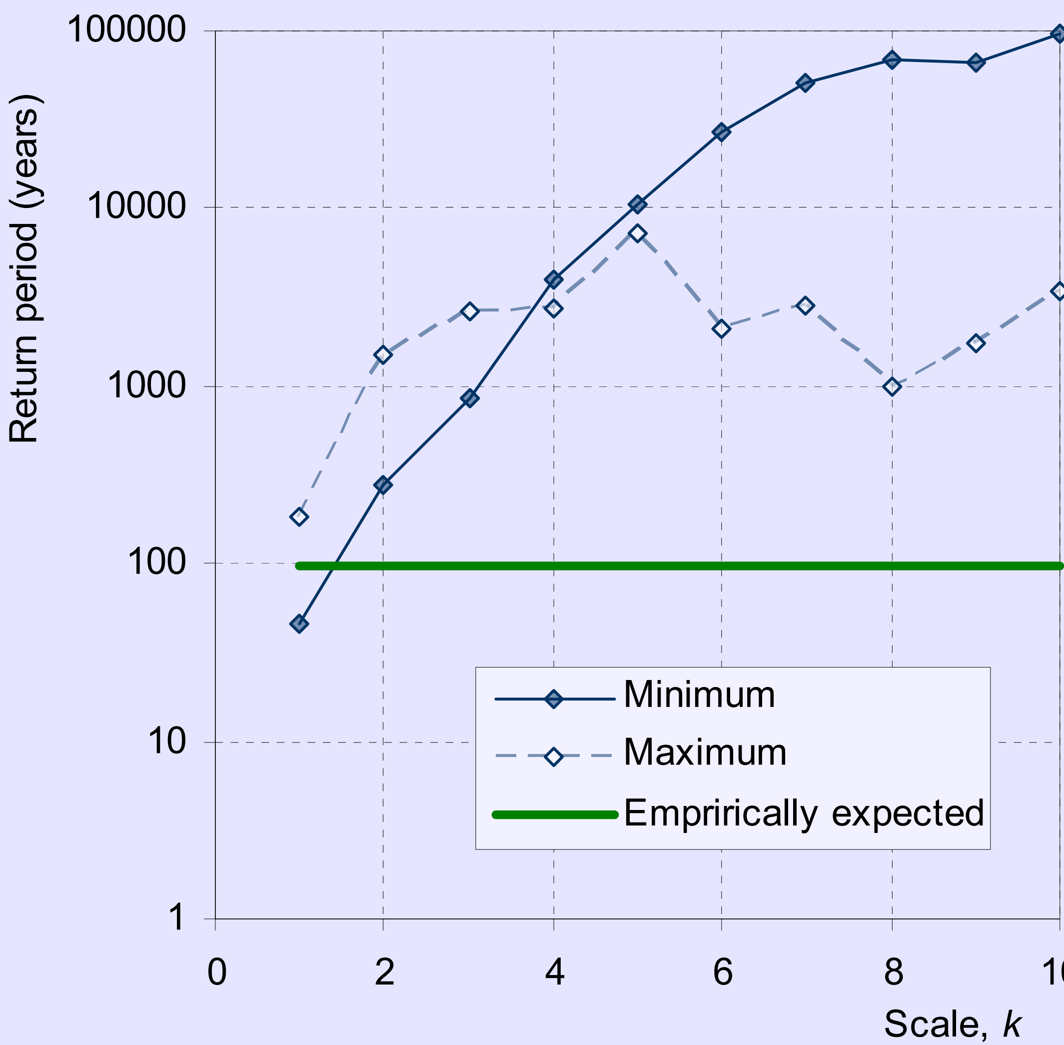
The next years were dry

Intense and persistent drought: Mean flow half that of the historic average, duration 7 years



Return period of the persistent drought

- Assessment was done using classic hydrologic statistics
- At the annual scale, the drought was a record minimum but with typical magnitude
- Aggregated at larger scales, it appeared something extraordinary
- Similar behavior was observed for maxima on aggregate scales



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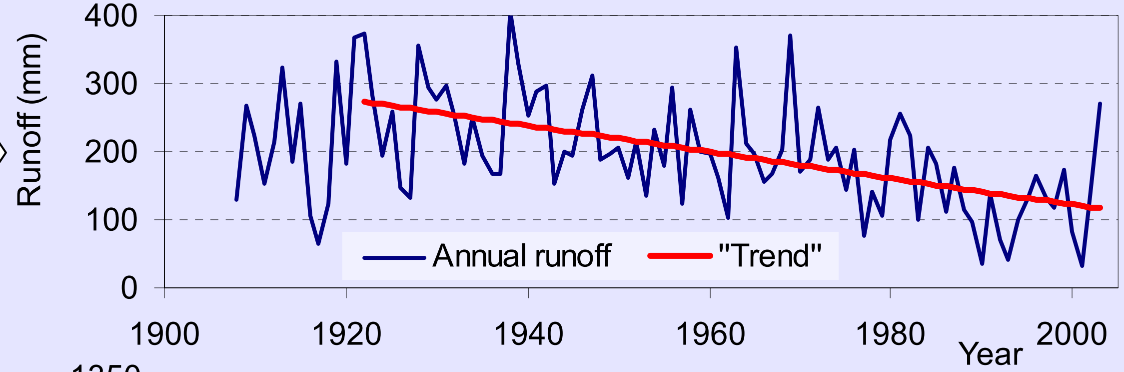
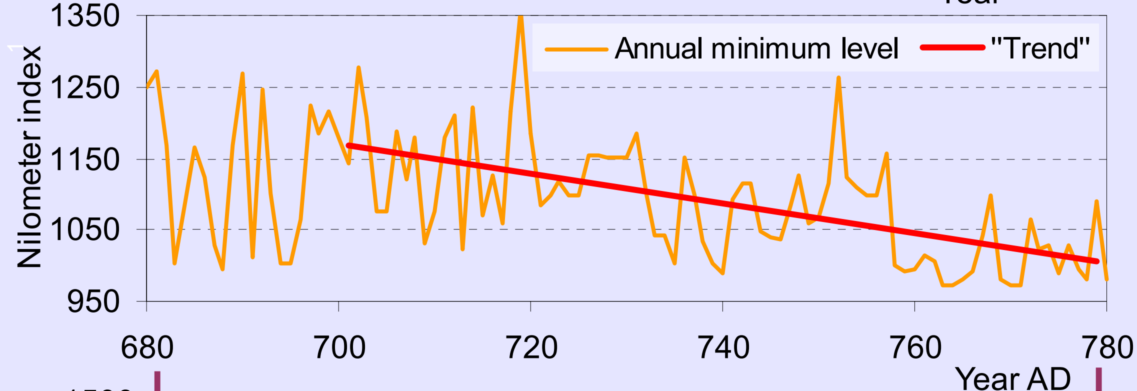
Comparisons with even longer series

The complete historic time series of Boeoticos Kephisos runoff



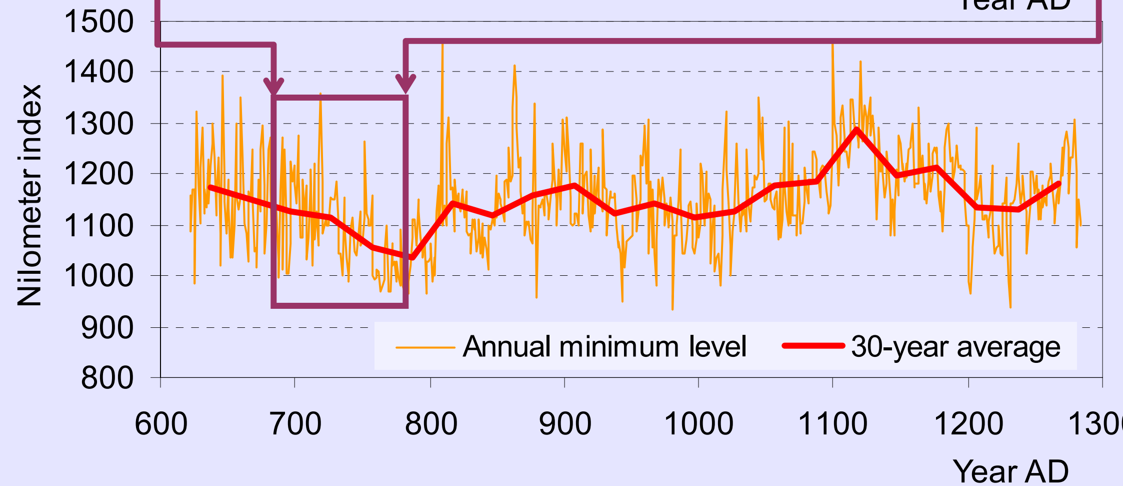
A part of the Nilometer series (the minimum annual water level in the Nile River, in cm)

A similar «trend»



The complete Nilometer series (622-1284 AD, 663 years)

Upward and downward fluctuations on all scales



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The fluctuations on many scales and the “Hurst phenomenon”

- The “weird” (as compared to purely random processes) behavior of hydrologic and other geophysical processes was discovered by the English engineer E. H. Hurst (1950) in the framework of the design of the High Aswan Dam in Nile $\Rightarrow$ **Hurst phenomenon**
- The Polish-French mathematician and engineer B. Mandelbrot (1965-1971) related it to the biblical story of the seven fat and the seven thin cows $\Rightarrow$ **Joseph effect**
- The behavior has been characterized with several other names $\Rightarrow$ **long-term persistence, long-term memory, long-range dependence, scaling behavior**
- Most of these names, even though correct, may be misleading for the conceptualization and understanding of the natural behavior and the causing mechanisms. Probably a better name $\Rightarrow$ **multi-scale fluctuation**
- The behavior was verified to be omnipresent, not only in geophysical processes (hydrologic, climatic), but also in biological (e.g. tree rings), technological (e.g. computer networks), social and economical (e.g. stock market)
- In water resources design and management, it has unfavorable effects (increase of uncertainty)

Easy detection and main effect of Hurst phenomenon

- Fundamental law of classic statistics

$$\mathrm{StD}[\bar{X}_n] = \frac{\sigma}{\sqrt{n}}$$

$\bar{X}_n$ = average of n variables

σ = standard deviation of each variable

n = aggregation scale (or sample size)

- Modified law in natural processes

$$\mathrm{StD}[\bar{X}_n] = \frac{\sigma}{n^{1-H}}, \quad H > 0.5$$

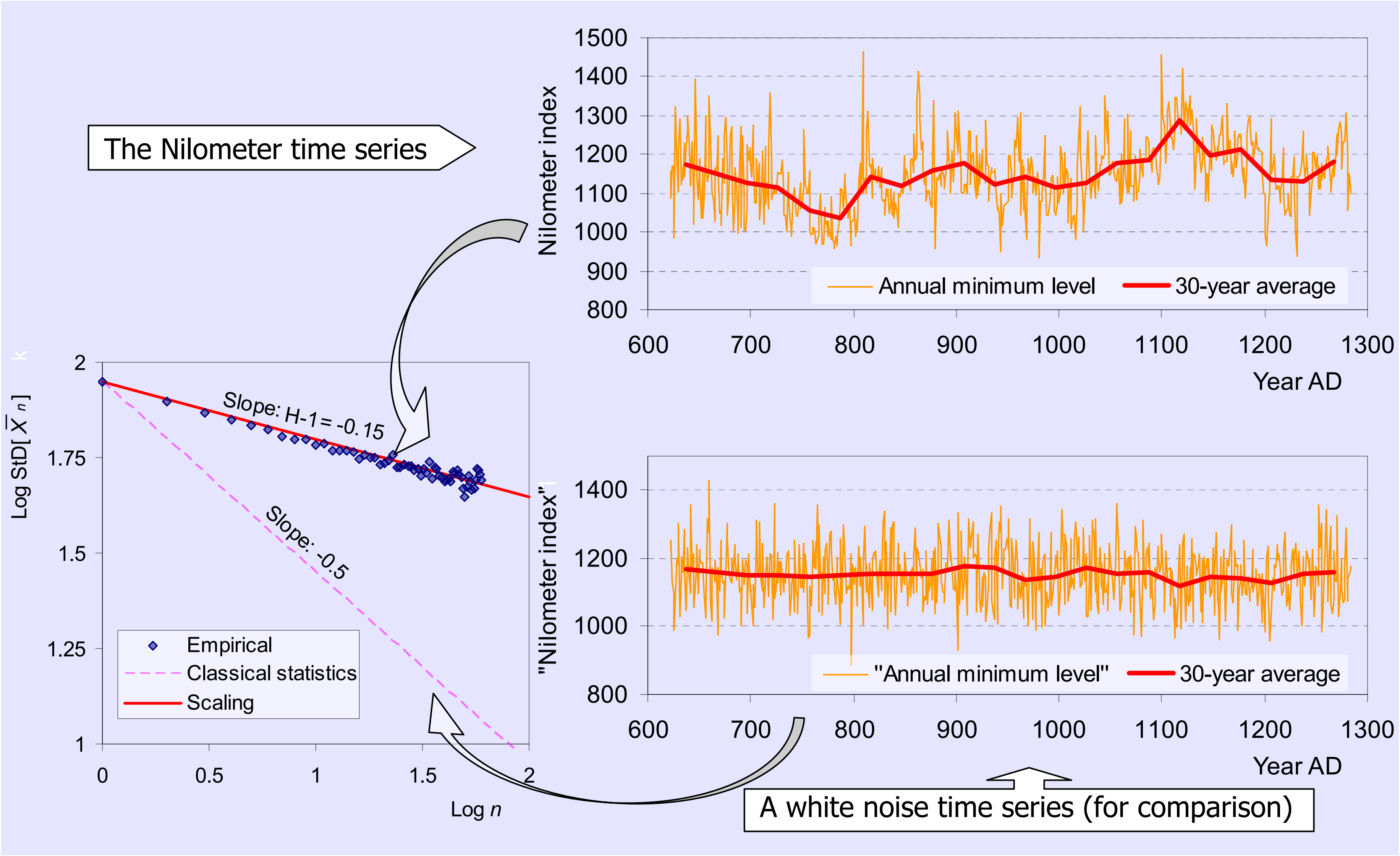
- Example

  To have $\mathrm{StD}[\bar{X}_n] / \sigma = 10\%$

  - n = 30 in classic statistics
  - n = 5 000 for the modified law with H = 0.8

Bomb in the foundation of climatology

Incongruity of natural processes with typical random processes : (a) The Nilometer series



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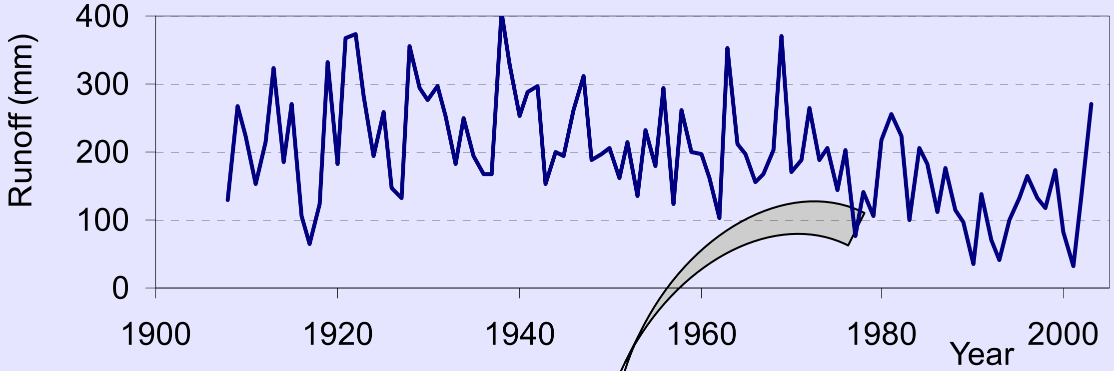
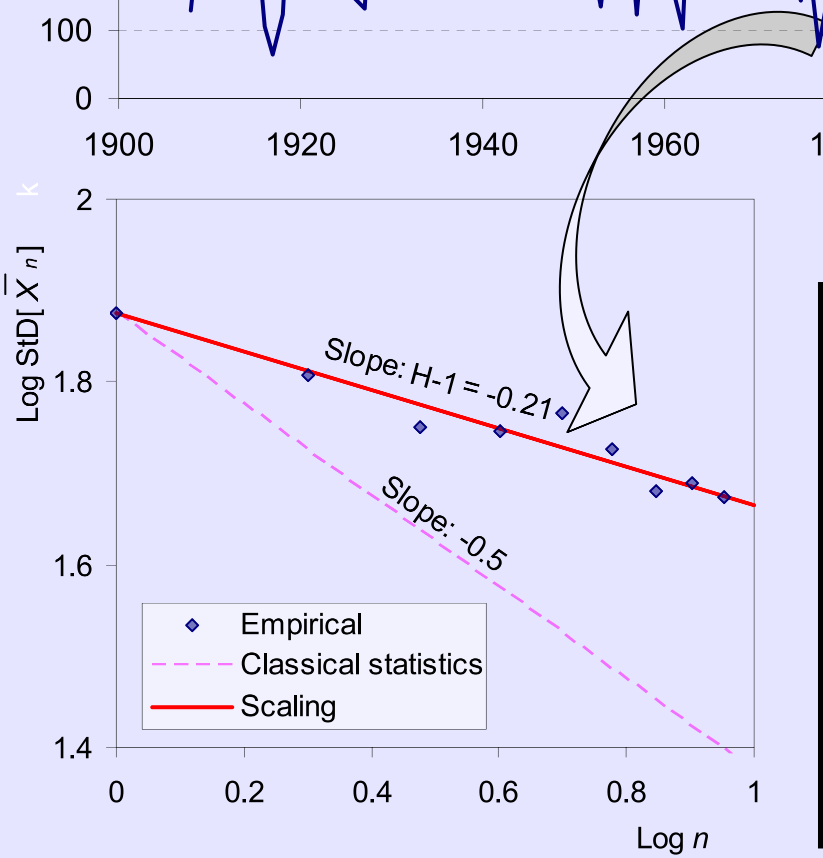
Incongruity of natural processes with typical random processes : (b) The Boeoticos Kephisos time series



The time series of the Boeoticos Kephisos runoff



Statistical characteristics of all processes			
Sample statistic	Runoff (mm)	Rainfall (mm)	Temperature (°C)
n	96	96	96
m (mm)	197.6	658.4	17.0
s (mm)	87.6	158.9	0.72
C_s	0.36	0.44	0.34
r_1	0.34	0.10	0.31
H	0.79	0.64	0.72

{2,14}

Mathematical description of the Hurst phenomenon

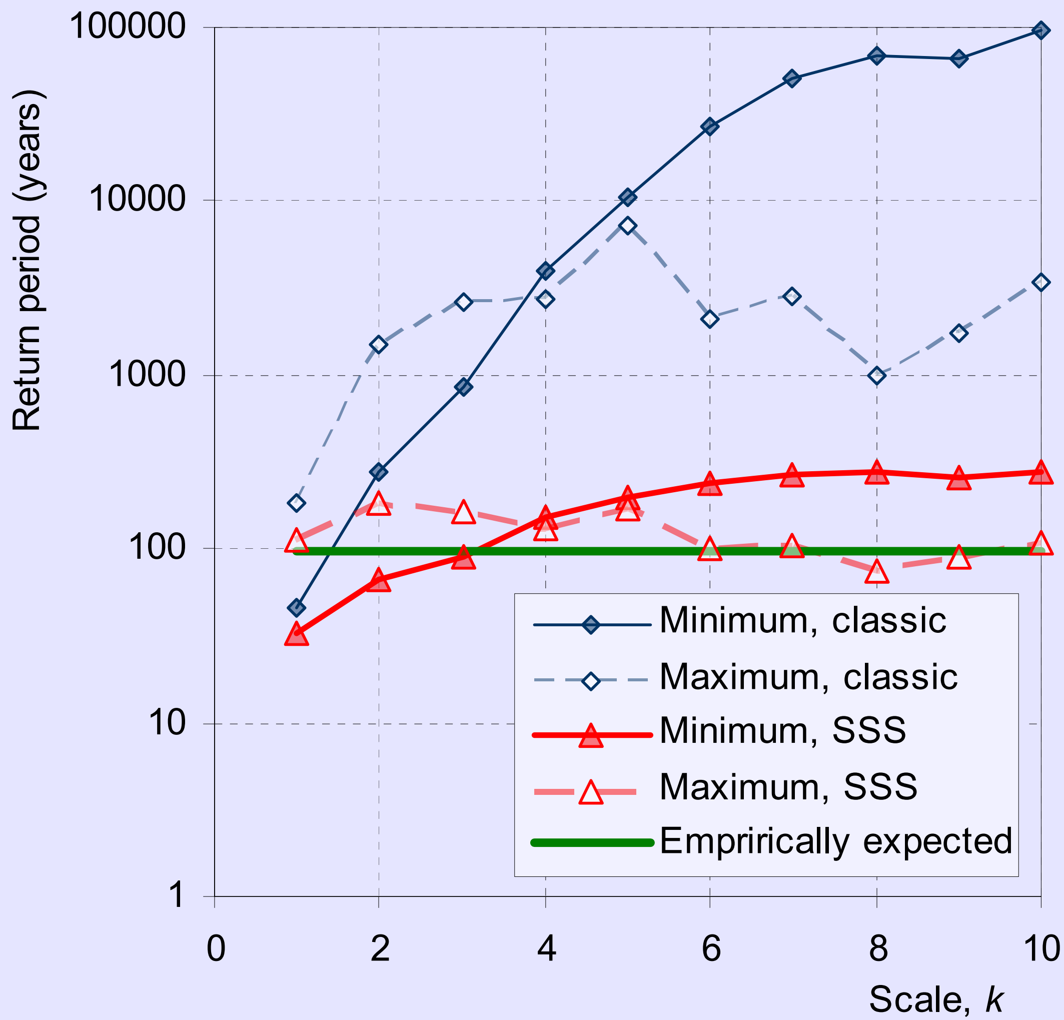
- The mathematical description of the Hurst phenomenon is done on grounds of the probability theory and particularly the theory of stochastic process
- The simple relationship

$$\mathrm{StD}[\overline{X}_n] = \frac{\sigma}{n^{1-H}}$$

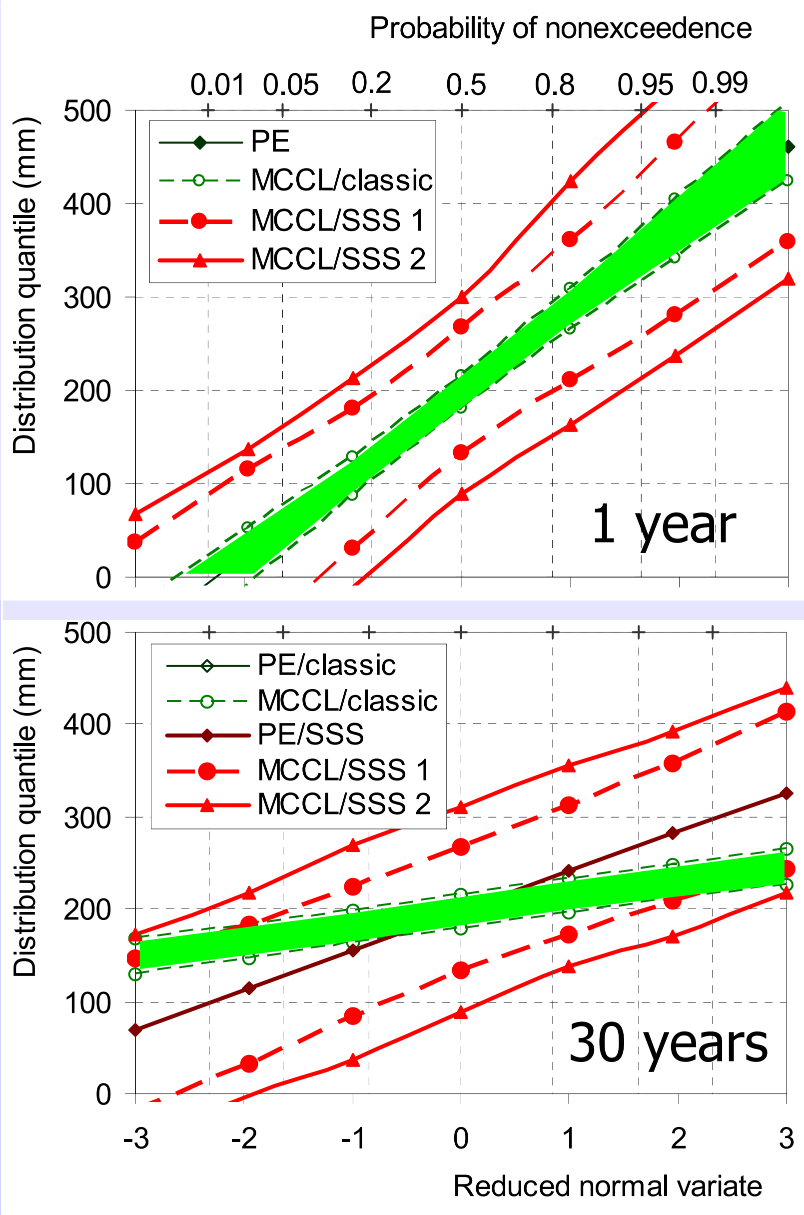
  entails a definition (good for our purposes) of a model (stochastic process) reproducing the Hurst phenomenon; *n* is meant as a scale of aggregation (rather than sample size)
- (Hurst used a different formalism, in terms of the so called rescaled range, which is complicated and probably misleading)
- Today the stochastic process with the above property is called a **S**elf-**S**imilar process with **S**tationary intervals or a **S**imple **S**caling **S**tochastic process (abbreviated as an **SSS** process)
- The SSS process was introduced by the Russian mathematician A. Kolmogorov (1940) who called it Wiener Spiral
- A significant contribution on the SSS process is due to the American mathematician J. Lamperti (1962) who called it a Semi-Stable Process
- The link of the SSS process with the Hurst phenomenon is due to B. Mandelbrot (1965), who called it Fractional Gaussian Noise

Back to Boeoticos Kephisos – Adoption of the SSS process

- The trend is a natural and usual behavior
- The persistent drought is not extraordinary; it is a natural and expected behavior



Implications on uncertainty: Boeoticos Kephisos runoff



Statistical model	Total uncertainty in runoff (due to variability and parameter estimation) % of average	
	Annual scale	30-year scale
Classic	200	50
SSS	270	200

Classic model
Climate is what you expect
Weather is what you get

SSS model
Weather is what you get … immediately
Climate is what you get
… if you keep expecting a long time

{2}

2. Explanation

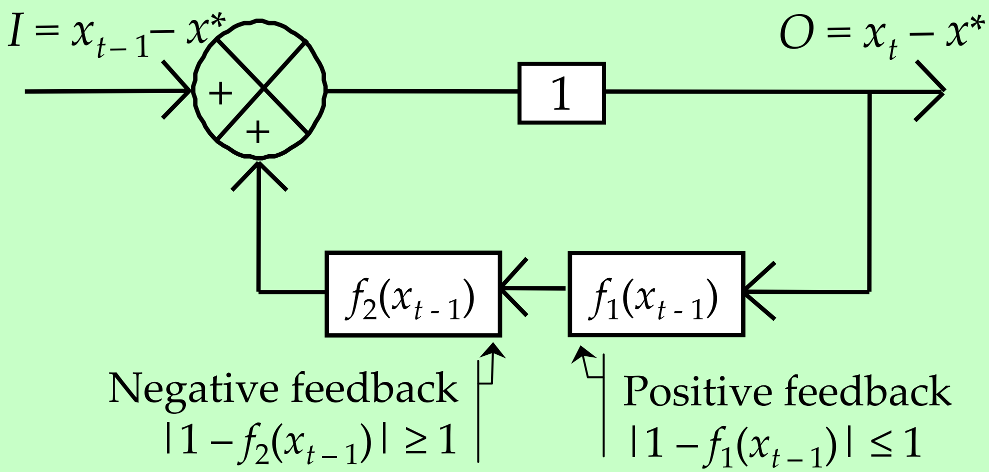
A climatic toy model: A simple system with nonlinear dynamics may produce the Hurst phenomenon

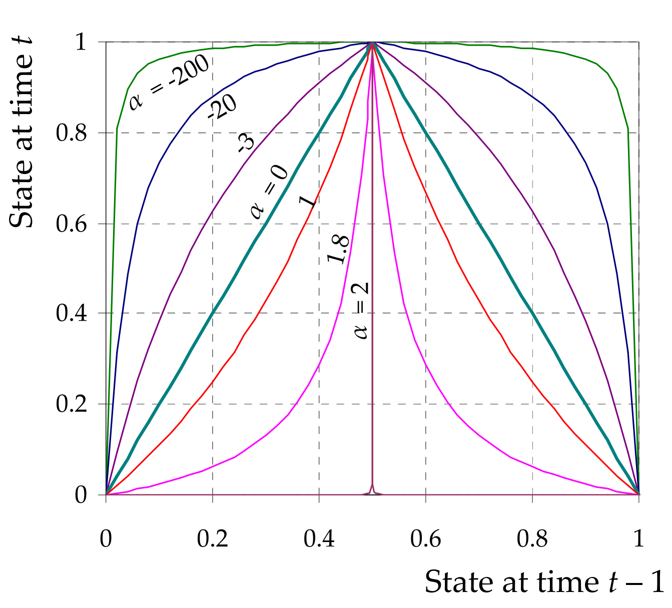
- A simplified climatic system is represented as a circuit with two feedback mechanisms, a positive (amplifying the departure from a stationary state x^*) and a negative (reducing this departure)
- The combined action of the two mechanisms could be represented by a generalized tent transform:

$$x_t = \frac{(2-\alpha)\min(x_{t-1}, 1-x_{t-1})}{1-\alpha \min(x_{t-1}, 1-x_{t-1})}$$

where $0 \le x_t \le 1$, $\alpha < 2$

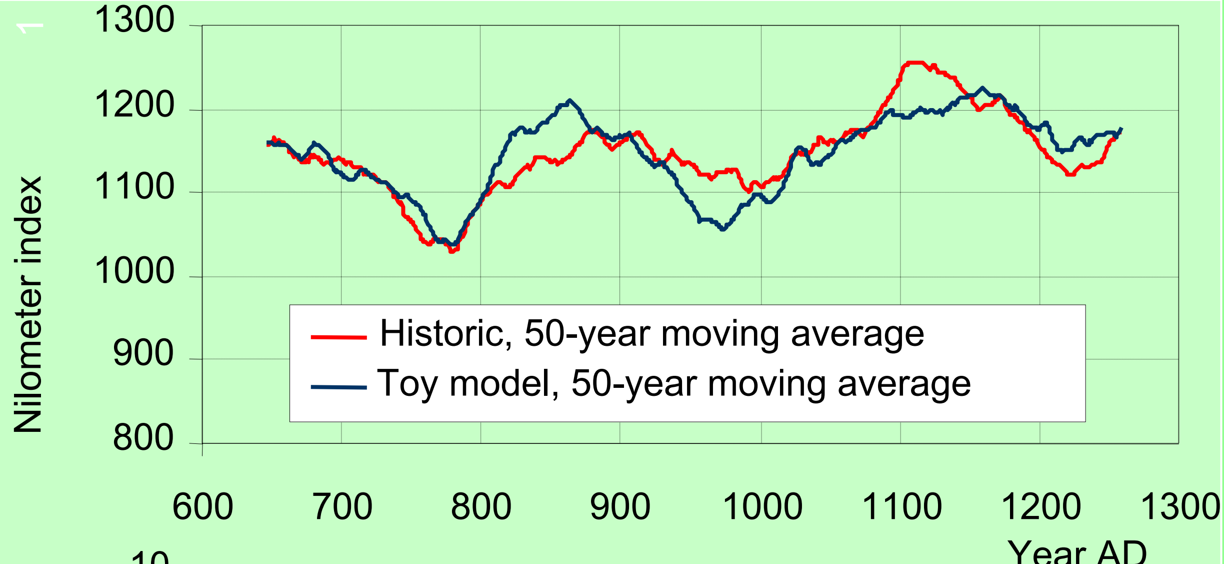
- The parameter α could be assumed to vary in time, following the same tent transform with a constant parameter β



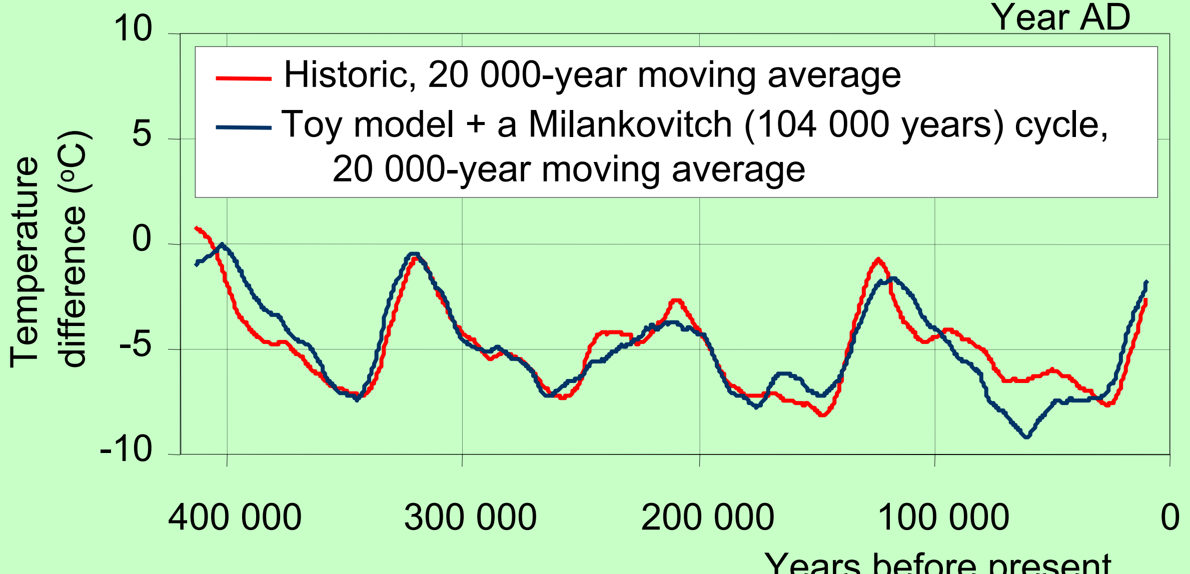


Demonstration: Toy model fitted to two long time series

The Nilometer time series



The Vostok (Antarctica) ice core deuterium data set going back to 422 766 years before present
Reconstructed temperature difference with reference to the mean recent time value



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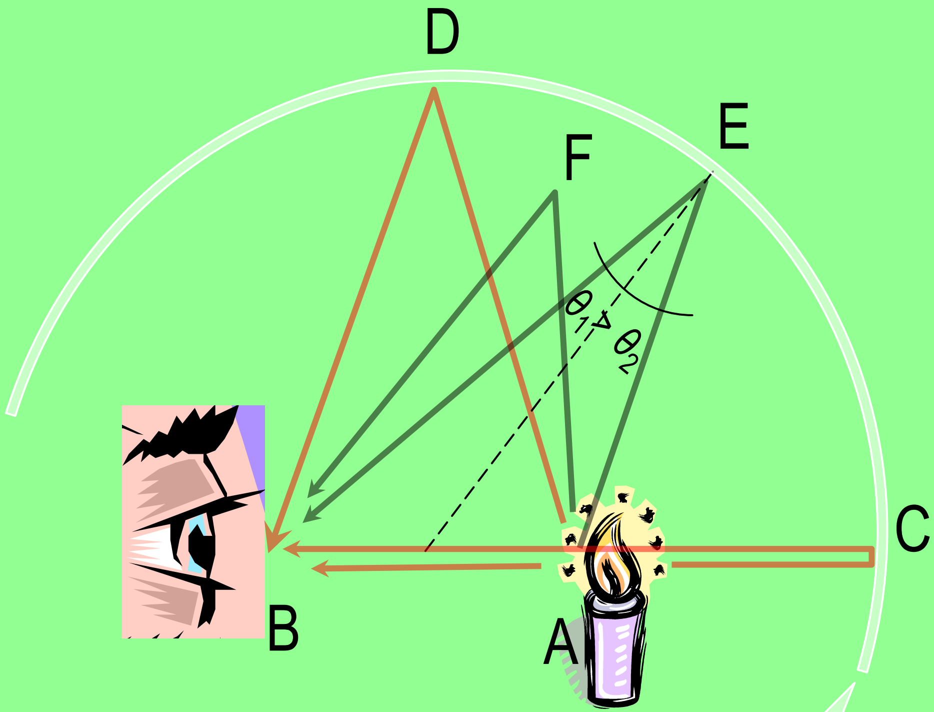
Towards a more general explanation: Nature loves extremes ...



A semi-cylindrical mirror

Why light follows the red paths from A to B (AB, ACB, ADB) and not other (the black) ones (e.g. AEB, AFB)?

- The red paths are those that (a) reach the mirror and (b) form an angle of incidence equal to the angle of reflection

(True for most cases; not true for AB; not general or informative)

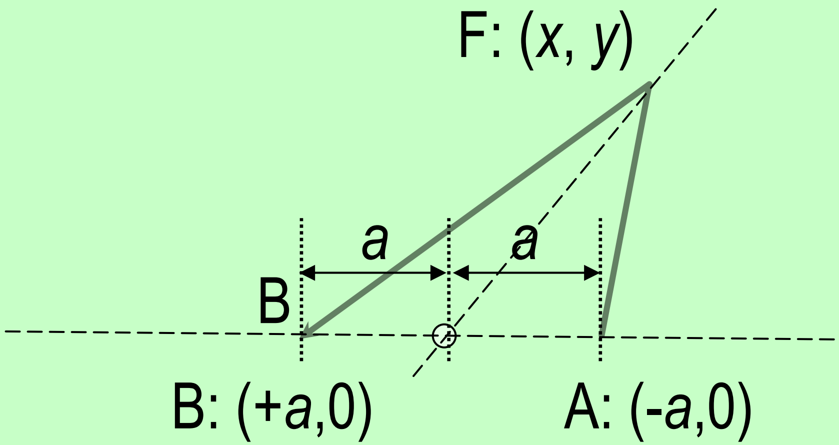
- The red paths have minimum travel time (or length)

(Fermat's principle – Not true for ADB)

- The red paths have extreme (stationary, i.e. minimum or maximum) travel time (or length)

(True)

The light example – no mirror

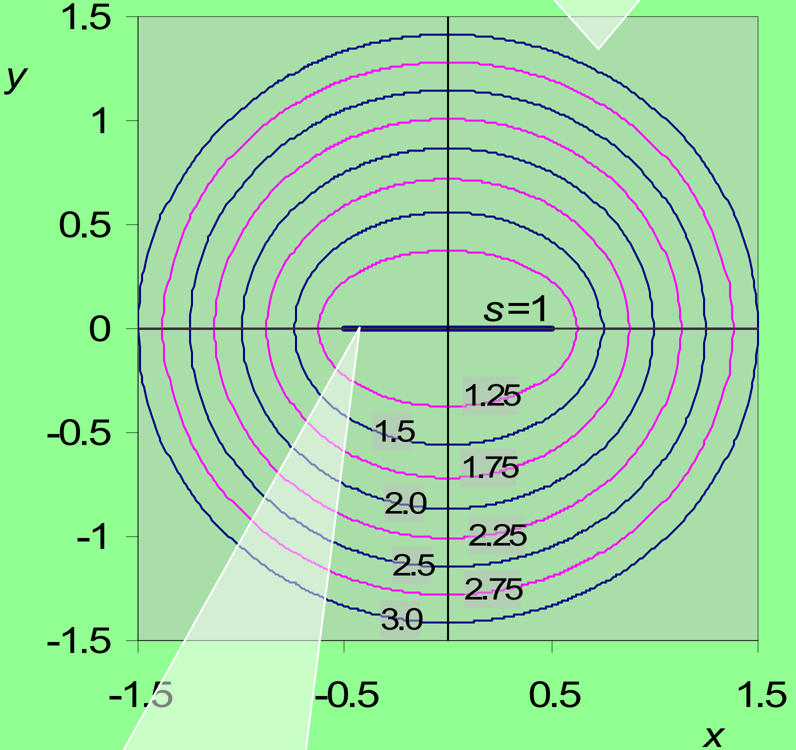


Assume that light can travel from A to B along a broken line with a break point F with coordinates (x, y).
(This is not restrictive: later we can add a second, third, … break points)
The travel distance is $s(x, y)$ = AF + FB where

$$AF = \sqrt{(x - a)^2 + y^2}$$

$$FB = \sqrt{(x + a)^2 + y^2}$$

Contours of the distance $s(x, y)$ assuming a = 0.5



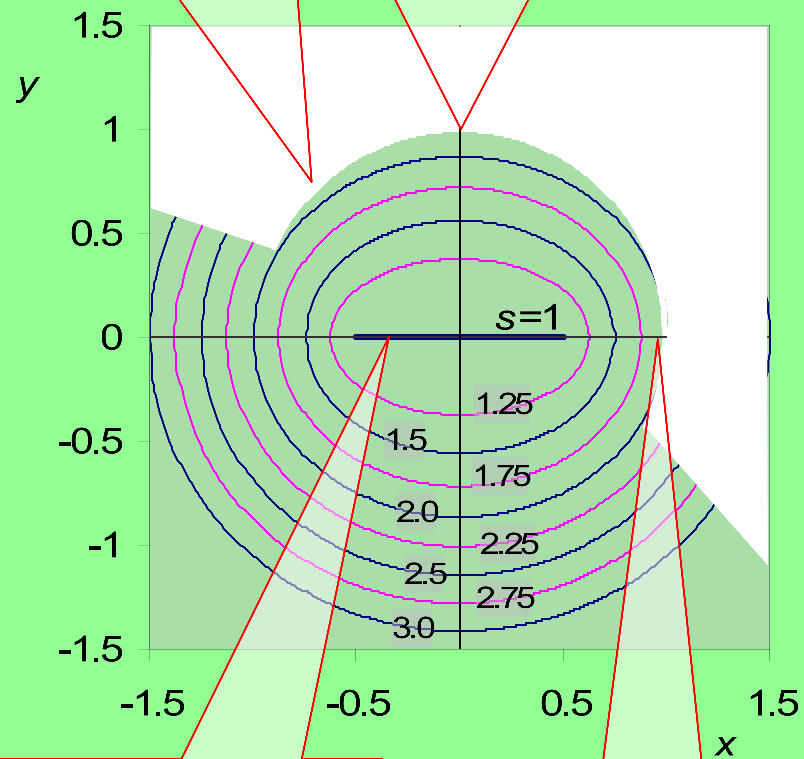
Line of minimum distance $s(x, y)$ = 1
Infinite points F essentially describing the same path

The light example with mirror

- The mirror introduces an inequality constraint in the optimization: the point F should not be behind the mirror
- Two points of local optima emerge on the mirror surface (the curve where the constraint is binding)

The mirror assuming radius r = 1

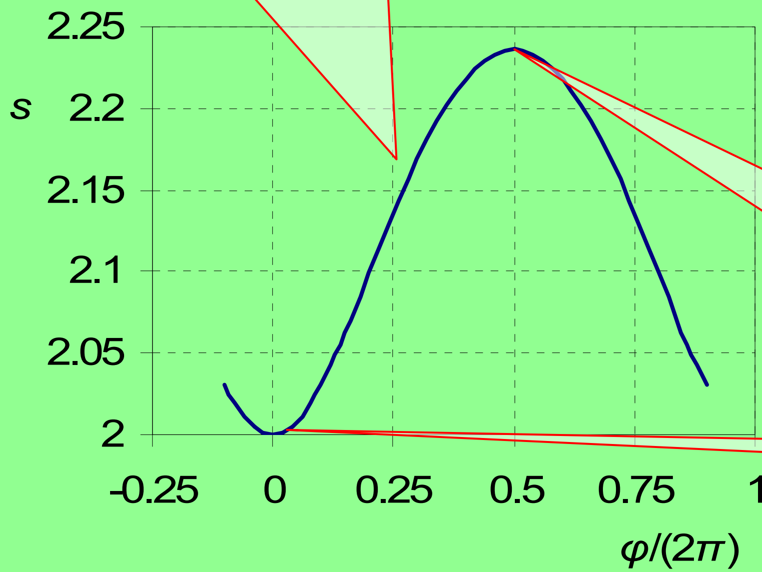
Local maximum: s = 2.24



Global minimum: s = 1

Local minimum: s = 2

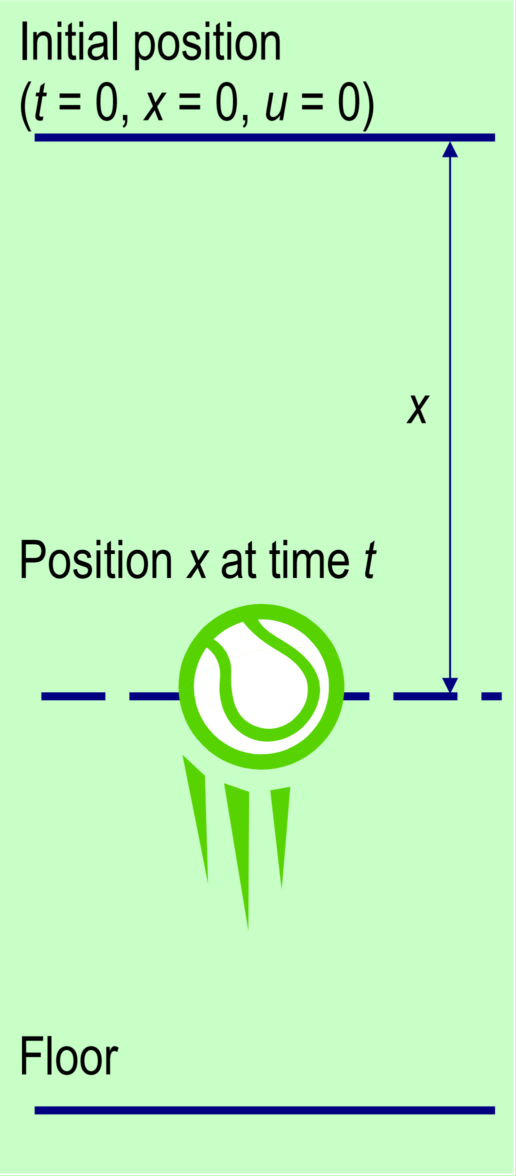
Close up along the mirror



Local maximum: s = 2.24

Local minimum: s = 2

A second example: a falling weight



Quantities involved

- Potential energy: $V = -m\,g\,x$
- Kinetic energy: $T = (1/2)\,m\,u^2 = (1/2)\,m\,(dx/dt)^2$
- Total energy: $E = T + V$
- Lagrangian: $L = T - V$
- Action: $S = \int L\,dt$

Alternative methodologies to find equations for the movement

1. Directly by integrating $d^2x/dt^2 = g$
2. From conservation of total energy
3. From minimization of action (more difficult)

- All methodologies result in same solution ($x = g\,t^2/2$, $u = dx/dt = g\,t$)

Principle of least action (Hamilton's principle – applicable both in classical and in quantum physics)

- From all possible motions between two points, the true motion has least action
- More correct to substitute "extreme" (or "stationary") for "least"

How nature works? (a hypothesis ...)

Property

- She preserves a few quantities (mass, momentum energy, ….)
- She optimizes a single quantity (Dependent on the specific system - Difficult to find what this quantity is)
- She disallows some states (Dependent on the specific system – Maybe difficult to find)

Mathematical formulation

- One equation per preserved quantity:

  $g_i(\mathbf{s}) = c_i, \quad i = 1, \ldots, k$

  where c_i constants; **s** the size n vector of state variables ($n \geq k$, sometimes $n = \infty$)

- A single "optimation":

  optimize $f(\mathbf{s})$

  [i.e. maximize/minimize $f(\mathbf{s})$] **This is equivalent to many equations** (as many as required to determine **s**)
  Conversely, many equations can be combined into an "optimation"

- Inequality constraints:

  $h_j(\mathbf{s}) \geq 0, \quad j = 1, \ldots, m$

- In conclusion, we may find how nature works solving the problem:

  optimize $f(\mathbf{s})$
  s.t. $g_i(\mathbf{s}) = c_i, \quad i = 1, \ldots, k$
  $h_j(\mathbf{s}) \geq 0, \quad j = 1, \ldots, m$

The typical “optimizable” quantity in complex systems …

- … is entropy – entropie – Entropie – entropia – entropía – entropi – entrópia – entroopia – entropija – энтропия – ентропія – 熵 – エントロピー – مقياس – אנטרופיה– εντροπία
- The word is ancient Greek (εντροπία, a feminine noun meaning: turning into; turning towards someone’s position; turning round and round)
- The scientific term is due to Clausius (1850)
- The entropy concept was fundamental to formulate the second law of thermodynamics
- Boltzmann (1877), then complemented by Gibbs (1948), gave it a statistical mechanical content, showing that entropy of a macroscopical stationary state is proportional to the logarithm of the number *w* of possible microscopical states that correspond to this macroscopical state
- Shannon (1948) generalized the mathematical form of entropy and also explored it further. At the same time, Kolmogorov (1957) founded the concept on more mathematical grounds on the basis of the measure theory

What is entropy?

- Entropy is defined on grounds of probability theory
- For a discrete random variable X taking values x_j with probability mass function $p_j \equiv p(x_j)$, j = 1,…,w, the Boltzmann-Gibbs-Shannon (or extensive) entropy is defined as

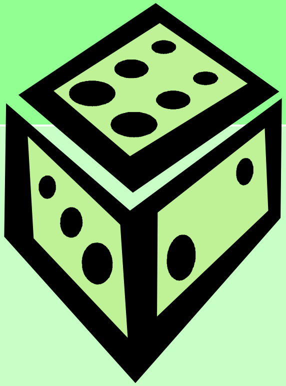
$$\varphi := E[-\ln p(X)] = -\sum_{j=1}^{w} p_j \ln p_j , \qquad \text{where } \sum_{j=1}^{w} p_j = 1$$

- For a continuous random variable X with probability density function $f(x)$, the entropy is defined as

$$\varphi := E[-\ln f(X)] = -\int_{-\infty}^{\infty} f(x) \ln f(x)\, dx, \qquad \text{where } \int_{-\infty}^{\infty} f(x)\, dx = 1$$

- In both cases the entropy φ is a measure of **uncertainty** about X and equals the **information** gained when X is observed.
- In other disciplines (statistical mechanics, thermodynamics, dynamical systems, fluid mechanics), entropy is regarded as a measure of **order** or **disorder** and **complexity**.
- Generalizations of the entropy definition have been introduced more recently (Renyi, Tsallis)

Entropy maximization: The die example



- What is the probability that the outcome of a toss of a die will be i? (i = 1, …, 6)
- The entropy is:

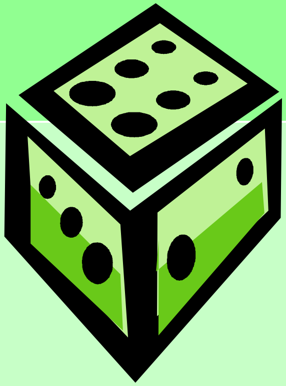
$$\varphi := E[-\ln p(X)] = -p_1 \ln p_1 - p_2 \ln p_2 - \ldots - p_6 \ln p_6$$

- The equality constraint (mass preservation) is

$$p_1 + p_2 + \ldots + p_6 = 1$$

- The inequality constraint is $p_i \geq 0$
- Solution of the optimization problem (e.g. by the Lagrange method) yields a single maximum: $p_1 = p_2 = \ldots = p_6 = 1/6$
- This method, the application of the Maximum Entropy Principle (mathematically, an "optimation" form) is equivalent to the Principle of Insufficient Reason (Bernoulli-Laplace; mathematically, an "equation" form)

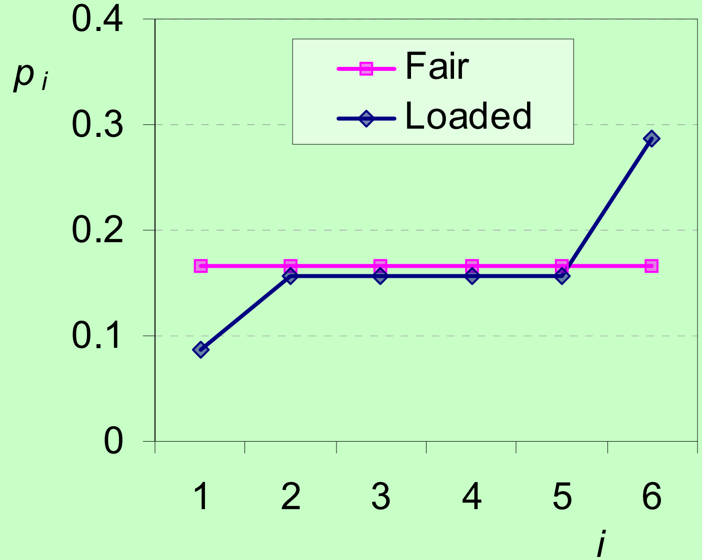
Entropy maximization: The loaded die example



- What is the probability that the outcome of a toss of a die will be i (i = 1, …, 6) if we know that it is loaded, so that $p_6 - p_1 = 0.2$?
- The IS principle does not work in this case
- The ME principle works. We simply pose an additional constraint:

$$p_6 - p_1 = 0.2$$

- The solution of the optimization problem (e.g. by the Lagrange method) is a single maximum:



Entropy maximization: The temperature example

- What will be the temperature in my house (T_H), compared to that of the environment (T_E)? (Assume an open window and no heating equipment)
- Take a space of environment (E) in contact to the house (H) with volume equal to that of the house

- Partition the continuous range of kinetic energy of molecules into several classes i = 1 (coldest), 2, …, k (hottest)
- Denote p_i the probability that a molecule belongs to class i, and partition it to $p_{\mathrm{H}i}$ and $p_{\mathrm{E}i}$, if the molecule is in the house or the environment, respectively
- Form the entropy in terms of $p_{\mathrm{H}i}$ and $p_{\mathrm{E}i}$
- Maximize entropy conditional on $p_{\mathrm{H}i} + p_{\mathrm{E}i} = p_i$
- The result is $p_{\mathrm{H}i} = p_{\mathrm{E}i}$
- Equal number of molecules of each class are in the house and the environment, so $T_H = T_E$
- This could be obtained also from the IR principle

Formalization of the principle of maximum entropy

- In a probabilistic context, the principle of ME was introduced by Janes (1957)
- In a probabilistic context, the principle of ME is used to infer unknown probabilities from known information
- In a physical context, it determines thermodynamical states
- The principle postulates that the entropy of a random variable should be at maximum, under some conditions, formulated as constraints, which incorporate the information that is given about this variable
- Typical constraints used in a probabilistic or physical context are:

Mass; Mean/Momentum; Non-negativity

$$\int_{-\infty}^{\infty} f(x)\, dx = 1, \quad E[X] = \int_{-\infty}^{\infty} x\, f(x)\, dx = \mu \qquad x \geq 0$$

Variance/Energy; Dependence/Stress

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx = \sigma^2 + \mu^2, \; E[X_i\, X_{i+1}] = \int_{-\infty}^{\infty} x_i\, x_{i+1}\, f(x_i, x_{i+1})\, dx_i\, dx_{i+1} = \rho\, \sigma^2 + \mu^2$$

Some results of ME interesting to hydrology

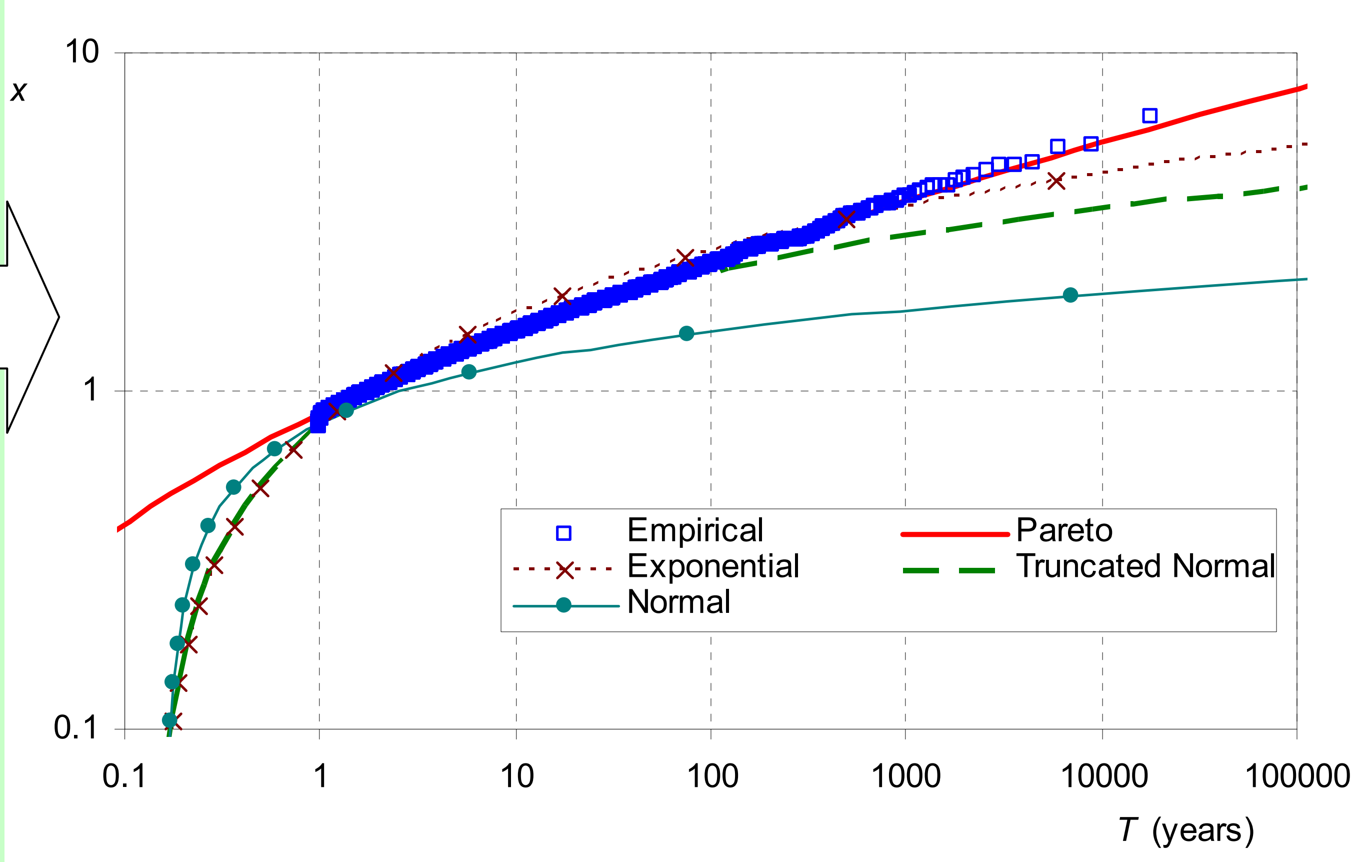
- Assume that a hydrometeorological variable X (e.g. temperature, rainfall, runoff) is continuous and positive, has known mean μ and known variation σ/μ. Estimate the distribution function with only this information, applying the ME principle
- The results are:
  - Maximum entropy + Low variation → (Truncated) normal distribution
  - Maximum entropy + High variation → Power-type (Pareto) distribution
  - Maximum entropy + High variation + High return periods → State scaling
- The celebrated state scaling ($x_T \sim T^{\kappa}$, where T is the return period and x_T the corresponding quantile) is only:
  - a consequence of the ME principle,
  - an approximation, good for high return periods and for variables with high variation
- Real world time series (especially long ones) validate the applicability of the ME principle in hydrometeorological processes

ME application to extreme daily rainfall worldwide

Data set: Daily rainfall from 168 stations worldwide each having at least 100 years of measurements; series above threshold, standardized by mean and unified; period 1822-2002; 17922 station-years of data

$\mu = 0.28$
(mean minus threshold)
$\sigma/\mu = 1.19 > 1$
ME distribution: Pareto
$\kappa = 0.15$
$\varphi_q = 1.16$

Conclusion: Scaling for $T > \sim 50$ yr



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Entropic quantities of a stochastic process

- The *order 1 entropy* (or simply *entropy* or *unconditional entropy*) refers to the marginal distribution of the process X_i:

$$\varphi := E[-\ln f(X_i)] = -\int_{-\infty}^{\infty} f(x) \ln f(x)\, dx, \quad \text{where} \int_{-\infty}^{\infty} f(x)\, dx = 1$$

- The *order n entropy* refers to the joint distribution of the vector of variables $\mathbf{X}_n = (X_1, \ldots, X_n)$ taking values $\mathbf{x}_n = (x_1, \ldots, x_n)$:

$$\varphi_n := E[-\ln f(\mathbf{X}_n)] = -\int_{D_n} f(\mathbf{x}_n) \ln f(\mathbf{x}_n)\, d\mathbf{x}_n$$

- The *order m conditional entropy* refers to the distribution of a future variable (for one time step ahead) conditional on known m past and present variables (Papoulis, 1991):

$$\varphi_{c,m} := E[-\ln f(X_1 | X_0, \ldots, X_{-m+1})] = \varphi_m - \varphi_{m-1}$$

- The *conditional entropy* refers to the case where the entire past is observed:

$$\varphi_c := \lim_{m \to \infty} \varphi_{c,m}$$

- The *information gain* when present and past are observed is:

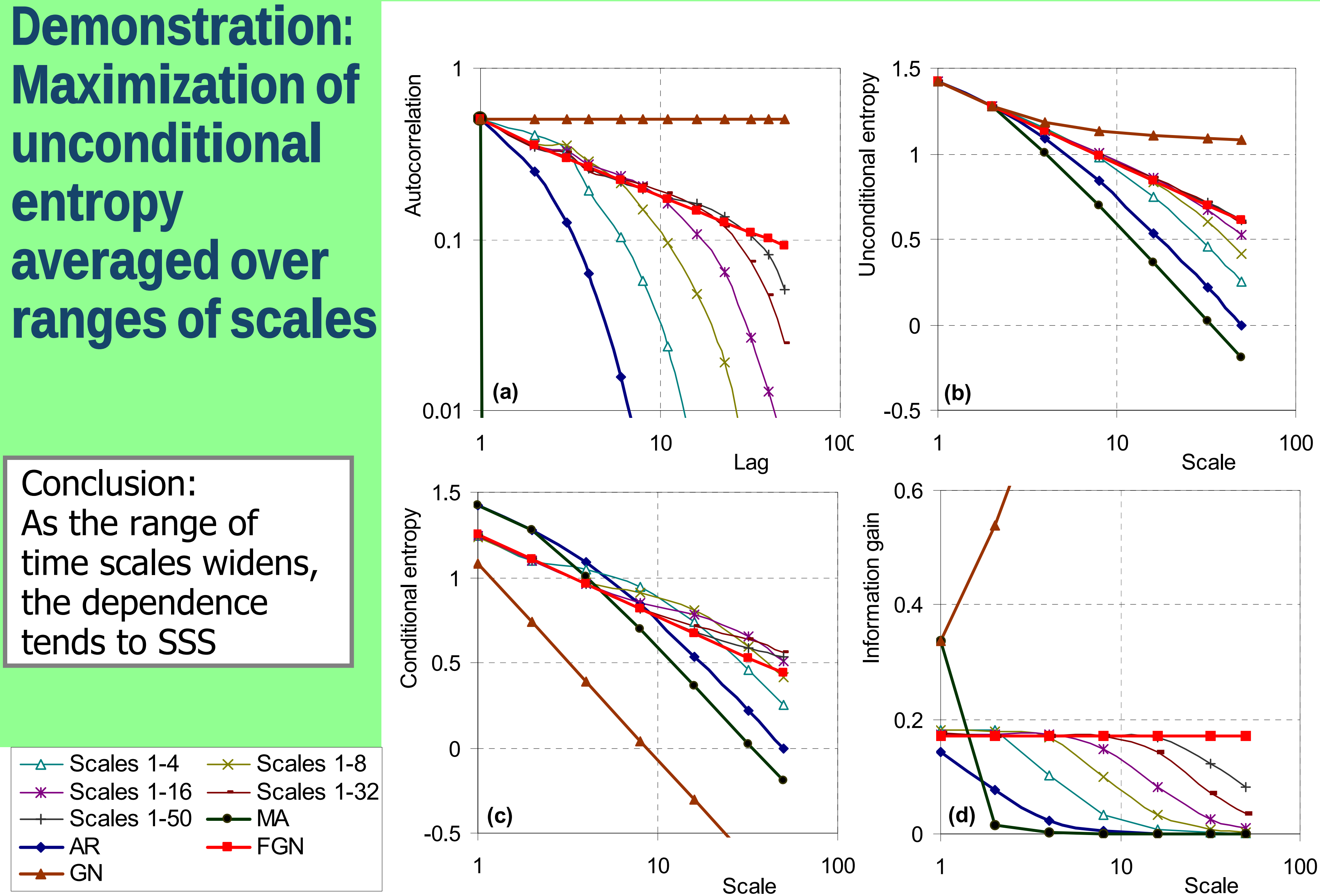
$$\psi := \varphi - \varphi_c$$

Note: notation assumes stationarity

Entropy maximization for a stochastic process

- The purpose is to determine not only marginal probabilities but the dependence structure as well
- All five constrains are used (mass/mean/variance/dependence/non-negativity)
- The lag one autocorrelation (used in the dependence constraint) is determined for the basic (annual) scale but the entropy maximization is done on other scales as well
- The variation is low (σ/μ << 1) and thus the process is virtually Gaussian (intermediate result). This is valid for annual and over-annual time scales
- For a Gaussian process the nth order entropy is given as $\varphi_n = \ln\sqrt{(2\pi e)^n \delta_n}$ where δ_n is the determinant of the autocovariance matrix $c_n := \mathrm{Cov}[\mathbf{X}_n, \mathbf{X}_n]$.
- The autocovariance function is assumed unknown to be determined by application of the ME principle. Additional constraints for this are:
  - Mathematical feasibility, i.e. positive definiteness of c_n (positive δ_n)
  - Physical feasibility, i.e. autocorrelation function (a) positive and (b) non increasing with lag and time scale
    (Note: periodicity that may result in negative autocorrelations is not considered here due to annual and over-annual time scales)

Demonstration: Maximization of unconditional entropy averaged over ranges of scales



Conclusion:
As the range of time scales widens, the dependence tends to SSS

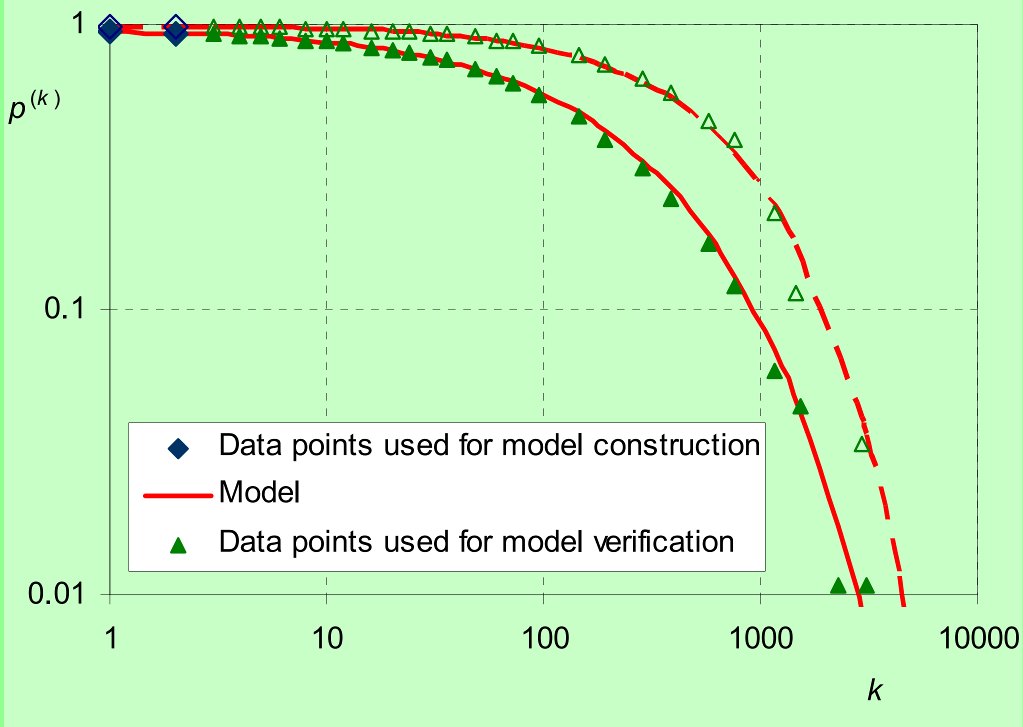
Results of the ME principle in stochastic processes

- Maximum entropy + Low variation + Dominance of a single time scale → Normal distribution + Time independence
- Maximum entropy + Low variation + Time dependence + Dominance of a single time scale → Normal distribution + Markovian (short-range) time dependence
- Maximum entropy + Low variation + Time dependence + Equal importance of time scales → Normal distribution + Time scaling (long-range dependence / Hurst phenomenon)
- The time scaling behavior is a result of the principle of maximum entropy
- The omnipresence of time scaling in numerous long hydrologic time series, validates the applicability of the ME principle

Another peculiar dependence explained by ME

- Rainfall at small scales is intermittent
- The dependence of the rainfall occurrence process is not Markovian neither scaling but in between; it has been known as clustering or overdispersion
- The models used for the rainfall occurrence process (point processes) are essentially those describing clustering of stars and galaxies
- The ME principle applied with the binary state rainfall process in more or less the same way as in the continuous state process explains this dependence

Probability $p^{(k)}$ that an interval of k hours is dry, as estimated from the Athens rainfall data set and predicted by the model of maximum entropy for the entire year (full triangles and full line) and the dry season (empty triangles and dashed line)



Interpretation of results

- The successful application of the ME principle in nature offers an explanation for of a plethora of phenomena (e.g. thermodynamic) and statistical behaviors including
  - the emergence of normal distribution, in many (but not all) cases
  - the scaling behavior in state, in other cases
  - the scaling behavior in time
  - the clustering behavior in rainfall occurrence
- This can be interpreted as dominance of uncertainty in nature
- It harmonizes with the Socratic view: «Ἕν οἶδα, ὅτι οὐδέν οἶδα» (One I know, that I know nothing)
- This view was not a confession of modesty – Socrates regarded the knowledge of ignorance as a matter of supremacy
- In this respect, the knowledge of the dominance of uncertainty can assist to safer design and management of hydrosystems

{3,8,9}

3. Operational synthesis

Stochastic simulation/forecasting of hydrologic processes

- Question: Why simulated series?
- Answer:
  - Analytical solutions for a hydrosystem as complex as that of Athens are not feasible or would assume oversimplification of the system
  - Of numerical methods, Monte Carlo simulation (stochastic simulation) is the most convenient
  - Detailed inflow and other (rainfall, evaporation) hydrologic series are needed at many sites simultaneously and at several time scales for Monte Carlo simulation the hydrosystem
  - The acceptable failure probability level for Athens is of the order of 10^{-2}: one failure in 100 years on the average
  - For a reasonable estimation error in the failure probability we need 1000-10 000 years of data
  - Historic hydrologic records are too short

Requirements for stochastic simulation

1. Multivariate model
2. Multiple time scales of operation: annual to monthly or sub-monthly
3. Multiple time scales of preservation: multi-year (reproduction of the Hurst phenomenon) to sub-monthly (reproduction of sub-annual periodicity)
4. Preservation of essential marginal statistics up to third order (skewness)
5. Preservation of joint second order statistics
   - autocorrelations of any type and any lag
   - concurrent cross-correlations
6. Parsimony of parameters
7. Performance in simulation mode (steady state simulations) and in forecast mode, given the current and historic values (terminating simulations)

Models with such features did not exist (particularly, the ARMA type models were not useful)

Stochastic simulation strategy

- Stage 1: Generate annual time series
  - Use a parsimonious model yet capable of describing over-annual scaling
  - No need to describe sub-annual periodicity
- Stage 2: Disaggregate the annual into sub-annual time series
  - Use a parsimonious model structure such as PAR(1)
  - Couple it to the annual model
  - So, no need to describe over-annual scaling explicitly
- A one stage procedure to handle over-annual and sub-annual properties simultaneously has also been studied but not implemented operationally so far

{7,16,17}

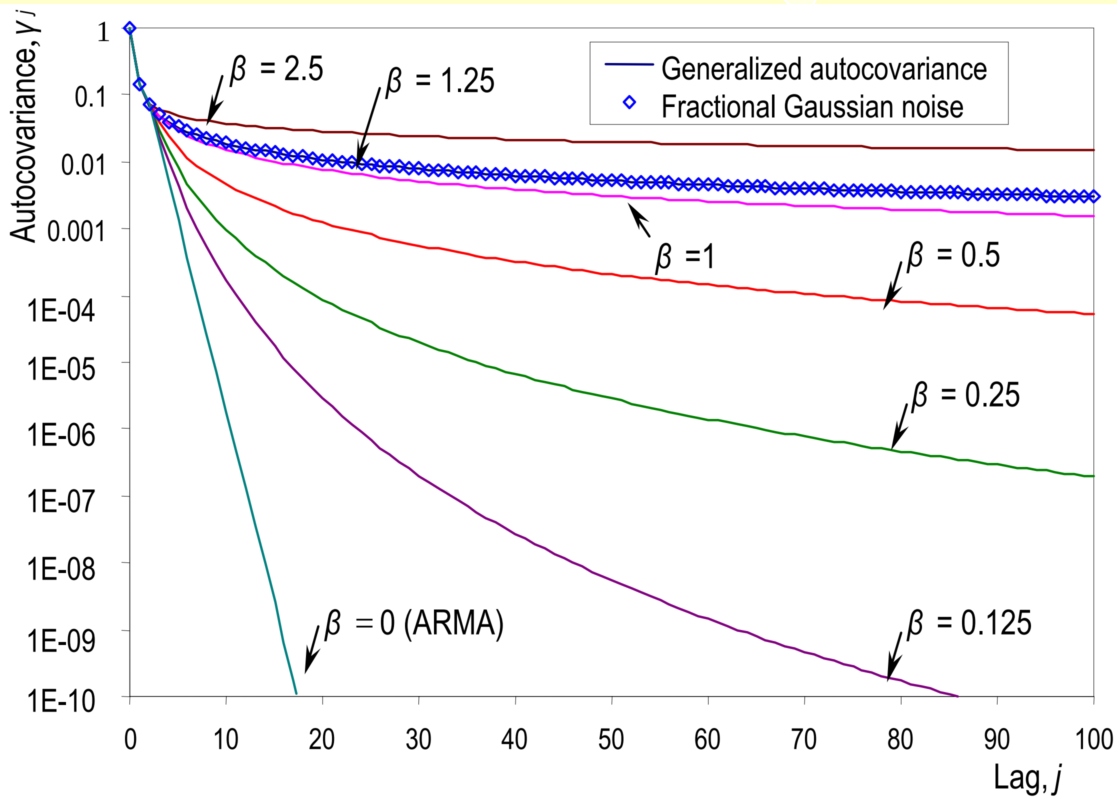
Annual model: The generalized autocovariance function (GAS)

- General GAS expression

  $\gamma_j = \gamma_0 \, (1 + \kappa \, \beta \, |j|^{\alpha})^{-1/\beta}$

  where γ_j: lag j autocovariance; γ_0: variance; κ, α, β: parameters
- Fittings options
  - Optimize parameters to best fit historic autocorrelograms
  - Preserve explicitly γ_1, γ_2 and Hurst exponent
  - Explicit preservation of more γ_i is also possible
- GAS behavior
  - For $\beta = 0 \Rightarrow$ ARMA: $\gamma_j = \gamma_0 \exp(-\kappa \, |j|^{\alpha})$
  - For $\kappa = (1/\beta)\,(1 - 1/\beta)^{-\beta}\,(1 - 1/2\beta)^{-\beta}$ and $\alpha = 1 \Rightarrow$ FGN

Demonstration of GAS for α = 1 and several values of β



Annual model: Generalized generating scheme for any covariance structure

Typical (backward) moving average (**BMA**) scheme: $X_i = \ldots + a_1 V_{i-1} + a_0 V_i$
where V_i independent random variables and a_i numerical coefficients

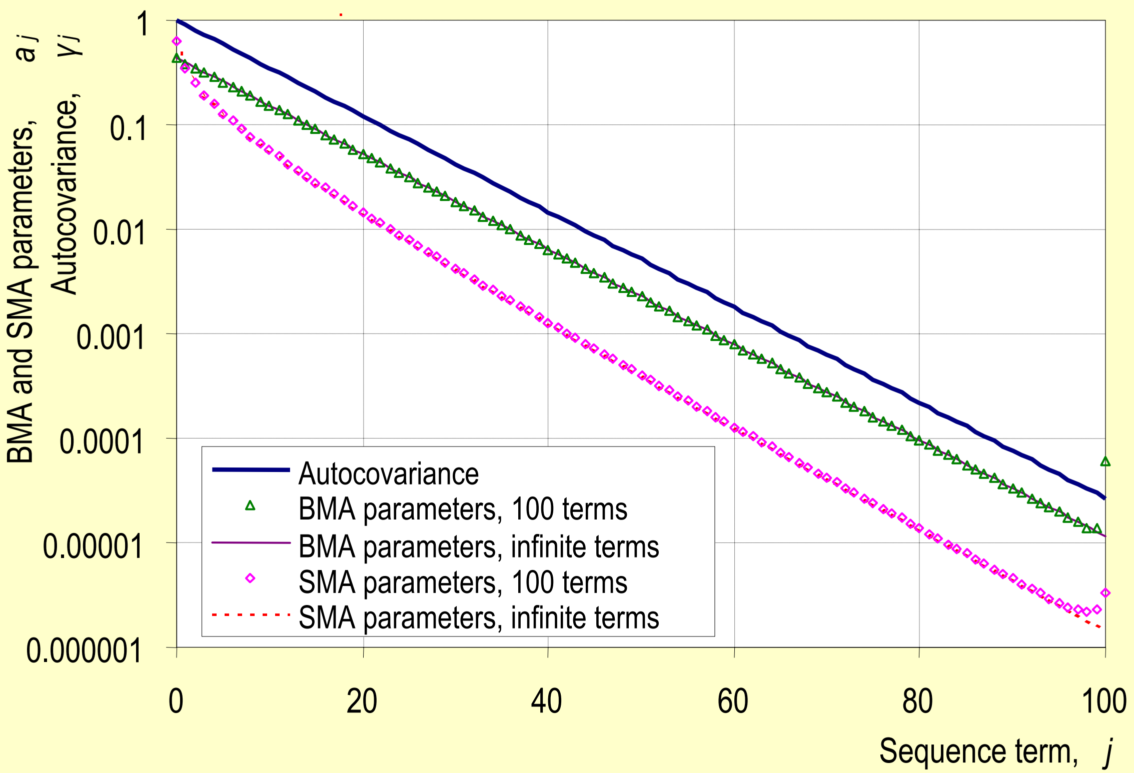
Symmetric moving average (**SMA**) scheme $X_i = \ldots + a_1 V_{i-1} + a_0 V_i + a_1 V_{i+1} + \ldots$

SMA has several advantages over BMA. Among them, it allows a closed solution for a_i:

$s_a(\omega) = [2\, s_\gamma(\omega)]^{1/2}$

where $s_a(\omega)$ and $s_\gamma(\omega)$ the Discrete Fourier Transforms of the series a_j and γ_j, respectively.

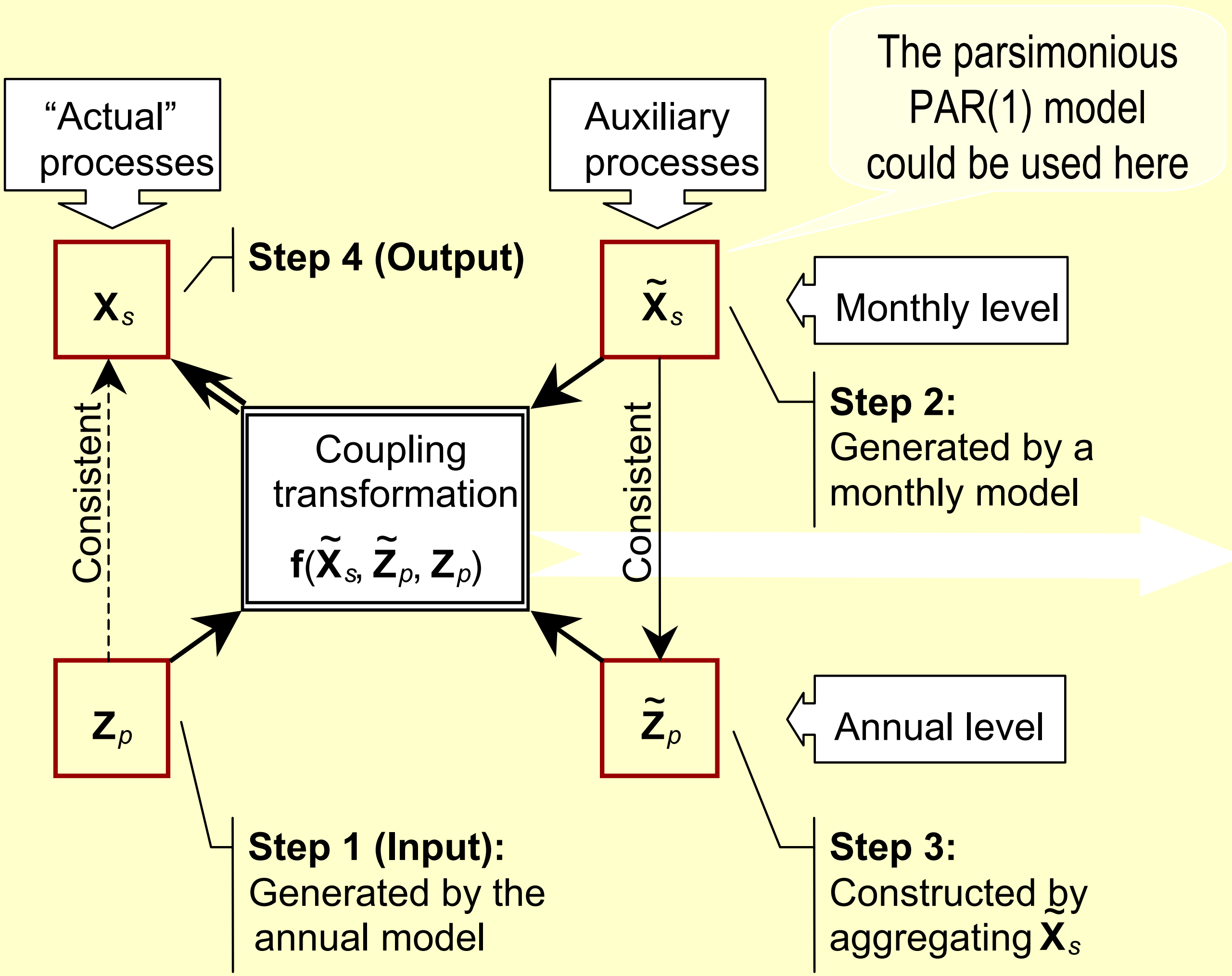
Both schemes are applicable for multivariate problems



Annual model: Stochastic simulation in forecast mode

- In forecast mode, the observed present and past values must condition the hydrologic time series of the future
- This is attainable using a two-step algorithm
  1. Generate future time series without reference to the known present and past values
  2. Adjust future time series using the known present and past values and a linear adjusting algorithm
- The linear adjusting algorithm:
  1. is expressed in terms of covariances among variables
  2. preserves exactly means, variances and covariances
  3. is easily implemented

Coupling stochastic models of different time scales



The linear transformation

$$\mathbf{X}_s = \tilde{\mathbf{X}}_s + \mathbf{h}\,(\mathbf{Z}_p - \tilde{\mathbf{Z}}_p)$$

where

$$\mathbf{h} = \mathrm{Cov}[\mathbf{X}_s, \mathbf{Z}_p] \cdot \{\mathrm{Cov}[\mathbf{Z}_p, \mathbf{Z}_p]\}^{-1}$$

preserves the vectors of means, the variance-covariance matrix and any linear relationship that holds among $\mathbf{X}_s$ and $\mathbf{Z}_p$.

{16,19}

Handling of skewness in multivariate problems: Optimized decomposition of covariance matrices

- Consider any linear multivariate stochastic model of the form

$$\mathbf{Y} = \mathbf{a}\,\mathbf{Z} + \mathbf{b}\,\mathbf{V}$$

  where **Y**: vector of variables to be generated, **Z**: vector of variables with known values, **V**: vector of innovations, and **a** and **b**: matrices of parameters
- The parameter matrix **b** is related to a covariance matrix **c** by

$$\mathbf{b}\,\mathbf{b}^{T} = \mathbf{c}$$

- This equation may have infinite solutions or no solution (if **c** is not positive definite)
- The skewness coefficients $\boldsymbol{\xi}$ of innovations **V** depend on **b**
- The smaller the values of $\boldsymbol{\xi}$, the more attainable the preservation of the skewness coefficients of the actual variables **Y**
- Therefore, the problem of determination of **b** can be seen as an optimization problem that combines
  - minimization of skewness $\boldsymbol{\xi}$, and
  - minimization of the error $||\mathbf{b}\,\mathbf{b}^{T} - \mathbf{c}||$
- A fast optimisation algorithm has been developed for this problem
- The algorithm works even for **c** that are not positive definite

{18}

An alternative modelling strategy – forecast oriented

- Consider the prediction of a single variable Y, conditional on several (even too many) other variables with known values. Use the linear model:

$$Y = \mathbf{a}\,\mathbf{Z} + V$$

  where **Z**: vector of variables with known values, **a**: matrix of parameters and V: prediction error, assumed independent of **Z**
- Assume that all variables (Y and **Z**) are normally distributed with zero mean (a prior normalizing transformation may be needed for this). Obtain:

$$\mathbf{a} = \boldsymbol{\eta}^T \mathbf{h}^{-1} \text{ and } \mathrm{Var}[V] = \gamma_0 - \boldsymbol{\eta}^T \mathbf{h}^{-1} \boldsymbol{\eta} \text{ where } \boldsymbol{\eta} := \mathrm{Cov}[Y, \mathbf{Z}],\ \mathbf{h} := \mathrm{Cov}[\mathbf{Z}, \mathbf{Z}],\ \gamma_0 = \mathrm{Var}[Y]$$

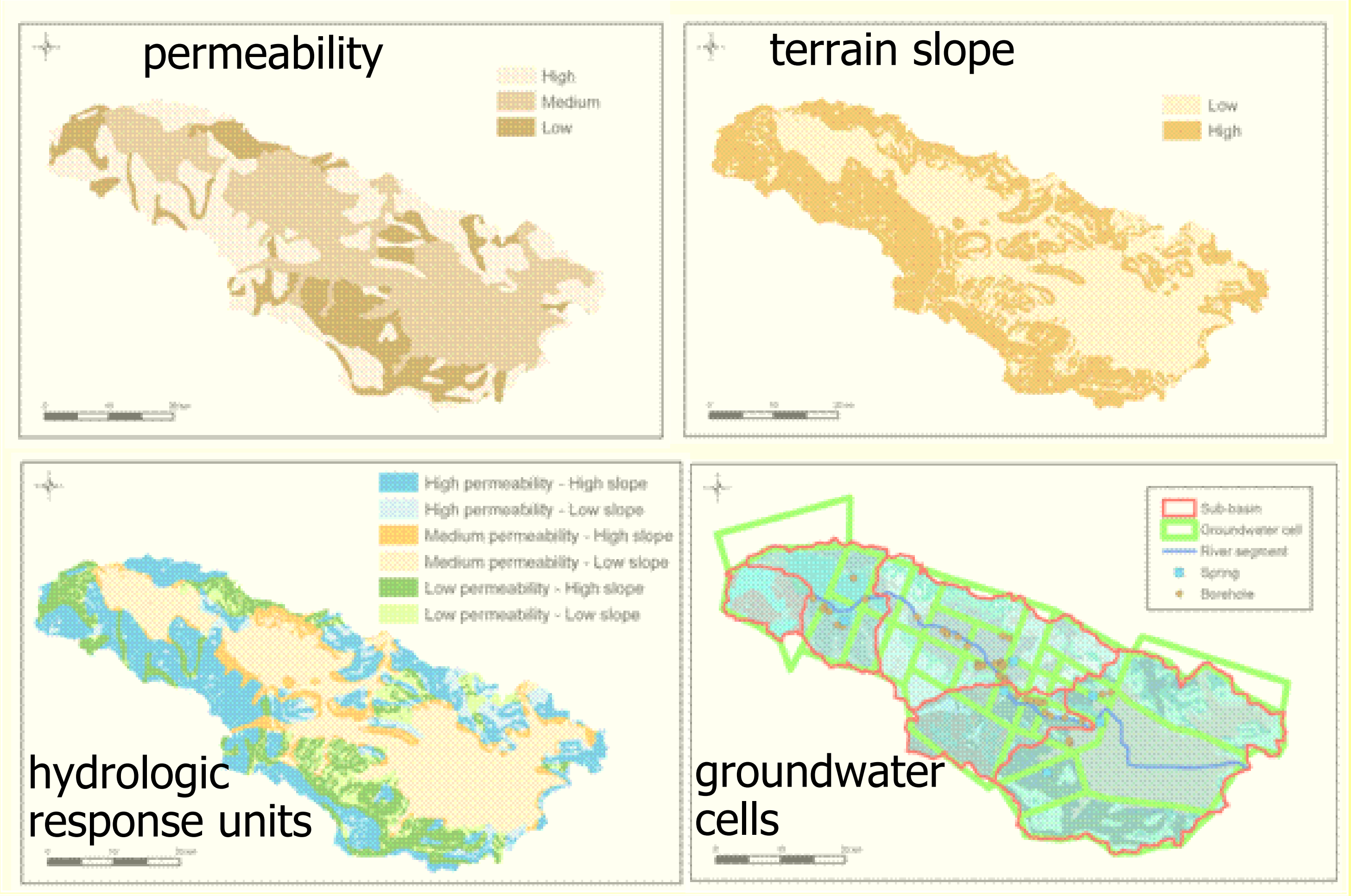
- Assume that all items of vector **η** and matrix **h** can be expressed in terms of a handful of parameters **π** (e.g. autocorrelation coefficients) that describe either long- or short-term dependence
- Use multi-scale entropy optimization to obtain (numerically, in terms of **π**) the vector **η** and those items of **h** that represent long-term dependence
- Use single-scale entropy optimization to obtain (in terms of **π**) the remaining items of **h** that do not represent long-term dependence. This can be done analytically using an auxiliary lower triangular matrix **b** such that:

$$\mathbf{b}\,\mathbf{b}^T = \mathbf{h}$$

  It can be shown that unspecified items of **h** correspond to zero items in **b**; all nonzero items of **b** can be obtained then by Cholesky decomposition
- For forecasts use the above model with $V = 0$
- For stochastic simulation use the same model with V generated from the normal distribution with the specified variance

Models developed are not only stochastic ...

In the Boeoticos Kephisos River basin a hydrologic model of the entire hydrologic cycle had to be developed, which was demanding due to the extended karstic activity and the intensive withdrawals for irrigation



{5,12}

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Early stage

The Hadrianean aqueduct

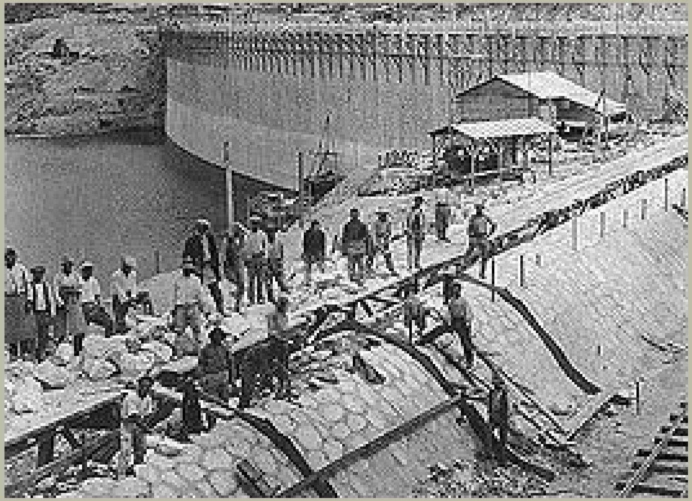
Supplementary water collection and distribution in Athens (early 20th century until 1930s)

Milestones

Marathon dam

Today

Construction of dam, 1928

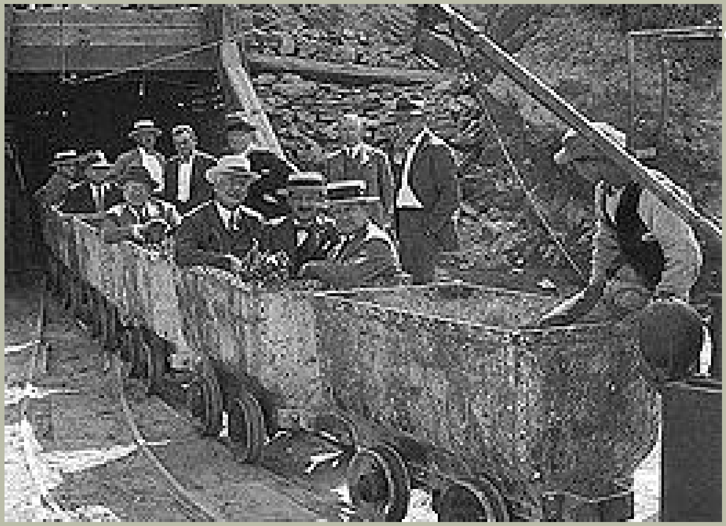


Construction of spillway, 1928

Hydrosystem
More pictures

Marathon dam (2)

Devastating flood, 1926



Inauguration of Boyati tunnel, 1928

Marathon spillway in action, 1941

Hydrosystem
Previous pictures

Hylike lake and pumping stations

Hylike lake

Hylike, floating pumping stations

Hylike, main pumping station

Kiourka pumping station

Hydrosystem

Mornos reservoir and aqueduct

Mornos canal at Delphi

Mornos reservoir

Mornos canal at Thebes plain

Siphon at Distomo

Hydrosystem

Control of Mornos aqueduct

Canal flow control construction

Aqueduct supervizing & control centre

Hydrosystem

Evinos dam and tunnel

Evinos dam during construction

Construction of the Evinos-Mornos connection tunnel

Hydrosystem

Treatment plants

Perissos water treatment plant

Asporpyrgos water treatment plant

Hydrosystem