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The Hurst phenomenon and Monte Carlo simulation to forecast reliability of an Australian reservoir

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Abstract: The issue of water supply reliability from Australian reservoirs has recently been the subject of increased scientific and media debate. 'Drought Persistence' or prolonged sequences of low inflows has driven reservoirs to seriously low levels. Statistical justification for these persistent droughts is often difficult to find if classical statistics and typical stochastic approaches are used. Therefore persistent droughts may be overlooked in reliability of supply calculations. This can result in dramatically underestimated risk of failure. The Hurst phenomenon offers a consistent basis to remedy this and the Hurst coefficient can be a simple measure to quantify the amount of persistence in a time series.

For this study, the Hurst coefficient was calculated for the historical flow data of the Boyne River, Queensland. Based on the coefficient and the probability distribution of the historical inflow data, synthetic reservoir inflow sequences were generated preserving the persistence. Using this data and Monte Carlo simulation, a tool was developed to forecast the reliability of supply into the future from the current storage level. This is used for planning risk reduction strategies by providing valuable information such as the lead-time available to implement contingencies.

Keywords: Hurst Phenomenon, drought persistence, reservoir reliability, Long-term dependence.

1. INTRODUCTION

Like many reservoirs in Australia, Awoonga Dam near Gladstone in Central Queensland has experienced declining water levels over the last decade due to an extended or persistent drought. This has lead to concerns for the operation of billions of dollars of existing and proposed industrial developments that rely on the dam for their water supply.

Due to this, the Gladstone Area Water Board (GAWB) commissioned Hydro Tasmania Consulting (HTC) who have extensive experience in managing their 60 reservoirs, to develop a planning tool to assist in the medium to long term (3-20yr) management and planning of Awoonga Dam in relation to drought risk. The tool was called named AWSIM-O which is short for Awoonga Dam Simulation - Operational Model).

The tool uses Monte Carlo simulations to estimate the water level in Awoonga Dam for various probabilities over the next 20 years. The methodology incorporates a water balance on monthly time step taking into account variability and probability of factors such as:
- Reservoir inflows
- Reservoir surface area losses/gains to evaporation/rainfall.
- Environment releases
- Demand growth into the future
- Water restrictions regime
- Augmentation of supply from other sources

This paper details the methodology and results from the tool.

2. METHODOLOGY

2.1 Overview

The water balance runs on monthly time step taking into account the components shown in Figure 1. The treatment of each of these components is given in the subsections below.

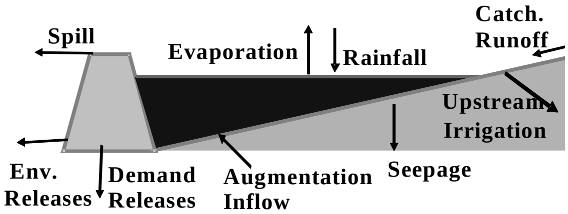

Figure 1 - Reservoir water balance components.

Note, for this study the 'catchment runoff' is differentiated from the reservoir inflow by the upstream irrigation.

2.2 Catchment Runoff Sequences

The catchment runoff sequences are the most important components of the whole model in determining probability of failure and low dam levels. Due to this, great effort was taken to get the sequences as representative of the historical sequence as possible.

2.2.1 Probability Distribution Function (PDF)

The first step in the data generation process is to define the probability distribution function of the catchment runoff. The PDF was defined for the lower critical flows using an empirical approach and for higher flows a Gamma Distribution was used.

The historical annual catchment runoff sequence for Awoonga Dam for 1938 to 2005 (68 years) was used as the basis for deriving the PDF. Figure 2 shows the histogram of this sequence. The graph shows an unusual behaviour characterized by a bimodal distribution, with the main peak being very low (around 60,000 ML/yr) and a secondary peak about six times higher (around 360,000 ML/yr). This indicates that the river flow alternates between at least two general regimes, the most frequent low/regular flow regime and a high flow one manifesting cyclonic/low pressure system events, which generally produce 200,000 ML or more runoff.

Due to the added complexity of a bimodal distribution it was decided to empirically derive the PDF for flows less than 960,000 ML/yr. This is simply done by using the histogram to derive the probabilities.

For flows greater than 960,000 ML/yr the Gamma Distribution was used. As the current capacity of Awoonga Dam is 777,000 ML, the Gamma distribution would only produce annual inflows that would overflow the dam and therefore in terms of this application, have no real impact on the model results.

The adopted PDF for the annual catchment runoff based on the methodology above is shown in Figure 3.

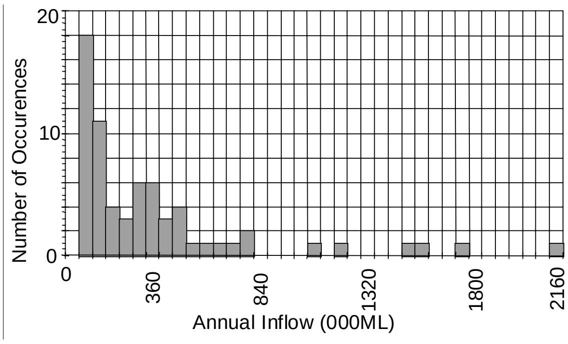

Figure 2 - Historical annual catchment runoff histogram

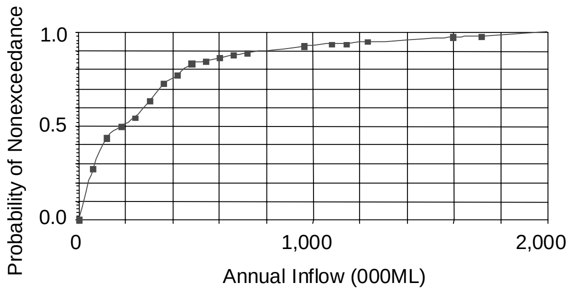

Figure 3 - Adopted probability distribution function for the annual catchment runoff

2.2.2 Drought Persistence

Initial reviews of the autocorrelation in the historical catchment runoff sequence showed that the autocorrelation was not statistically significant (see Table 1). This does not suggest long term persistence or dependence between runoff in consecutive years. Based on this, catchment runoff sequences were generated by randomly sampling from the PDF. Testing these sequences in the AWSIM-O model produced results with lower probabilities of failure than expected. That is, at the 1% probability the calculated reservoir level was not near Minimum Operating Level (MOL). Closer analysis of the generated catchment runoff sequences showed that the 5 and 10 year moving averaged flows were larger than in the historical sequence at the drier end or higher exceedance probabilities. This indicated there was some persistence or dependence in the historical catchment runoff sequence that was not being preserved in the generated sequence. This problem is very common in hydrological statistics as it is known that classical statistical estimators tend to hide the long range dependence of processes (Koutsoyiannis, 2003). To uncover this behaviour much longer records (e.g. more than 100 years) are needed.

Table 1 Autocorrelation statistics of historical catchment runoff data (1939-2005)

Lag (yrs)	Correlation	T stat.	T signif.*	Conclusion
1	0.11	0.88	2.36	Not signif.
2	0.08	0.61	2.36	Not signif.
3	0.19	1.49	2.36	Not signif.
5	0.10	0.76	2.36	Not signif.
10	-0.19	-1.45	2.37	Not signif.

*95% significance

Long-term persistence or long range dependence in time series can be characterized by the Hurst coefficient, H (Hurst, 1951). It is named after the British hydrologist H. E. Hurst, who investigated persistence of such hydrological phenomena as levels of the river Nile. Purely random data or white noise (with no persistence) has a Hurst Coefficient of 0.5, while highly persistent data such as a straight line with non-zero gradient has a Hurst Coefficient of 1. Hurst (1951) showed that water level of the Nile River in Egypt has a Hurst Coefficient of 0.85. This indicates a long range dependence that is well beyond what could be adequately explained assuming random data.

Analysis of Awoonga Dam's annual historical catchment runoff sequence gave a Hurst Coefficient of 0.67. This indicates that the dam's catchment runoff has a moderate amount of long range persistence. This persistence can explain to some extent why the current drought in the Awoonga Dam Catchment seems prolonged and why the wet periods of 1950s and 1970s also appear prolonged. Perhaps the actual Hurst coefficient is underestimated because of the small sample size (68 years) and the high variability of the flows. A greater Hurst coefficient would produce such prolonged droughts with higher probability. Meteorological phenomena such as El Nino and the more recently discovered Inter Pacific Oscillation Index (IPO) provide scientific climatic explanations and support for this multiyear and multiscale fluctuations, which are the conceptual basis of long range dependence (Koutsoyiannis, 2005).

Koutsoyiannis (2005) provides a number of techniques to generate time series data respecting the persistence. Here we have adopted his technique that uses the sum of three independent Auto-Regressive Order 1 (AR1) processes to give an acceptable approximation of the persistence. Using the PDF and the Hurst Coefficient of 0.67, 10,000 replicates of 21 years sequences were generated. Testing of these sequences in the AWSIM-O model produced failure probabilities consistent with previous hydrological analysis for Awoonga Dam.

2.2.3 Disaggregation to Monthly Data

Historical catchment runoff patterns were used to disaggregate the generated annual catchment runoff down to monthly flows. Visual analysis of the historical monthly catchment runoff pattern against the historical annual catchment runoff volume showed no obvious correlation between annual catchment runoff magnitude and the monthly pattern. In particular, patterns that were dominated by one or two months were scattered throughout the low and high flow years. Based on this, the historical monthly catchment runoff patterns were randomly sampled for each year of the simulation.

The results of the model are not very sensitive to the monthly patterns due to the multiyear capacity of this storage.

2.3 Upstream Irrigation

1,536 ML was the approximate annual irrigation allocation in the catchment upstream of Awoonga Dam in 2004 (Bartlett and Bryden, 2004). This value is subtracted from the Catchment Runoff value for each year to calculate the inflow to the dam. Negative dam inflows produced by this calculation are set to zero.

2.4 Demand Releases

The AWSIM-O user sets the user demand (ML/yr) time series for the next 20 years. The water restriction rules defined by GAWB reduce the demands as the model runs.

2.5 Reservoir Surface Rainfall

For each month during the simulation, a rainfall value is generated and multiplied by the reservoir surface area at the start of the month to estimate storage gain for the current month. The first estimate for the rainfall is the monthly average rainfall for the closest gauge.

It is desirable to preserve the correlations between catchment runoff and rainfall for the simulations to correctly determine the probabilities of dam failure. That means during drought periods with low catchment runoff, rainfall on the reservoir surface is likely to be lower than average. This correlation is determined by plotting the residual of the first estimate (Residual1 = Mean Monthly Rain -

Actual Rain) against historical catchment runoff and fitting a regression equation. The second estimate for rainfall is determined by subtracting the regression equation from the first estimate (monthly averages).

The errors/residuals of the second estimate were found to be random with a Gaussian Distribution that has a mean of zero and a standard deviation of 57.9. During the simulation runs, this random term is added stochastically to the rainfall estimate for each month to incorporate this variation into the simulations runs.

Based on the above information the monthly rainfall estimates can be summarised as:
Rainfall (mm/month) = AveRain (month)
- (-0.00055 * MonthInflow_ML + 13.62175)
+ Random Error (Mean=0, Std. Dev.= 57.9)

If the resulting rainfall estimate is less than zero then it is set to zero.

2.6 Reservoir Surface Evaporation

For each month during the simulation, an evaporation value is generated and multiplied by the reservoir surface area at the start of the month to estimate storage loss for the current month. The method for calculating this was similar to that for rainfall as described above except that the correlation was preserved between rainfall and evaporation.

The monthly evaporation estimates can be summarised as:
Evaporation (mm/month) = AveEvap (month)
+ (0.08433 * rainfall_mm - 6.44591)
+ Random Error (Mean=0, Std. Dev.=16.44)

If the resulting evaporation estimate is less than zero then it is set to zero.

2.7 Reservoir Seepage

For each month during the simulation, a seepage depth is multiplied by the reservoir surface area at the start of the month to estimate seepage loss for the current month. The seepage depth (mm/month) is user definable. 30 mm/month is typically assumed for reservoirs in Queensland. This parameter is generally not that significant to the model results.

2.8 Environmental Flow Releases

Environmental Flow releases for any year in the simulation are calculated based on the generated inflow for that year. 'Historical' Trigger Flows were calculated using daily historical catchment runoff data between 1939 and 2004 and the release rules as defined in the Boyne River Basin Resource Operations Plan. These were plotted against Historical Catchment Runoff. An equation of best fit was found of the form:
ER = (a * b + c * AF ^ d) / (b + AF ^ d)

where: ER = Env. Flow Release (ML/yr), AF = Annual Flow (ML/yr), a = -31201.916, b = 11617.236, c = 402629.39 and d = 0.60928.

The errors/residuals of the equation were found to be random with a Gaussian Distribution that has mean of zero and a standard deviation that increased as the annual flow increased. By trial and error, the standard deviation was found to increase by $1.36668 * AF^{0.75}$. This random term is added to the trigger flow estimate for each month to incorporate this variation into the simulations runs.

2.9 Augmentation Inflow

One of the main reasons that GAWB commissioned the development of AWSIM-O was to be able to examine the effect of different contingency plans to augment the inflow to the dam (e.g. Fitzroy River pipeline or desalination plant). The model has the ability to augment the inflow of the dam from another source at a certain time in the future. This is done by entering the annual volume (ML/yr) of the augmentation and the date it is expected to come online.

2.10 Spill

If at the end of each month, the storage volume is greater than the full storage volume, then the storage volume is set to the full storage volume. The difference is spill.

3. RESULTS

On a regular basis (e.g. monthly) GAWB engineers run the AWSIM-O tool. The user enters the latest water level in the Awoonga Dam, checks the other settings and runs the program. AWSIM-O then conducts the 10,000 replicated model runs each going out 20 years from the current month.

A summary of the typical model results is given as per Table 2. This table gives the probability of failure or the probability of Awoonga Dam reaching Minimum Operating Level at certain dates in the future.

Table 2 A Typical 'Results Summary' Table

Probability Of Failure (%)	Date	Months Ahead
0.1%	Sep/2011	67
1%	Feb/2013	84
2%	Sep/2022	201
5%	-	-
10%	-	-
25%	-	-
50%	-	-

There are also two output graphs from the model run. See Figure 4 for an example 'Sample Of Simulation Run Results' plot. This provides water level traces for the first 250 model runs. It is intended to give the user a feel for the variation in water level traces for different runs. This can be useful when explaining the methodology of the model to non experts (public, accountants, lawyers).

See Figure 5 for an example of the 'Water Level Probability Over Time' plot. This graph shows the probability of the water level in Awoonga Dam being less than various levels into the future.

4. CONCLUSIONS AND DISCUSSION

A probabilistic approach to reservoir reliability is now recognized to be the most rational and effective one (e.g. against deterministic alternatives). The Monte Carlo method is an easily implemented numerical procedure that makes the probabilistic approach feasible and a faithful representation of the real system. The integration of this methodology into a decision support system for reservoir management, as demonstrated in the Awoonga Dam case, provides the user with an easily operated tool. The experience with this case study shows that non experts involved in the decision process (thanks to user friendly environment with visualization options and to the faithful system representation) do not have severe difficulties in operating the program, interpreting its results and finally getting insights and understanding of the natural system.

A key point in the model development in this case study, also met in all similar cases of management of reservoirs performing overyear regulation, is the faithful representation of prolonged droughts such as the current and older droughts in the Awoonga Dam Catchment. Classical statistical approaches based on a temporal independence hypothesis and even typical stochastic models with short range dependence are not able to reproduce such droughts. An effective solution to this, also followed in this study, is to admit the presence of the Hurst phenomenon and to reproduce it with appropriate stochastic techniques.

Some behaviours detected in the Awoonga inflows are very interesting and may trigger additional theoretical analyses, which are out of the scope of the present study but will eventually improve the developed system. One of these is the bimodality of the flow distribution which should be studied in conjunction with the presence of long range dependence. A conditional simulation technique, which would generate future inflows taking account of the past information (measurements) would improve the estimate of inflows in the 0-10 year forecast range. In this respect, the study of the influence of the Interdecadal Pacific Oscillation (IPO) index on inflows could potentially result in another improvement of the same type.

The variability in water demand projections could be incorporated into the Monte Carlo simulations to take them into account in a probabilistic fashion.

It is recommended that a 'financial module' be incorporated into the AWSIM-O Model. This module could calculate the probable cost of water restrictions\failure into the future. This would allow financial comparison between contingency strategies and their timing.

6. ACKNOWLEDGMENTS

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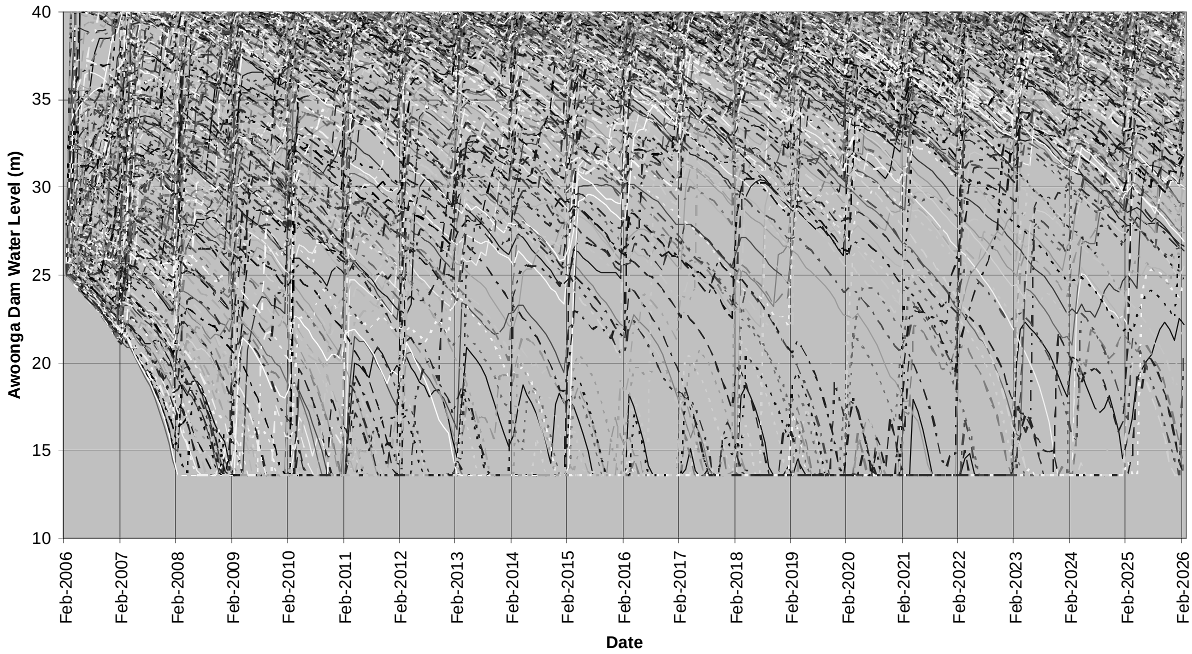

Figure 4: Typical 'sample of simulation run results' spaghetti plot

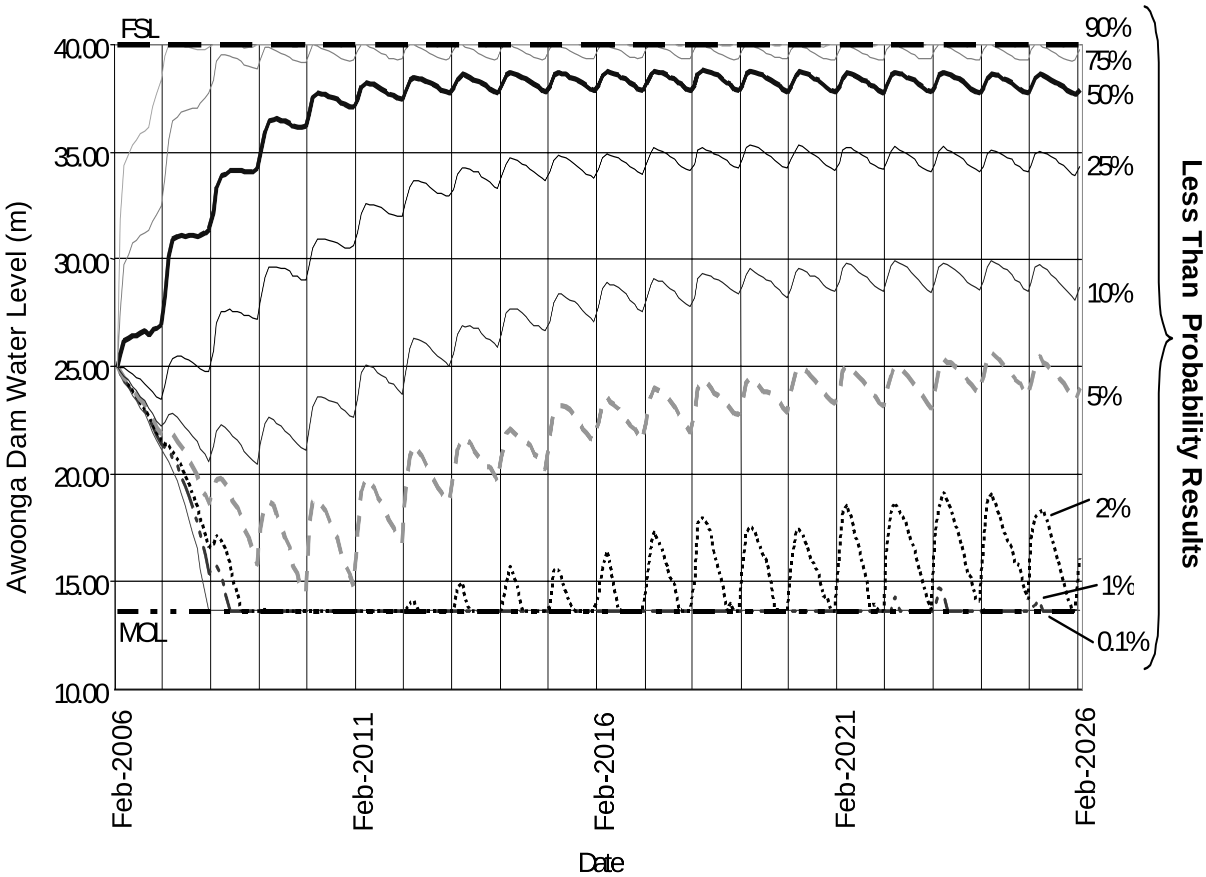

Figure 5: Typical 'water level probability over time' plot