European Geosciences Union General Assembly 2007
Vienna, Austria, 15-20 April 2007
Session NP4.02 Statistical analysis of paleoclimate time series

A stochastic methodological framework for uncertainty assessment of hydroclimatic predictions

D. Koutsoyiannis and Andreas Efstratiadis
Department of Water Resources, National Technical University of Athens, Greece
Konstantine P. Georgakakos
Hydrologic Research Center, San Diego, CA., USA

1. Abstract

In statistical terms, the climatic uncertainty is the result of at least two factors, the climatic variability and the uncertainty of parameter estimation. Uncertainty is typically estimated using classical statistical methodologies that rely on a time independence hypothesis. However, climatic processes are not time independent but, as evidenced from accumulating observations from instrumental and paleoclimatic time series, exhibit long-range dependence, also known as the Hurst phenomenon or scaling behaviour. A methodology comprising analytical and Monte Carlo techniques is developed to determine uncertainty limits for the nontrivial scaling case. It is shown that, under the scaling hypothesis, the uncertainty limits are much wider than in classical statistics. Also, due to time dependence, the uncertainty limits of future are influenced by the available observations of the past. The methodology is tested and verified using a long instrumental meteorological record, the mean annual temperature at Berlin. It is demonstrated that the developed methodology provides reasonable uncertainty estimates whereas classical statistical uncertainty bands are too narrow. Furthermore, the framework is applied with temperature, rainfall and runoff data from a catchment in Greece, for which data exist for about a century. The uncertainty limits are then compared to deterministic projections up to 2050, obtained for several scenarios from several climatic models combined with a hydrological model. Climatic model outputs for rainfall and the resulting runoff do not display significant future changes as the projected time series lie well within uncertainty limits assuming stable climatic conditions along with a scaling behaviour.

2. Rationale and main hypotheses

- Climate is not constant but rather varying in time and expressed by the long-term (e.g. 30-year) time average of a natural process, defined on a fine scale;
- The evolution of climate is represented as a stochastic process; if X_i denotes an atmospheric or land surface variable at the annual scale, then climate is represented by the moving average process, with a typical time window $k = 30$:

$$X_i^{(k)} := (X_i + \dots + X_{i-k+1})/k$$

- The distributional parameters of the process, marginal and dependence, are estimated from an available sample by statistical methods
- The climatic uncertainty is the result of at least two factors, the climatic variability and the uncertainty of parameter estimation (sampling uncertainty)
- A climatic process exhibits scaling behaviour, also known as long-range dependence, multi-scale fluctuation or the Hurst phenomenon (Koutsoyiannis, 2002, 2003, 2005); this has been verified both in long instrumental hydrometeorological (and other) series and proxy data
- Because of the dependence, the uncertainty limits of the future are influenced by the available observations of the past
- Future climate trends suggested by general circulation models (GCM) should be viewed in the framework of uncertainty bands estimated by statistics accounting for scaling behaviour

3. Test case and data sets

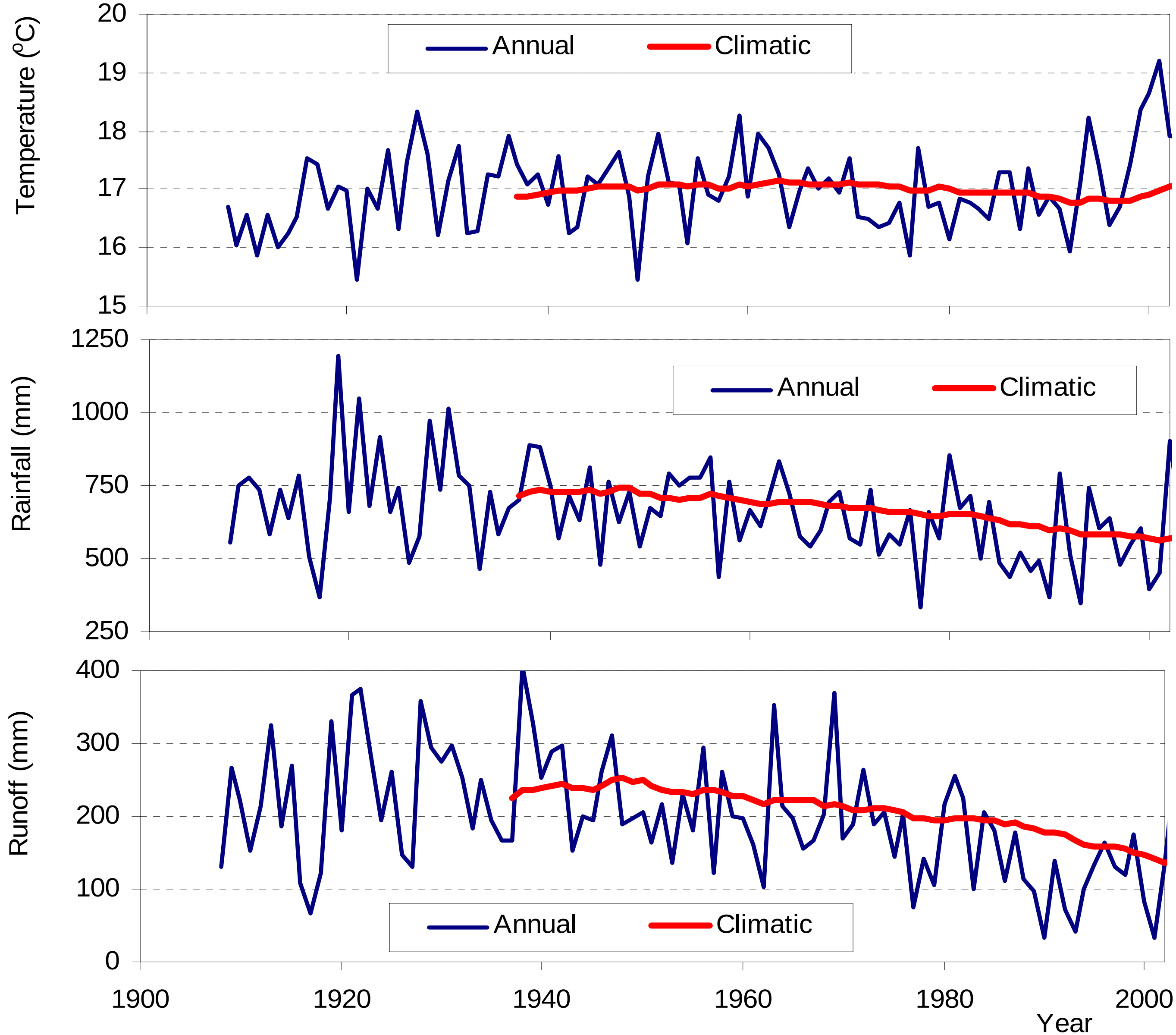
- Boeoticos Kephisos River basin: a closed basin (i.e. without outlet to the sea), with an area of 1955.6 km^2, mostly formed over a karstic subsurface; an important basin in Greece, part of the water supply system of Athens whose history, as regards hydraulic infrastructure and management, extends backward to at least 3500 years
- Data availability extends for about 100 years (the longest data set in Greece) and modelling attempts with good performance have already been done on the hydrosystem (Rozos et al., 2004).
- The relatively long records made possible the identification of the scaling behaviour of rainfall and runoff (Koutsoyiannis, 2003)

Sample statistics of the three long time series of the case study on an annual basis

Sample statistic	Temperature (°C)	Rainfall (mm)	Runoff (mm)
Size, n	96	96	96
Mean, $x_0^{(n)}$	17.0	658.4	197.6
Standard deviation, s	0.72	158.9	87.6
Variation, $C_v = s/m$	0.04	0.24	0.44
Skewness, C_s	0.34	0.44	0.36
Lag-1 autocorrelation, r_1	0.31	0.10	0.34
Hurst coefficient, H	0.72	0.64	0.79

4. Observed behaviour in the test basin

Plots of the annual time series and their 30-year climatic values of Aliartos temperature, Aliartos rainfall and Boeoticos Kephisos annual runoff at Karditsa (see locations in panel 3); notice that the climatic time series are not centred in time (see definition in panel 2)



5. Probabilistic quantification of hydroclimatic uncertainty

- **Parameter** (or **sampling**) **uncertainty**: The estimate of a climatic **parameter** β (e.g. the mean annual rainfall at a certain location) has some uncertainty due to limited observation record $\mathbf{x}_{0,n} = [x_0, \ldots, x_{1-n}]'$; this is defined assuming that $\mathbf{x}_{0,n}$ is realization of a vector of identically distributed random variables $\mathbf{X}_{0,n} = [X_0, \ldots, X_{1-n}]$ and determined in terms of confidence limits for confidence coefficient α:

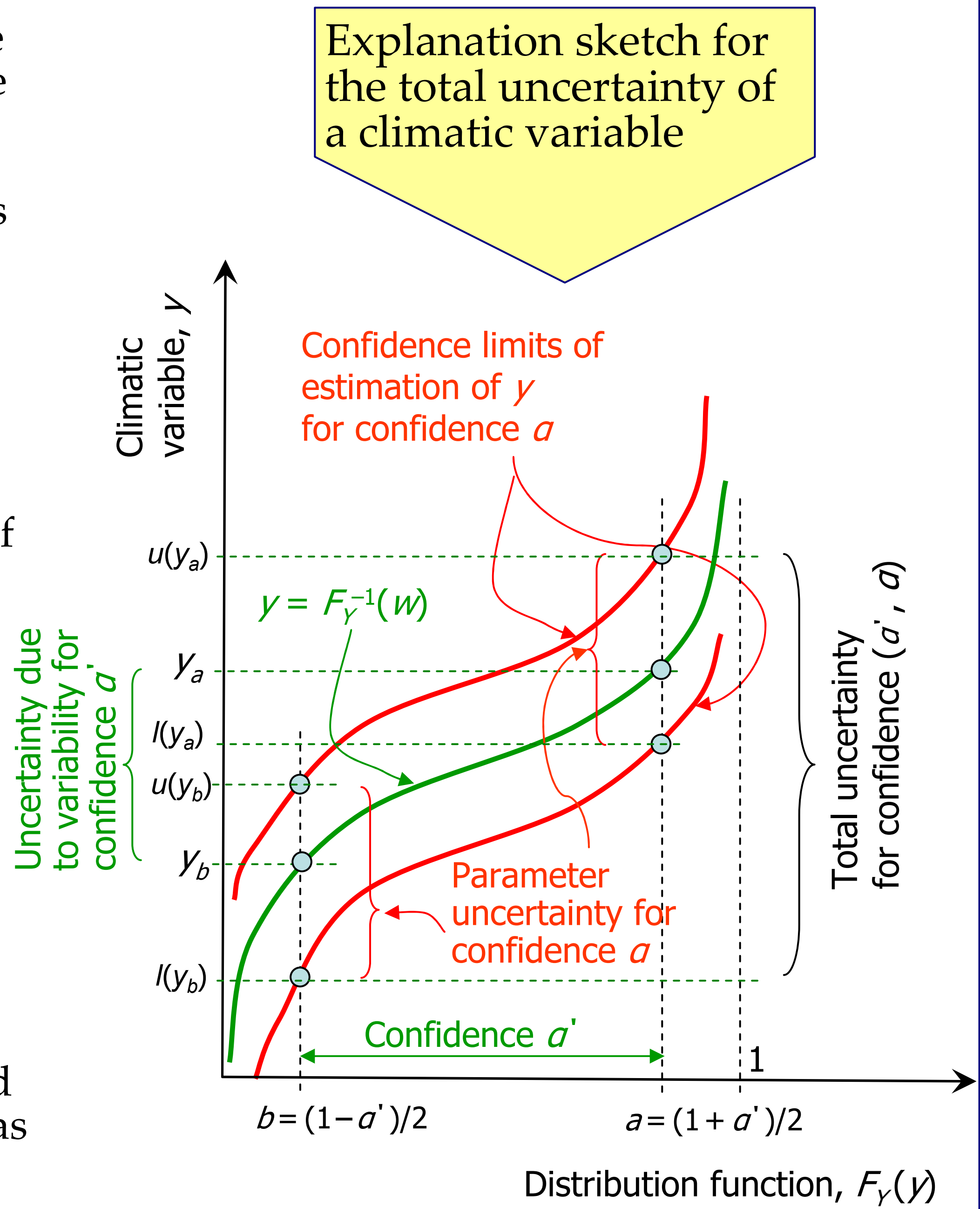
$$P(L \le \beta \le U) = \alpha$$

where L and U are estimators (functions of $\mathbf{X}_{0,n}$) of the lower and upper limits with estimates l and u (functions of $\mathbf{x}_{0,n}$)

- **Uncertainty due to variability:** A climatic **variable** (e.g. the mean annual rainfall at a certain location for a 30-year period) in addition to parameter uncertainty, has also the uncertainty due to (natural) temporal variability; this is determined in terms of distribution quantiles for confidence coefficient α':

$$P\{y_b < X < y_a\} = \alpha'$$

- The quantiles y_b and y_a are parameters and entail parameter uncertainty determined as above



6. The scaling property (the Hurst phenomenon)

- In classical statistics we have the fundamental law

$$\sigma^{(k)} = \sigma / k^{1/2}$$

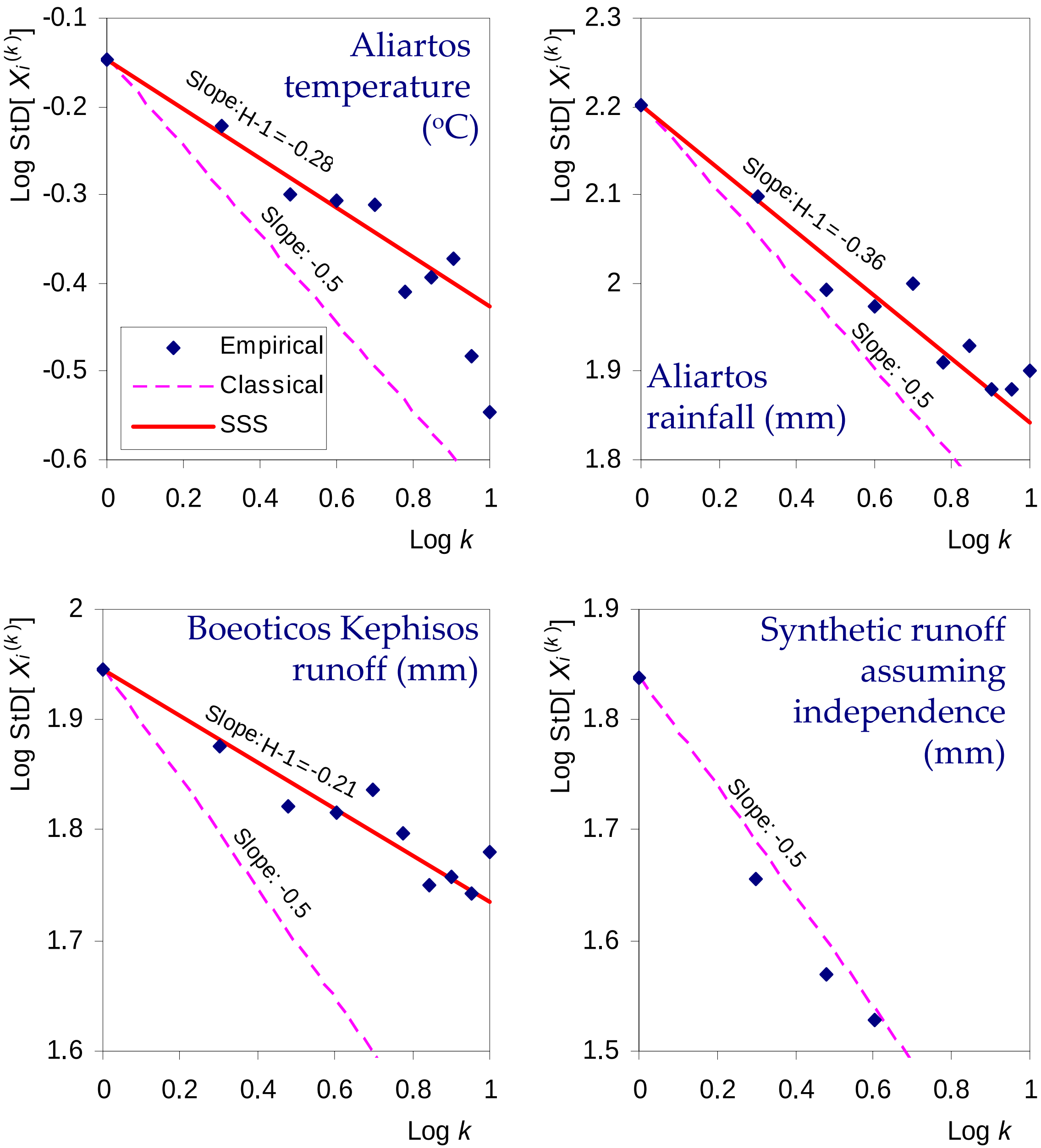
where $\sigma^{(k)} := \mathrm{StD}[X^{(k)}]$ is the standard deviation of the random variable $X^{(k)}$ at scale k and $\sigma \equiv \sigma^{(1)}$ is the standard deviation of each of X_i

- As shown in the figure, the historical time series follow not this law, but the generalized law

$$\sigma^{(k)} = \sigma / k^{1-H}$$

where the constant H is known as the Hurst coefficient

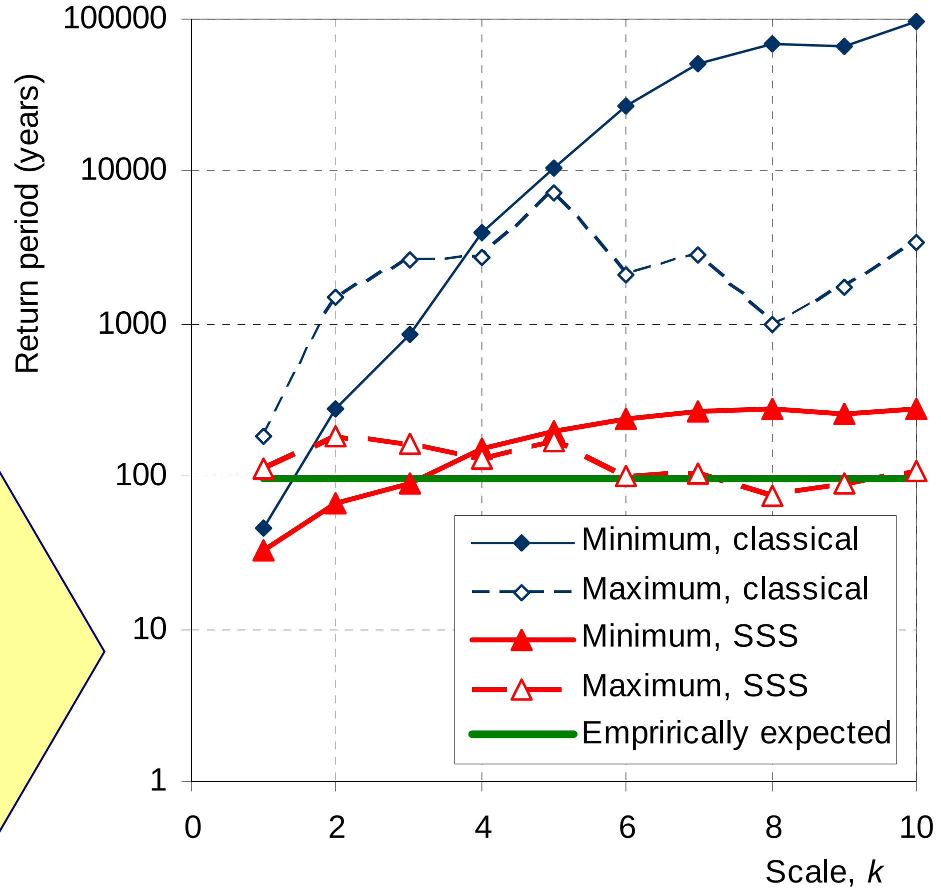
- The latter law:
  - (a) defines the time scaling behaviour (or Hurst behaviour);
  - (b) defines a stochastic process known as simple scaling stochastic (SSS) process (or stationary increment of a self similar process)



7. The importance of the scaling behaviour in typical hydrological tasks

- In the observed 96-year flow record of Boeoticos Kephisos, there are multi-year periods of high flows and low flows (persistent droughts, such as a recent drought that lasted 7 years); this is observed in flow records of other rivers as well
- If such behaviour is modelled with classical statistics, return periods of 10^3-10^5 years are obtained, whereas for the given record length we would expect return periods of the order of 10^2 years
- In contrast, SSS statistics (Koutsoyiannis, 2003) give reasonable results (close to expectation)

Return periods of the minimum and maximum of the average, over scale $k = 1$ to 10 years, runoff of Boeoticos Kephisos from the 96-year runoff record; the return periods were calculated assuming that the distribution is normal for all scales and that the standard deviation over scale k is given by the classical and SSS laws (panel 6), respectively



8. Unconditional uncertainty for the SSS case

- For confidence coefficient α', the distribution quantiles of $X_i^{(k)}$ defining the **uncertainty due to variability** are $y_b^{(k)}$ and $y_a^{(k)}$ where $b = (1 - \alpha')/2$, $a = (1 + \alpha')/2$; for the SSS case assuming normal distribution, these are given as

$$y_b^{(k)} = \mu + \zeta_b \, \sigma / k^{1-H}$$

where ζ_b is the b quantile of the standard normal distribution; a similar expression is obtained for $y_a^{(k)}$

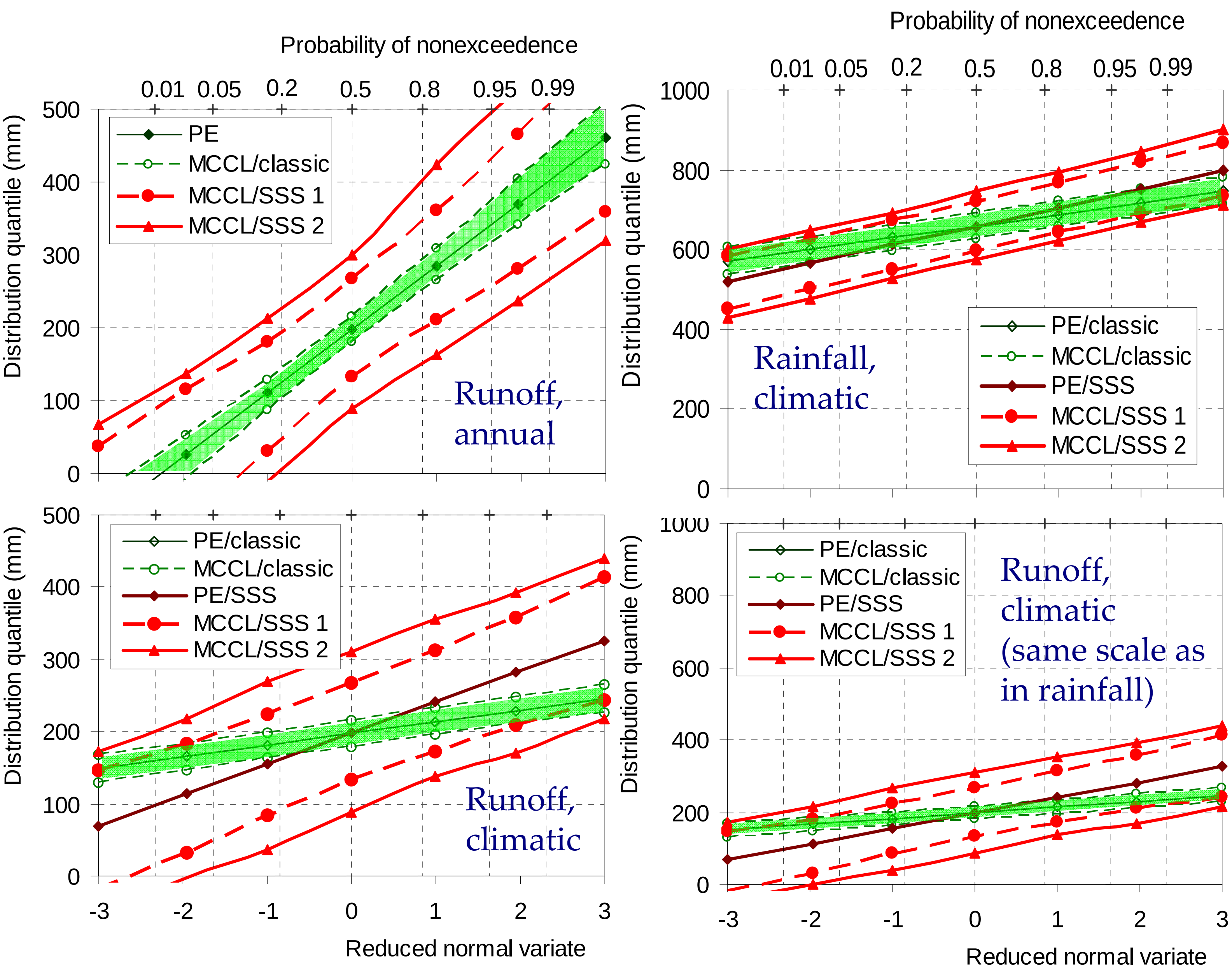
- The **parameter uncertainty** is due to uncertainty in the estimation of μ, σ and H
- Assuming that the true values of σ and H are known without uncertainty (the former being equal to the sample estimate s of standard deviation), the following semi-analytical expressions have been derived for the upper and lower a-confidence limits $u(y_b^{(k)})$ and $l(y_b^{(k)})$ of $y_b^{(k)}$ for the SSS case (Koutsoyiannis, 2003):

$$u(y_b^{(k)}),\, l(y_b^{(k)}) = x_0^{(n)} + \zeta_b \, s / k^{1-H} \pm \zeta_{(1+\alpha)/2} \, \varepsilon_b \, s$$

$$\varepsilon_b = \frac{1}{n^{1-H}} \sqrt{1 + \frac{\varphi(n, H)}{2\, n^{2H-1}} \left(\frac{\zeta_b}{k^{1-H}} \right)^2}, \quad \varphi(n, H) = (0.1\, n + 0.8)^{0.088(4H^2 - 1)^2}$$

- It can be verified that if $H = 0.5$ (independence case), these switch to known expressions in classical statistics; as H grows away from 0.5 the differences from the classical statistics increase drastically
- In the more realistic case that all μ, σ and H are unknown and estimated from the sample, the confidence limits can be estimated numerically by Monte Carlo simulation

9. Demonstration of differences: classical vs. SSS case



Explanation

Climatic values are for **30 years**

PE: point estimate

MCCL/classic: Monte Carlo confidence limits assuming independence as in classical statistics

MCPL/SSS 1: Monte Carlo confidence limits assuming scaling and known *H*

MCPL/SSS 2: Monte Carlo confidence limits assuming scaling and unknown *H*

10. Conditional uncertainty (for the observed past)

- For observed past ($\mathbf{X}_{0,n} = \mathbf{x}_{0,n}$), the parameters whose conditional confidence limits are sought are the distribution quantiles

$$y_{b,i}^{(k)} = E[X_i^{(k)} \mid \mathbf{x}_{0,n}] + \zeta_b \, \mathrm{StD}[X_i^{(k)} \mid \mathbf{x}_{0,n}]$$

and $y_{a,i}^{(k)}$ that is given by a similar expression, where

$$X_i^{(k)} \mid \mathbf{x}_{0,n} = \frac{i}{k} X_i^{(i)} \mid \mathbf{x}_{0,n} + \left(1 - \frac{i}{k}\right) X_{i-k}^{(i-k)} \mid \mathbf{x}_{0,n}, \quad i \geq k$$

$$X_i^{(k)} \mid \mathbf{x}_{0,n} = \frac{i}{k} X_i^{(i)} \mid \mathbf{x}_{0,n} + \left(1 - \frac{i}{k}\right) x_0^{(k-i)}, \quad i \leq k$$

- The final equations for calculating the relevant quantities, derived in Koutsoyiannis et al. (2007) are

$$E[X_i^{(k)} \mid \mathbf{x}_{0,n}] = \left[\frac{i}{k}\phi_{i,n}(H) + \left(1 - \frac{i}{k}\right)\phi_{i-k,n}(H)\right]\mu + \left\{\frac{i}{k}[1-\phi_{i,n}(H)] + \left(1 - \frac{i}{k}\right)[1-\phi_{i-k,n}(H)]\right\} x_0^{(n)}, \quad i \geq k$$

$$E[X_i^{(k)} \mid \mathbf{x}_{0,n}] = \frac{i}{k}\phi_{i,n}(H)\,\mu + \frac{i}{k}[1-\phi_{i,n}(H)]\,x_0^{(n)} + \left(1 - \frac{i}{k}\right) x_0^{(k-i)}, \quad i \leq k$$

$$\mathrm{StD}[X_i^{(k)} \mid \mathbf{x}_{0,n}] = k^{H-1}\,\sigma\sqrt{\psi_{i/k}(H)}, \quad i \geq k \qquad \mathrm{StD}[X_i^{(k)} \mid \mathbf{x}_{0,n}] = \frac{i^H}{k}\,\sigma\sqrt{\psi_1(H)}, \quad i \leq k$$

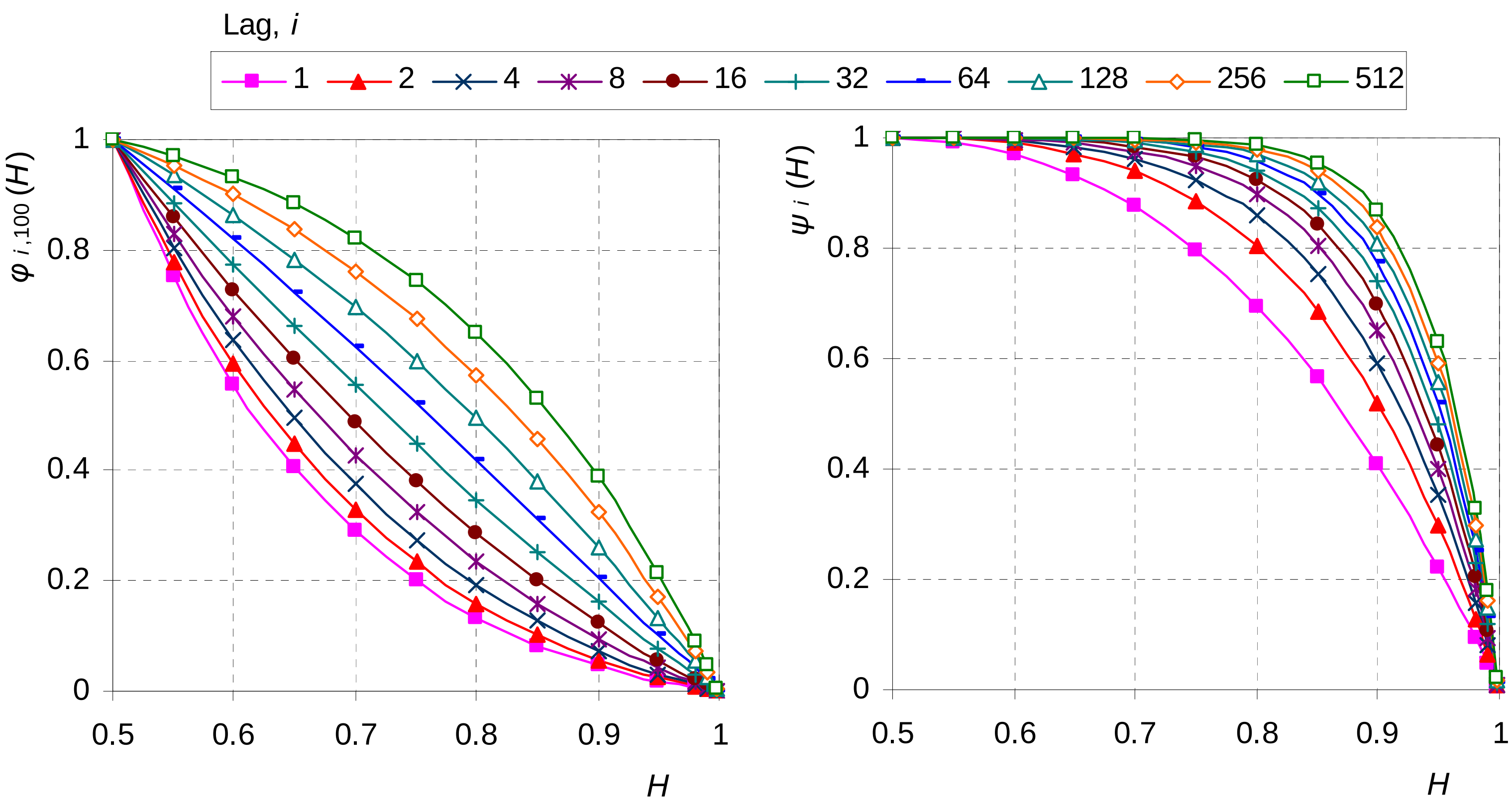
where $\phi_{i,n}(H)$ is a function of the lag i, the sample size n and the Hurst coefficient H, whereas $\psi_i(H)$ is a function of the lag i and the Hurst coefficient H; both are defined in Koutsoyiannis et al. (2007) and demonstrated in panel 11

11. Demonstration of quantities involved in the estimation of conditional climatic confidence limits

- The function $\phi_{i,n}(H)$ was evaluated for sample size $n = 100$ (rounding off 96, which is the historical sample size)
- Both $\phi_{i,100}(H)$ and $\psi_i(H)$ were evaluated numerically for different values of i and H and then approximate analytical expressions were established, which are

$$\varphi_{i,100}(H) = 1 - (2H - 1)^{c_1}\,[1 - c_2\,(1 - H)]),\ c_1 = 0.75 + 0.1 \ln i,\ c_2 = 2 - 3.3 \exp[-(0.18 \ln i)^{3.7}]$$

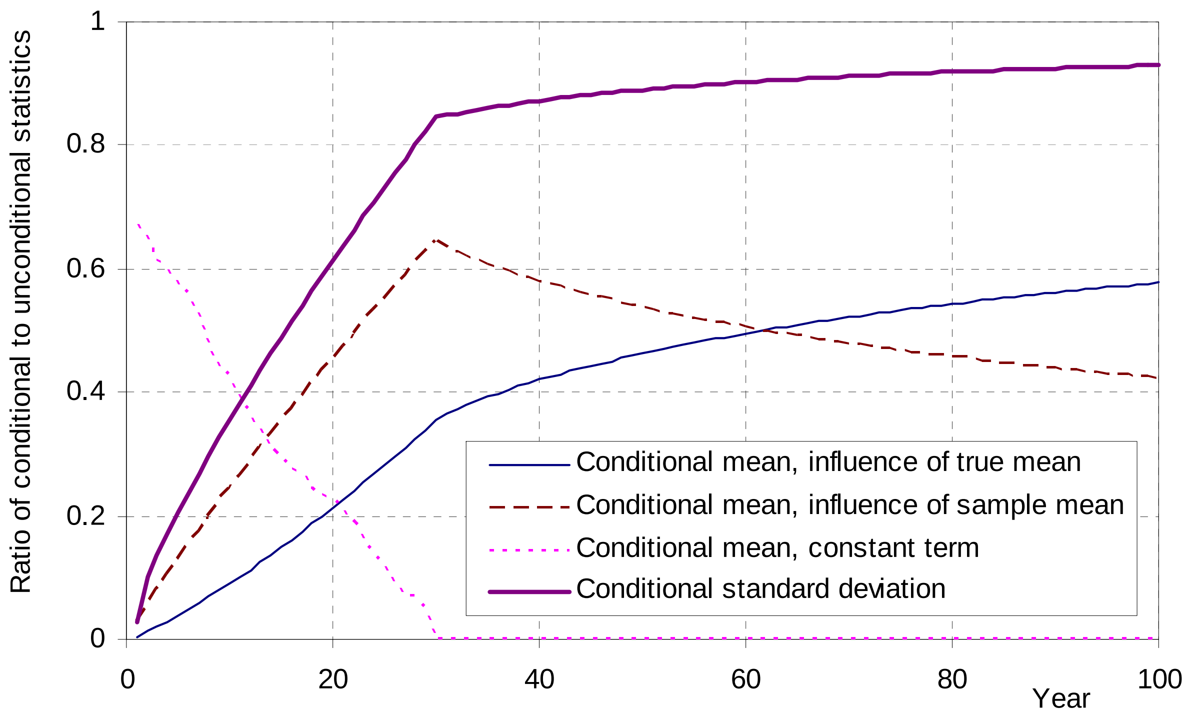
$$\psi_i(H) = 1 - (2H - 1)^{2 + \ln i}\,[1 - (2 - 1.28 / i^{0.25})\,(1 - H)]$$



12. Variation of conditional statistics with lead time

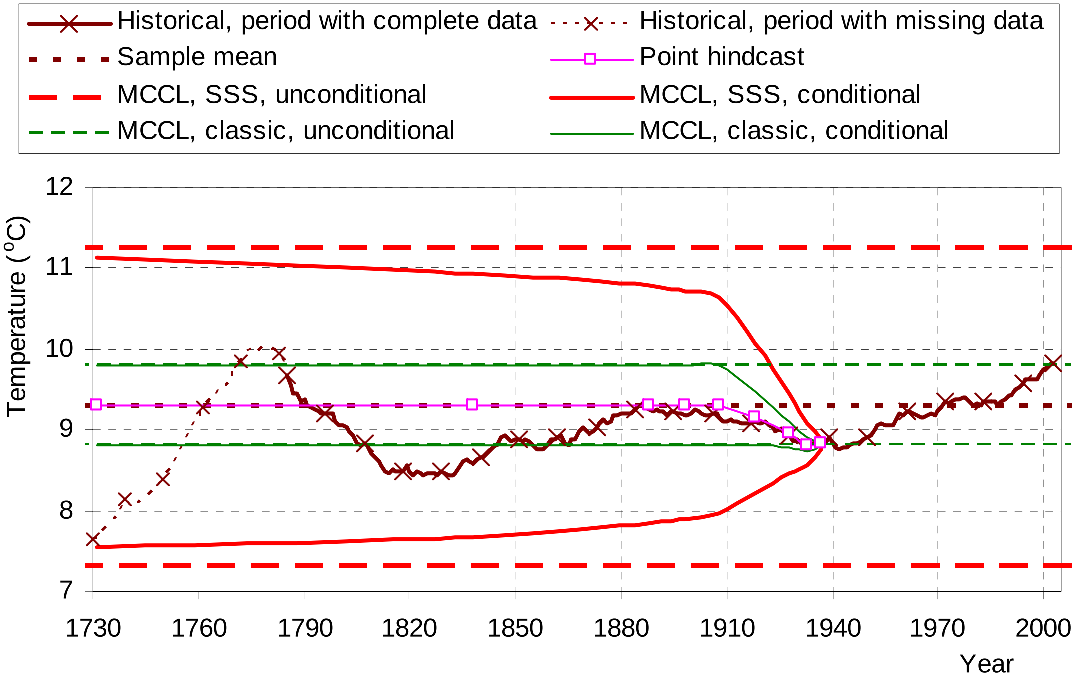
- In the calculation of the conditional mean, three terms are involved (panel 10)
  - Term 1 = coefficient of the true mean μ: it is an increasing function of the lead time i
  - Term 2 = coefficient of the sample mean $x_0^{(n)}$: it is an increasing function of the lead time i up to $i = k = 30$ and then becomes a decreasing function
  - Term 3 = the last (constant) term in the second equation in panel 10: it is a decreasing function of i and vanishes off at $i = k = 30$
- Even for lead time as high as 100, the influence of the **known** sample average is significant and the influence of the **unknown** true mean is smaller than 60%; this has an attenuating effect to the width of the confidence band
- The conditional standard deviation increases quickly with lead time, reaching 85% at $i = k = 30$; then it increases slowly and becomes 93% at a lead time 100

Ratios of conditional mean (each of its three additive components) and standard deviation to the unconditional quantities; the true parameters were assumed equal to their sample estimates



13. Method validation using a longer data set

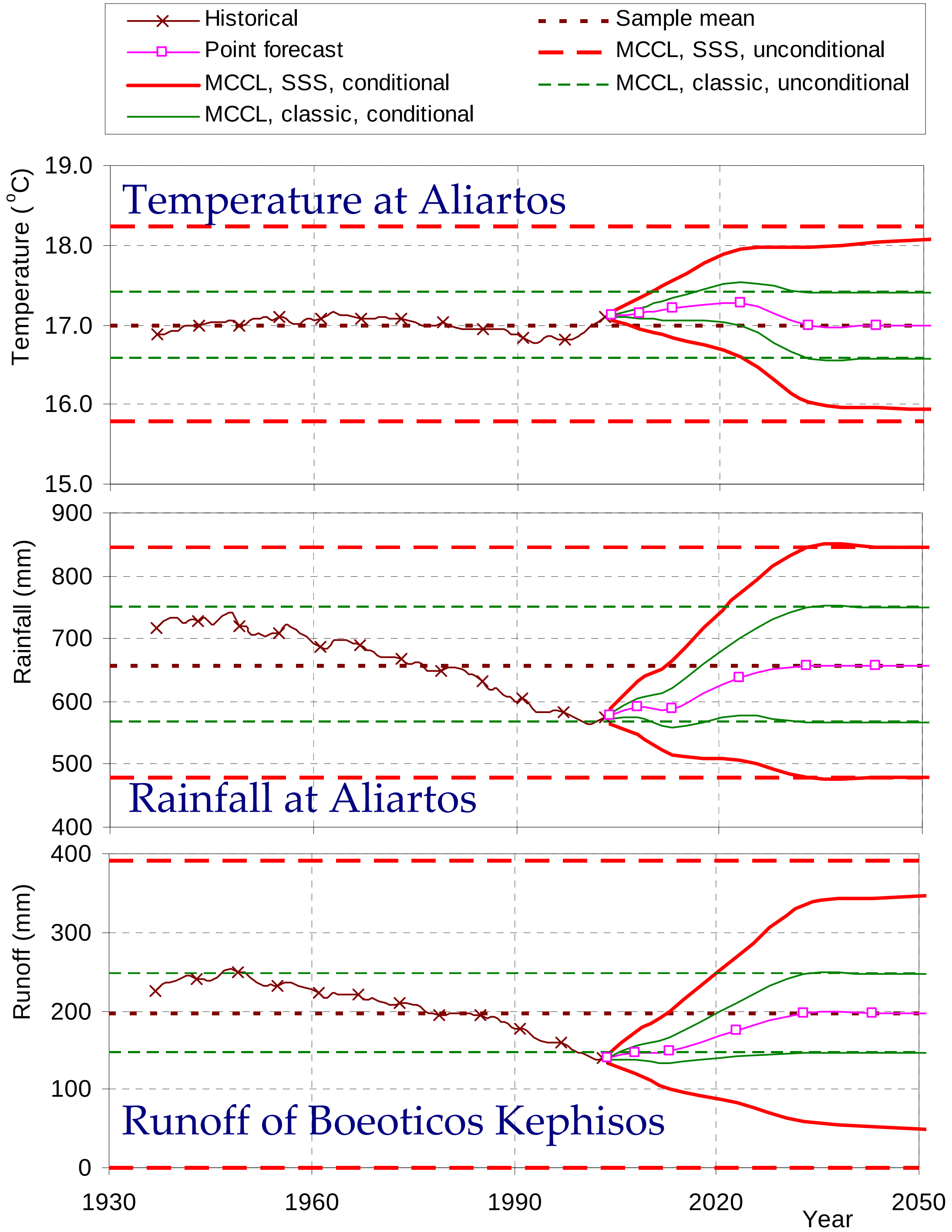
- Validation of the proposed method is done against the potential arguments that:
  - (a) the 20th century data used in this study are already affected by anthropogenic influences,
  - (b) the natural variability would be less than observed in the 20th century, and
  - (c) as a consequence, the uncertainty limits estimated by the proposed method are artificial (not representing the natural variability) and too wide
- To put light to these arguments, a longer data set, not related to the case study, was used: the mean annual temperature record of Berlin/Tempelhof, one of the longest instrumental meteorological records going back to 1701 (with missing data before 1756)
- The data of the period 1908-2003 (as in the Boeoticos Kephisos case) were used to estimate the parameters of the scaling model ($H = 0.78$) and then climatic hindcasts were calculated in terms of conditional point estimates and confidence bands (for $\alpha = \alpha' = 95\%$ and climatic time scale of 30 years)
- This was done for both the scaling and the classical statistical model; the non used part of the series was compared with the confidence limits



14. Case study: Resulting conditional SSS confidence limits and comparison to classical statistical estimates

Historical climate and (conditional) point estimates and confidence limits of future climate (for $\alpha = \alpha' =$ 95% and climatic time scale of 30 years)

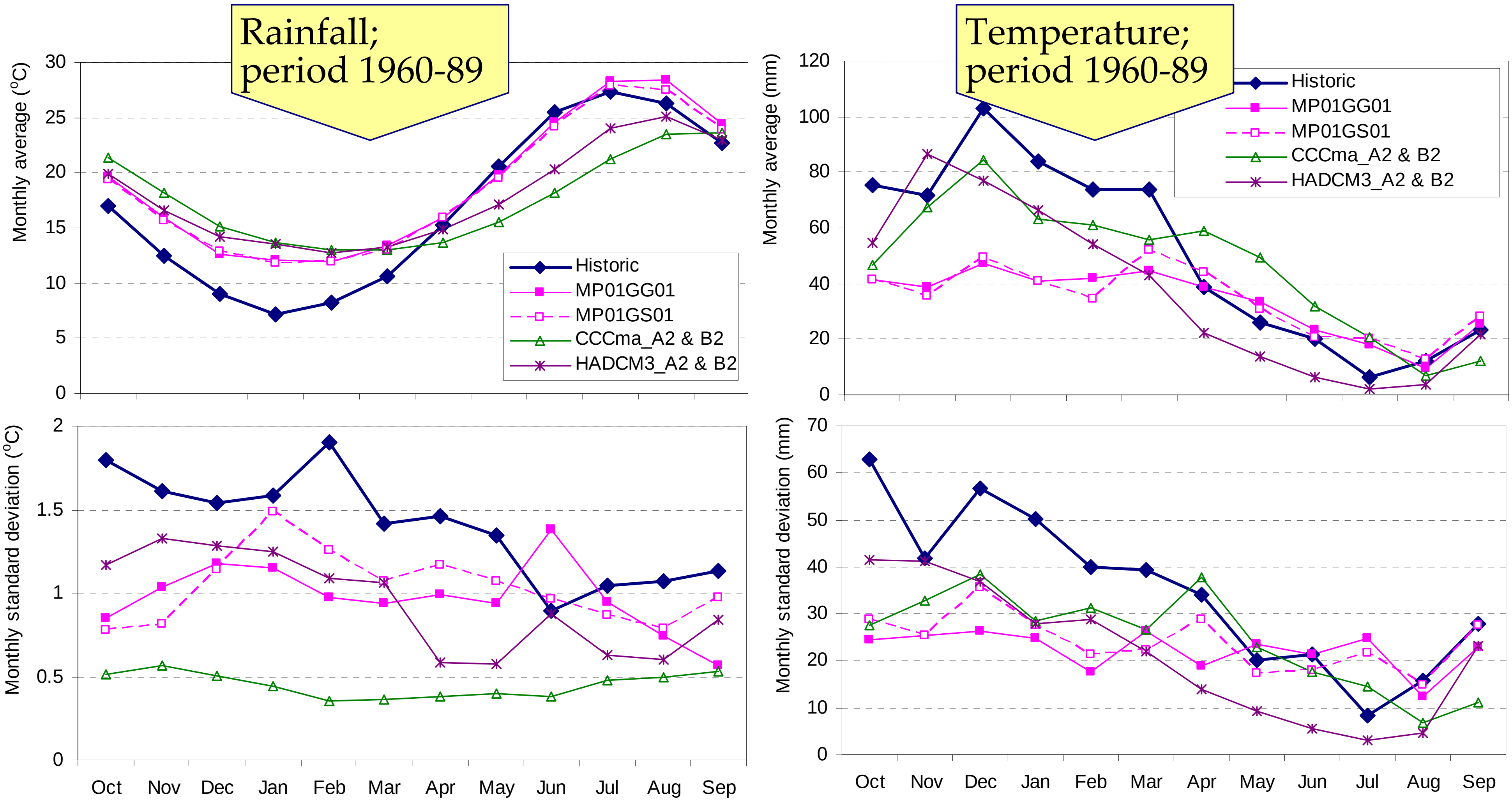
Comparison shows that SSS confidence bands are 2.5-3 times wider than the classical statistical bands



15. Scenario-based analysis of uncertainty: IPCC scenarios, models and data sets

- **IPCC scenarios for future climatic projections by GCM**
  - SRES A2: high population growth (15.1 billion in 2100), high energy and carbon intensity, and correspondingly high CO_2 emissions (concentration 834 cm^3/m^3 in 2100)
  - SRES B2: lower population (10.4 billion in 2100), energy system predominantly hydrocarbon-based but with reduction in carbon intensity (CO_2 concentration 601 cm^3/m^3 in 2100)
  - IS92a (older): in between the above two (population 11.3 and CO_2 concentration 708 cm^3/m^3 in 2100)
- **GCM coupled atmosphere-ocean global models**
  - ECHAM4/OPYC3: developed in co-operation between the Max-Planck-Institute for Meteorology and Deutsches Klimarechenzentrum in Hamburg, Germany; mean resolution 2.81° both in latitude and longitude (a total of 64 latitudes × 128 longitudes) [M1 in panel 3]
  - CGCM2: developed at the Canadian Centre for Climate Modeling and Analysis; resolution 3.75° both in latitude and longitude (a total of 48 latitudes × 96 longitudes) [M2 in panel 3]
  - HADCM3: developed at the Hadley Centre for Climate Prediction and Research; resolution 2.5° in latitude and 3.75° in longitude (a total of 73 latitudes × 96 longitudes) [M3 in panel 3]
- **GCM outputs** (from http://ipcc-ddc.cru.uea.ac.uk/ddc_gcmdata.html)
  - MP01GG01: output of ECHAM4/OPYC3 with historical inputs for 1860-1989 and inputs from IS92a beyond 1990
  - MP01GS01: same as in 1 but also considering the sulphate concentration
  - CCCma_A2: output of CGCM2 with historical inputs for 1900-1989 and inputs from scenario A2 beyond 1990
  - CCCma_B2: same as in 3 but for scenario B2 beyond 1990
  - HADCM3_A2: output of CGCM2 with historical inputs for 1950-1989 and inputs from scenario A2 beyond 1990
  - HADCM3_ B2: same as in 5 but for scenario B2 beyond 1990

16. Comparison of GCM outputs with historical data

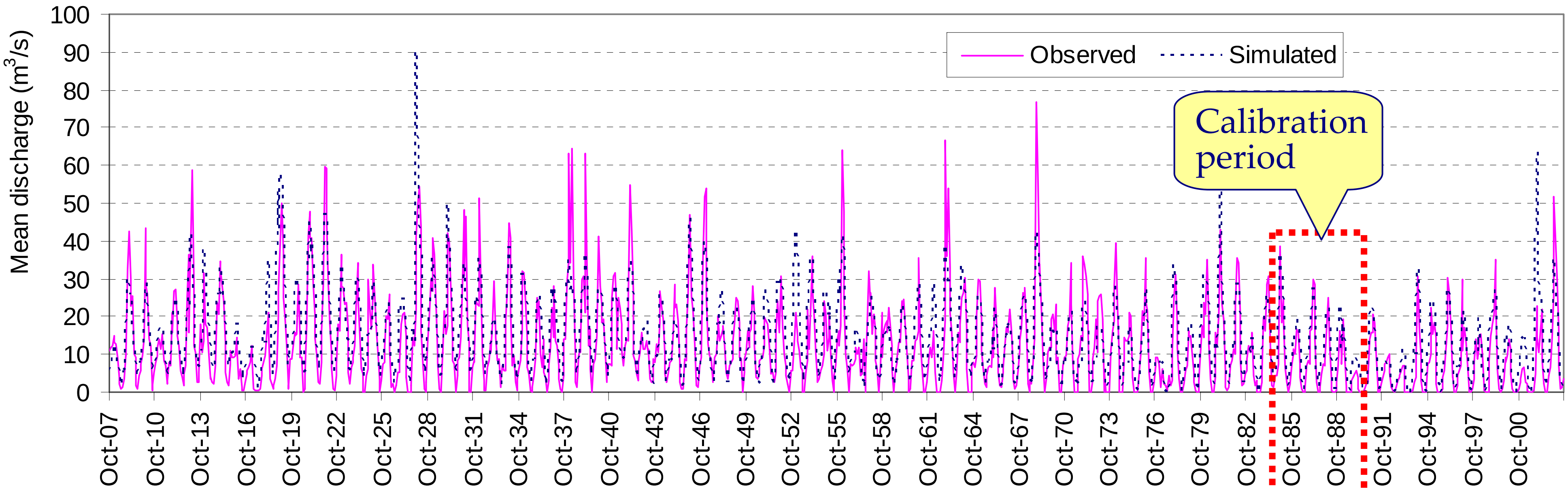


Remarks: Positive bias in temperature and negative bias in the rainfall; underdispersion both in temperature and rainfall, on sub-annual, annual and over-annual scales

Rectification (except for over-annual scale): Rescaling of GCM output time series on monthly basis so as to match historical means and standard deviations of period 1960-89

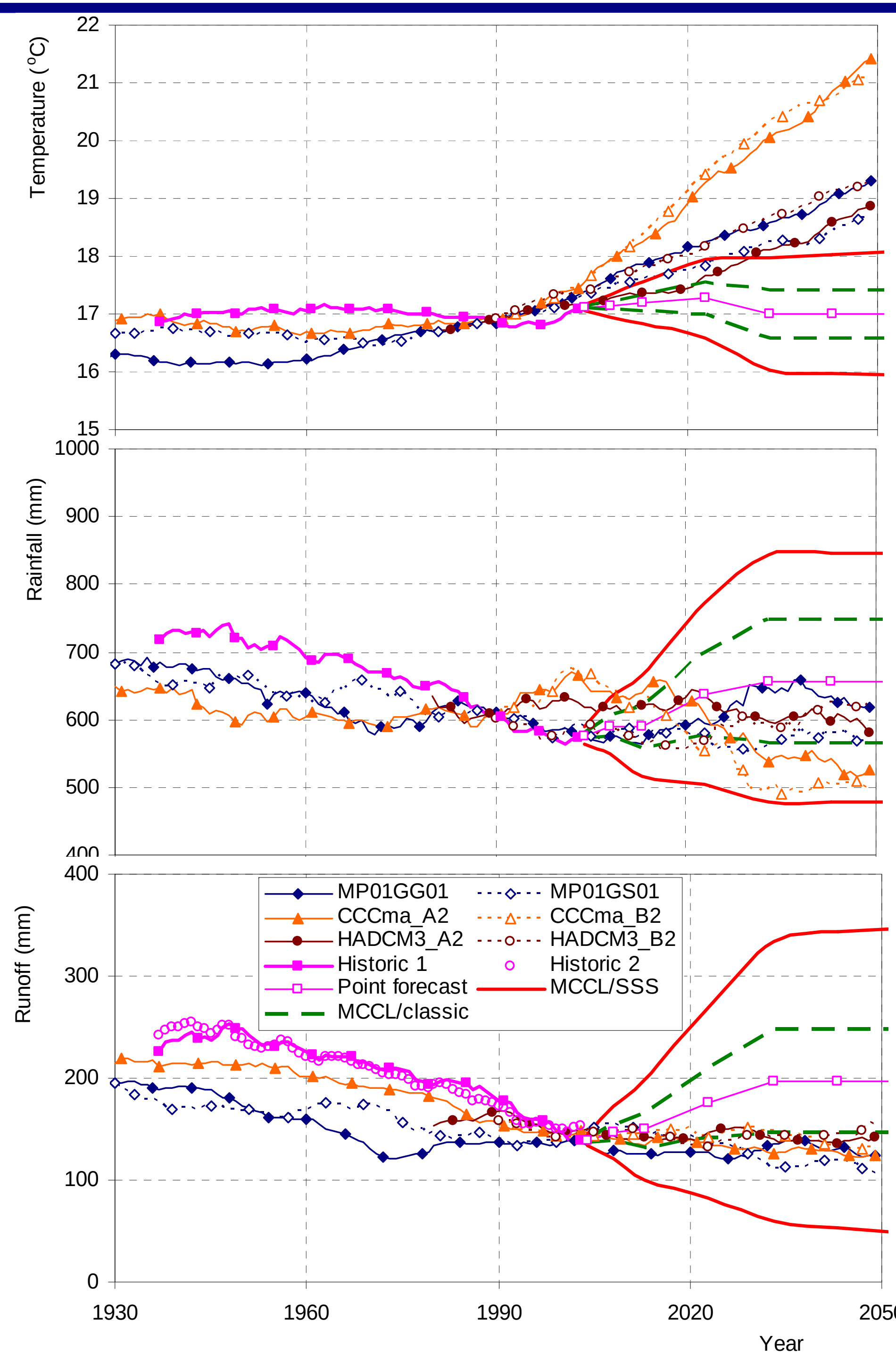
17. Generation of runoff from GCM outputs

- The hydrological response that is used in this climatic study is runoff but, in order to estimate it, the entire water cycle had to be simulated
- Given the great extend of karst, both surface and ground water processes had to be modelled simultaneously
- Given that the basin is not in natural condition, the model had to take into account both natural processes and anthropogenic influences on the catchment
- All these requirements were handled with an advanced hydrological modelling scheme, called Hydrogeios (Efstratiadis et al., 2005)
- As shown in the diagram, the model performance in comparison to historical data is excellent, even though its calibration was done on a very small (6-year) period with detailed data (Efstratiadis, 2006)



18. Comparison of GCM projections with climatic confidence limits

- All GCM projections agree that sooner or later the temperature will depart from the natural uncertainty limits
- In contrast, GCM rainfall and the resulting runoff fall within the SSS uncertainty limits for the whole examined period up to 2049; this means that traces such as the ones projected by the GCM can be readily obtained by stochastic simulation assuming stationarity
- This more or less harmonizes with some earlier studies (e.g. a comprehensive climatic study for United States by Georgakakos and Smith, 2001)
- The proposed stochastic method suggests that the uncertainty of runoff, even assuming natural variability, is in fact much larger than projected by GCM



19. Conclusions

- Classical statistics, applied to climatology and hydrology, describes only a portion of natural uncertainty and underestimates seriously the risk if long-range dependence is present
- Simple scaling stochastic (SSS) processes offer a sound basis to adapt hydro-climatic statistics so as to capture interannual variability
- The uncertainty bands obtained from the SSS framework are significantly wider (about 3 times) than those obtained by classical statistics
- The detailed case study involving three important hydrometeorological processes (temperature, rainfall and runoff) in a catchment in Greece and elsewhere (mean annual temperature at Berlin) provides evidence that the SSS, rather than the classical, uncertainty bands are compatible to reality
- To capture anthropogenic climate changes, climatic model outputs should be incorporated in an uncertainty analysis and it can be anticipated that future uncertainty is even greater than produced by the SSS framework
- However, for the known past, GCM do not capture climatic variability, i.e. they result in monthly, annual and over-annual variability that is too weak; obviously, this raises questions for their performance in predicting future climate variability
- Even for the future, GCM predict too small changes in hydroclimatic conditions (except for temperature that is predicted to increase significantly) in comparison to SSS uncertainty bands under a stationarity assumption
- Thus, it can be recommended that SSS uncertainty bands offer a safer and more reasonable basis for planning and management in comparison to GCM projections and classical statistics

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