A power-law approximation of the turbulent flow friction factor useful for the design and simulation of urban water networks

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Abstract. An approximation of the friction factor of the Colebrook-White equation is proposed, which is expressed as a power-law function of the pipe diameter and the energy gradient and is combined with the Darcy-Weisbach equation, thus yielding an overall power-law equation for turbulent pressurized pipe flow. This is a generalized Manning equation, whose exponents are not unique but depend on the pipe roughness. The parameters of this equation are determined by minimizing the approximation error and are given either in tabulated form or as mathematical expressions of roughness. The maximum approximation errors are much smaller than other errors resulting from uncertainty and misspecification of design and simulation quantities and also much smaller than the errors in the original Manning and the Hazen-Willians equations. Both these can be obtained as special cases of the proposed generalized equation by setting the exponent parameters constant. However, for large roughness the original Manning equation improves in performance and becomes practically equivalent with the proposed generalized equation. Thus its use, particularly when the networks operate with free surface flow is absolutely justified. In pressurized conditions the proposed generalized Manning equation can be a valid alternative to the combination of the Colebrook-White and Darcy-Weisbach equations, having the advantage of simplicity and speed of calculation both in manual and computer mode.

Keywords*:* urban water networks; pipe flow; friction factor; Darcy-Weisbach equation; Colebrook-White equation; Manning equation; power law.

1. Introduction

Turbulent flow in circular cross section pipes is of great practical interest, particularly in the design and simulation of urban water pipe networks. The Darcy-Weisbach equation (named after two great engineers of the 19th century) is by far the most widely used in flow calculations. Its use is accompanied with the calculation of the friction factor by the Colebrook–White equation (see section 2). The latter equation (Colebrook and White, 1937, Colebrook, 1939) is an implicit function that needs iterations to solve. Thus, soon after its appearance, it was regarded too complex to be of practical use (Rouse, 1943, from Brown, 2004). Evidently, the specific form of the formula was dictated by the need to better fit laboratory results rather than to make it convenient for engineering application. The earliest remediation of this disharmony of the origin and the target of the equation was provided by its graphical depiction, the famous Moody diagram (Moody, 1944).

With the advancement of computers, the use of the Moody diagram receded, without being totally abandoned. Still the diagram enjoys a good place in water engineering textbooks and handbooks (e.g. Mays, 1996, 2001; Butler and Davies, 2000) as well as in other engineering fields (e.g. ASHRAE, 2001). On the other hand, computer based approaches gain ground. In addition to the computational implementation of the Colebrook–White equation, which requires a few repetitions, several explicit approximations have been proposed (Moody, 1947; Wood, 1966; Jain, 1976; Chen, 1979; Churchill, 1977; Round, 1980; Barr, 1981; Zigrang and Sylvester, 1982; Haaland, 1983; Manadili, 1997). An excellent review and comparison of all approximations has been compiled by Romeo et al. (2002) who also provided another explicit approximate formula. The most recent approximation of this type has been proposed by Sonnad and Goudar (2006).

All of the above approximations aimed at converting the implicit friction factor formula to an explicit one and used the same reference variables (the Reynolds number and relative roughness). They differ in terms of the level of accuracy depending upon the complexity of their functional forms. The more complex ones usually provide estimates of high accuracy, while the simpler ones can result in maximum absolute error that exceeds 15% (Zigrang and

Sylvester, 1982). It can be argued that the contribution of all these approximations in practice is not very significant. Typically, the friction factor is not the final desideratum but an intermediate result for calculation of quantities such as velocity, discharge, diameter or energy gradient. In most of these calculations, substitution of an explicit formula of the friction factor results in a composite formula that may be again implicit in terms of final desiderata. Besides, the implicit setting of the original Colebrook-White formula is not a serious problem because of its quick convergence. For instance, the approximation by Zigrang and Sylvester (1982) is none other than the writing down in analytical terms of two or three iterations of the original Colebrook-White formula with an appropriately chosen initial value. From a programming point of view, this is more complicated that writing a small algorithmic loop.

Thus, despite the large number of approximate equations, the problem of simplifying the calculations still remains. A drastic simplification is the use of either the Manning or the Hazen–Williams equations. Both preceded the Colebrook-White equation: The Manning equation was introduced in 1867 by Philippe Gauckler and was validated by experimental data in 1887 by Robert Manning (Levi, 1995), whereas the Hazen–Williams equation was introduced in 1902 (Liou, 1998). Both have the advantage of providing convenient power-law correlations of all design quantities, easily solvable for each one. However, the accuracy of these equations can be low, as will be discussed later (see also Liou, 1998). Another alternative is provided by nomographs of charts that correlate all design quantities. Such charts, which evidently manifest power-law relationships, are either provided by pipe manufacturers or contained in engineering textbooks and handbooks (e.g. Butler and Davies, 2000; ASHRAE, 2001). However, its use has several weaknesses. They are rarely supported by methodological descriptions and documentation of their assumptions and derivation and thus one may have difficulty to trust them. They may not be representative for a spectrum of conditions or pipe materials; they must be too many of them to form a representative collection for different conditions and this negates their target to be convenient tools. Also, they do not comply with the need to eliminate the manual use of graphs.

In this study we propose a different simplification of the Colebrook-White formula, in a manner that, when combined with the Darcy-Weisbach equation, it enables the expression of the friction factor in terms of the final design quantities rather than intermediate ones. Theoretically, this could be done at no cost in terms of accuracy. However, as our target is to provide a convenient approximation for design and simulation purposes, we preferred to express our approximation in a simple power-law relationship, which, combined with the Darcy-Weisbach equation, yields an overall power-law equation. This is a generalized Manning equation, whose exponents are not unique but depend on the pipe roughness. This generalized equation resembles the original Manning and the Hazen-Williams equations (in fact both are obtained as special cases of the generalized equation) but it is much more accurate (more than five times) than them. Furthermore, the maximum relative error in approximating the Colebrook-White friction factor can be smaller than the simplest approximations of it discussed above.

It can be argued that small approximation errors can be accepted in practical problems, particularly if these are smaller than other unavoidable errors involved in calculations. Even the Colebrook-White formula and the Moody diagram are not fully correct for all conditions (Rouse, 1943) but perhaps accurate only to 15% (White, 1994; Brown, 2004). Furthermore, in calculations of this type there is substantial uncertainty, both in basic quantities such as the pipe roughness (which is difficult to define) and design quantities such as the design discharge (see section 5).

2. Rationale

The Darcy-Weisbach equation for turbulent flow in circular cross section pipes correlates the energy gradient J with the pipe diameter D and the average velocity V:

$$J = f \frac{1}{D} \frac{V^2}{2\mathrm{g}} \tag{1}$$

where g is the gravity acceleration (so that $V^2/2\mathrm{g}$ is the kinetic energy head) and f is the (dimensionless) friction factor. The latter is given by the Colebrook–White equation:

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{2}$$

where Re $:= VD/\nu$ is the Reynolds number and ε/D is the relative roughness, both dimensionless quantities, whereas ε is the absolute (surface) roughness of the specific pipe and ν is the kinematic viscosity. A third basic equation involved in pipe flow calculations is the one relating the discharge Q with the velocity:

$$Q = \frac{\pi D^2}{4} V \tag{3}$$

It is easily seen that the term $\mathrm{Re}\sqrt{f}$ that appears in the right-hand side of (2) can be calculated by solving (1) for f and then substituting it into the quantity $\mathrm{Re}\sqrt{f}$. The result is:

$$\mathrm{Re}\sqrt{f} = \frac{2^{1/2}\, g^{1/2}\, J^{1/2}\, D^{3/2}}{\nu} \tag{4}$$

Setting $D = \varepsilon\,/\,(\varepsilon/D)$ in (4) and then substituting the result into (2) we obtain

$$\frac{1}{\sqrt{f}} = -2\,\log_{10}\!\left(\frac{\varepsilon/D}{3.7} + \frac{1.775\,\nu}{g^{1/2}\,\varepsilon^{3/2}}\frac{(\varepsilon/D)^{3/2}}{J^{1/2}}\right) \tag{5}$$

Now we define a normalized roughness

$$\varepsilon_* := \varepsilon/\varepsilon_0, \text{ where } \varepsilon_0 := \left(\frac{\nu^2}{g}\right)^{1/3} \tag{6}$$

and we observe that ε_* is a dimensionless quantity always known in all practical problems (because g and ν are constants in any design or simulation and ε is also known given the pipe material and general technical conditions). The characteristic parameter ε_0 has units of length and assuming a standard value $\nu = 1.1 \times 10^{-6}$ m^2/s, it is easily seen that $\varepsilon_0 = 0.05$ mm.

By virtue of (6), equation (5) can be written as

$$\frac{1}{\sqrt{f}} = -2\,\log_{10}\!\left(\frac{\varepsilon/D}{3.7} + \frac{1.775}{\varepsilon_*^{3/2}}\frac{(\varepsilon/D)^{3/2}}{J^{1/2}}\right) \tag{7}$$

which shows that f is a function of the normalized roughness ε_*, the relative roughness ε/D and the energy gradient J, all dimensionless quantities. Because ε_* is always given in any practical problem, we seek a function f_* of ε/D and J determined for this specific ε_*:

$$f = f_*(\varepsilon/D, J) \tag{8}$$

For reasons stated in the Introduction, we abandon a requirement for perfect accuracy and seek an approximation by a power law:

$$f \approx \alpha \frac{(\varepsilon_0/D)^\beta}{J^\gamma} \tag{9}$$

where α, β and γ are coefficients depended on ε_*, i.e., $\alpha = \alpha(\varepsilon_*)$, $\beta = \beta(\varepsilon_*)$ and $\gamma = \gamma(\varepsilon_*)$. To avoid an infinite α when ε tends to zero we have formulated (9) in terms of $(\varepsilon_0/D) = (\varepsilon/D)/\varepsilon_*$. Had we used (ε/D) in lieu of (ε_0/D), $\alpha(0)$ (the value of α for $\varepsilon = \varepsilon_* = 0$) would be infinite (because f has a finite non zero value for $\varepsilon = \varepsilon_* = 0$). Thus, with the particular formulation used in (9) we avoid infinity problems and keep our expressions more convenient without loss of generality and consistency.

3. Formulation

The power-law approximation is perfectly convenient, because it results in simple power-law equations correlating all design quantities J, D, V and Q. Such equations can be written in a generalized Manning form

$$V = (1/N)\, R^{(1+\beta)/2}\, J^{(1+\gamma)/2} \tag{10}$$

where $R = D/4$ is the hydraulic radius and N is a generalized Manning coefficient:

$$N := \frac{\varepsilon_0^{\beta/2}}{2^{3/2+\beta}\, g^{1/2}}\, \alpha^{1/2} \tag{11}$$

Note that (10) and (11) are dimensionally homogeneous. For convenient reference, various forms of the power laws resulting from (10) and (11) and also involving Q are given in Box 1.

Box 1 Basic forms of the generalized Manning equation and its parameters, useful for application.

Definitions of the characteristic roughness ε_0, the normalized roughness ε_*, and the generalized Manning coefficient N:

$$\varepsilon_0 := \left(\frac{\nu^2}{g}\right)^{1/3} = 0.00005 \text{ m}, \;\; \varepsilon_* := \varepsilon/\varepsilon_0, \;\; N := \frac{\varepsilon_0^{\beta/2}}{2^{3/2+\beta}\, g^{1/2}}\, \alpha^{1/2} \qquad \text{(B1.1)}$$

Relationships among energy gradient J, velocity V and diameter D:

$$J = \left(\frac{4^{1+\beta} N^2 V^2}{D^{1+\beta}}\right)^{\frac{1}{1+\gamma}}, \;\; D = 4\left(\frac{N^2 V^2}{J^{1+\gamma}}\right)^{\frac{1}{1+\beta}}, \;\; V = \frac{1}{2^{1+\beta} N} D^{(1+\beta)/2} J^{(1+\gamma)/2} \qquad \text{(B1.2)}$$

Relationships among energy gradient J, discharge Q and diameter D:

$$J = \left(\frac{4^{3+\beta} N^2 Q^2}{\pi^2 D^{5+\beta}}\right)^{\frac{1}{1+\gamma}}, \;\; D = \left(\frac{4^{3+\beta} N^2 Q^2}{\pi^2 J^{1+\gamma}}\right)^{\frac{1}{5+\beta}}, \;\; Q = \frac{\pi}{2^{3+\beta} N} D^{(5+\beta)/2} J^{(1+\gamma)/2} \qquad \text{(B1.3)}$$

Optimal dimensionless parameters α, β, γ and the dimensional parameter N (in SI units m and s) from normalized roughness ε_* for the usual range (0.1 m $\le D \le$ 1 m, 0.2 m/s $\le V \le$ 2 m/s):

$$\beta = 0.3 + 0.0005\, \varepsilon_* + \frac{0.02}{1 + 6.8\, \varepsilon_*}, \;\; \gamma = \frac{0.096}{1 + 0.31\, \varepsilon_*} \qquad \text{(B1.4)}$$

$$\alpha = 0.0037\, (1 + 1.6\, \varepsilon_*)^{0.32}\, (80\,000)^{\beta}, \quad N = 0.00687\, (1 + 1.6\, \varepsilon_*)^{0.16} \qquad \text{(B1.5)}$$

Maximum relative errors in estimation of J, D, V, Q: 5%, 1%, 3%, 3%, respectively.

Having determined the form of the approximation, it is a matter of numerical optimization to determine its parameters β, γ and α (or equivalently N) for a specific ε (or ε_*) in a manner that the maximum relative error (precisely, its absolute value) within a range is minimized. The range of application can be defined in terms of the diameter D and velocity V. Obviously, the narrower the range, the most accurate the approximation is. A range of diameter 0.05 m $\le D \le$ 10 m, of velocity 0.1 m/s $\le V \le$ 10 m/s and of roughness $0 \le \varepsilon \le 5$ mm ($0 \le \varepsilon_* \le 100$) covers all values met in practical problems; this will be referred to as global range. A narrower range defined as 0.1 m $\leq D \leq$ 1 m, 0.2 m/s $\leq V \leq$ 2 m/s (with the same interval of roughness) covers most cases in urban networks; this will be referred to as usual range.

Determination of a set of triplets β, γ, α and the corresponding N for a specific value of ε is a routine task given the widespread modern computational tools. One forms a computation grid of values of D and V within the specified range, calculates the accurate values of f from equation (2) for each grid point, assumes some initial values of β, γ and N (e.g. $\beta = 0.33$, $\gamma = 0$, $N = 0.010$), computes the approximation of f from (9) and (11) for each grid point and determines the maximum, over all grid points, relative error. Then one lets an optimization procedure to modify the initial values so as to minimize the maximum relative error. Here an evolutionary commercial solver (by Frontline systems, http://solver.com/), was used. Due to the roughness of the surface representing the objective function (i.e. the maximum relative error) the evolutionary solver was proved to be superior to other tried options (based on gradient optimization methods) in locating the global minimum. Values of β, γ, α, N, determined in this way for the most typical values of ε that are used in urban water systems in Europe, are shown in Table 1 both for the global and usual application ranges.

After a set of triplets β, γ, α has been determined for several values of ε, it can be attempted to establish functions $\beta = \beta(\varepsilon_*)$, $\gamma = \gamma(\varepsilon_*)$ and $\alpha = \alpha(\varepsilon_*)$ or $N = N(\varepsilon_*)$, give them mathematical expressions, define their internal parameters, and determine the numerical values of the parameters by a global optimization, now considering all values of ε_* simultaneously for a specific application range. The laborious task in this problem is to find the appropriate mathematical expressions for the functions. To choose these expressions, graphical depictions of the values of β, γ, α, N, determined for a specific ε, versus ε_*, are helpful. Once the expressions have been defined, the estimation of their internal parameters can be done using the same solver as above. The established functions $\alpha(\varepsilon_*)$, $\beta(\varepsilon_*)$ and $\gamma(\varepsilon_*)$ for the usual range defined above, are shown in Box 1, along with the maximum relative errors in estimation of J, D, V, and Q. Similar functions for the global range as well as other useful sub-ranges are shown in the Appendix.

It can be seen that the errors increased in comparison to those in Table 1; this is because of an additional error in the fitting of the functions $\alpha(\varepsilon_*)$, $\beta(\varepsilon_*)$ and $\gamma(\varepsilon_*)$. Thus, the error in the estimation of the gradient J increases from 2.7% to 5%. The specific forms of the functions $\alpha(\varepsilon_*)$, $\beta(\varepsilon_*)$ and $\gamma(\varepsilon_*)$ are simple, with linear and hyperbolic components. The exponential term in $\alpha(\varepsilon_*)$ has been included to simplify the resulting expression of N in SI units, which in application replaces α. Indeed, as shown in Box 1, which contains the equations for final application in SI, N has a very simple expression. Graphical depictions of the variation of all parameters with roughness are given in Figure 1.

4. Comparison with the Manning and the Hazen-Williams equations

Apparently, the Manning equation can be obtained as a special case of the generalized equation (10) setting $\beta = 1/3$ and $\gamma = 0$. Similarly, the Hazen-Williams equation can be obtained setting $\beta = 0.26$, $\gamma = 0.08$. Then the parameter α (or $n = N$ for the Manning equation or $C = 1 / (0.85\, N)$ for the Hazen-Williams equation) can be estimated by minimizing the error as in the previous cases. The optimized parameters are shown in Box 2.

Box 2 Constants α and β and optimal n or C as functions of normalized roughness ε_* for the Manning and Hazen-Williams equations in the SI system (units m, s) for the usual range.

Manning equation

$$V = (1/n)\,(D/4)^{2/3}\,J^{1/2} \quad \text{(B4.1)}$$

$$\beta = 1/3, \quad \gamma = 0, \quad N = n = 0.009\,(1 + 0.3\,\varepsilon_*)^{1/6} \quad \text{(B4.2)}$$

Maximum relative errors in estimation of J, D, V, Q: 34%, 7%, 23%, 23%, respectively.

Hazen-Williams equation

$$V = 0.85\,C\,(D/4)^{0.63}\,J^{0.54} \quad \text{(B4.3)}$$

$$\beta = 0.26, \quad \gamma = 0.08, \quad N = 0.008\,(1 + 0.22\,\varepsilon_*)^{1/6}, \quad C = 1 / (0.85\,N) \quad \text{(B4.4)}$$

Maximum relative errors in estimation of J, D, V, Q: 36%, 8%, 27%, 27%, respectively.

The maximum relative errors are also shown in Box 2. It can be seen that the errors are high in both cases, about five times larger than those of the proposed generalized Manning equation. A graphical comparison of the approximated f values with the Manning and Hazen-Williams equations with the Colebrook-White values for the usual range of diameters and velocities and for ε = 0.5 mm is given in Figure 2. Values estimated by the proposed generalized Manning equation (10) are also given in this figure. Clearly, this figure demonstrates that the performances of the Manning and Hazen-Williams equations are not satisfactory but the approximation of the proposed generalized Manning equation (10) can be acceptable. This result harmonizes with earlier suggestions (e.g. Liou, 1998) to avoid the use of the Hazen-Williams equation and this is the case also for the Manning equation.

However, it can be seen from that the parameter γ of the proposed generalized equation (10) tends to zero for large roughness and simultaneously the parameter β becomes about 1/3. These are the values of the original Manning equation and thus this observation supports the use of this equation for large roughness. Indeed, it was found that for $\varepsilon \geq 1$ mm the maximum relative errors in estimation of J, D, V, Q by the original Manning equation become 9%, 2%, 4%, 4%, respectively. Interestingly, for large ε, neglecting the term 1 over 0.3 ε_* in estimation of n by equation (B4.2), we obtain $n = \varepsilon^{1/6} / 26$ (ε in m), which agrees with known earlier results (Meyer-Peter and Müller, 1948; Henderson, 1966, p. 98; Julien, 2002). All this discussion can support the use of the original Manning equation for large roughness. Particularly, this can be true for free surface flow in pipes as well as in lined or unlined open channels and natural channels. In this case the breaking of the perfect symmetry that is present in closed cylindrical pipe full flow makes the applicability of the Colebrook-White equation questionable. On the other hand, there exists a large body of experience for the successful applicability of the Manning equation.

On the other end, for small roughness the Manning equation has very poor performance, so it cannot be suggested for small ε. In contrast, the Hazen-Williams approximation can be acceptable this case. Indeed, for $\varepsilon \leq 0.1$ mm the maximum relative errors in estimation of J, D, V, Q by the Hazen-Williams equation become 10%, 2%, 5%, 5%, respectively (more than three times better than in Box 2). Again, however, we cannot suggest the use of the Hazen-

Williams equation even in this case: the proposed generalized Manning equation can perform more than three times better than this if fitted particularly for this range of roughness ($0 \le \varepsilon \le 0.1$ mm; the fitted equations are not reproduced here).

5. Error comparison

The judgment of whether an approximation error can be acceptable or not should be done by comparing with other alternatives and with other types of errors. As mentioned in the Introduction, the original Colebrook-White formula, which was the basis of this study, is perhaps accurate only to 15%. The simplest of the existing approximations (see Introduction) are accurate, as compared to the Colebrook-White formula, to about 15% too; obviously if these maximum errors happen to be simultaneous (at the same conditions) and on the same direction (e.g. both positive) the resulting total error could reach in this case 30%; however statistically this is very unlikely. Roughly, the percentage 15% can be regarded as an upper bound for an approximation to be acceptable. The Manning and Hazen-Williams equations exhibit errors that can exceed twice this value and thus they must not be regarded as acceptable. On the other hand, the proposed generalized Manning equation (10) gives errors smaller than 5% for the usual range of diameters and velocities and thus it can be acceptable with this logic. Even in the global range the errors are smaller than 10-12% (Table 1 and Appendix).

However, the comparison with other sources of errors is more enlightening. Uncertainty or error exists in all involved quantities. The pipe roughness is difficult to define and its value is very uncertain (Noutsopoulos, 1973). The discharge is uncertain too, particularly in the design phase of an urban network. The energy losses can be measured but in complex networks it is difficult to distinguish the friction losses (and thus the energy gradient) from form losses. Even the diameter of the pipe may by uncertain due to manufacturing defects or due to deformation and waving (Xanthopoulos, 1975), and particularly due to incrustation after long use. Thus, a maximum error 1-2% in diameter estimation, which is the maximum error of the proposed generalized equation, could be acceptable.

Particularly, the uncertainty in roughness can reach one to two orders of magnitude. For example, Butler and Davies (2000) suggest for concrete pipes ε in the range 0.06-1.5 mm if the pipes are new and 1.5-6 mm if the pipes are old. Also, Chaudhry (1996) reproduces experience charts (from USBR), according to which the roughness of steel pipes ranges from 0.03 mm (for new smooth pipes) to 6 mm (for pipes with severe tuberculation/incrustation). In both theses cases the highest value differs from the lowest by a factor of 100 or more. In this respect, Table 2 provides a set of error values in the estimation of the energy gradient due to misspecification of the roughness by a factor of 2, 5 and 10. It can be seen that these values are much greater than the maximum approximation error of the same quantity by the power-law equation.

6. Conclusions

Equation (9), which is a power-law approximation of the Colebrook-White equation, enables the expression of the friction factor in terms of the final design quantities. In turn, combined with the Darcy-Weisbach equation, it yields an overall power-law relationship (equation (10)) that is a convenient approximation for design and simulation purposes. This is a generalized Manning equation, whose exponents are not unique but depend on the pipe roughness. The exponent parameters β and γ and the generalized Manning coefficient N are given either in tabulated form (Table 1 for the most typical design roughness values) or as mathematical expressions of roughness (Box 1). The maximum approximation errors in estimating the energy gradient is no more than 5% for the most usual range of diameters and velocities in urban water networks. The corresponding error in the estimation of diameter is 1%. These are much smaller than other errors resulting from uncertainty and misspecification of design and simulation quantities. The small errors render the method a useful substitution of the Darcy-Weisbach and Colebrook-White equations for both design and simulation. In the design phase, it can be argued that the simplification of calculations by the proposed equation is considerable and that the cost is almost negligible if compared to the uncertainty of unknown future design quantities and conditions. But even in simulation of existing urban water systems, where uncertainties are smaller, it can be assumed that the proposed method could be worth trying, because of the expected reduction in computer time.

The original Manning and the Hazen-Willians equations have been also examined in this study as potential alternatives for simplification of calculations. In fact, both can be obtained as special cases of the proposed generalized equation by setting the exponent parameters constant. It turns out that the approximation errors of both equations are much higher than those of the generalized Manning equation and thus their use cannot be encouraged. However, for large roughness, the performance of the original Manning equation is significantly improved and thus its use, particularly when the networks operate with surface flow rather than in pressurized conditions, is absolutely justified.

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Appendix: Optimal parameters for additional diameter and velocity ranges

Equations (B1.4) and (B1.5) in Box 1 are optimized in terms of approximation error for the usual range of diameters and velocities. If the application range becomes wider, the error increases. Generally, the error in the energy gradient J by equations (B1.3)- (B1.5) remains smaller than 15% for a range wider than usual, i.e. for 0.05 m $\leq D \leq$ 3.5 m, 0.1 m/s $\leq V \leq$ 5 m/s. The error in other estimated quantities (D, V, Q) in this range is significantly lower.

For even higher diameters and velocities up to D = 10 m and V = 10 m/s the approximation error may reach 25%. However, it is possible to decrease this error by slightly changing the internal parameters of these equations, so that they become optimal for the new range of diameters and velocities. This can be done using the same method described in section 3. Here we provide equations optimized for the global range (as defined in section 3) as well as for two other sub-ranges of it, referred to as *usual* + *small* and *usual* + *large* ranges and graphically depicted in Figure A1. Note that the three different ranges examined here are deliberately wider than the usual range and overlap with each other as the purpose is to provide convenient additional information for wider spectra of applications that are not fit into the usual range. As D increases, V is expected to be larger and this was taken into account for the construction of sub-ranges in Figure A1.

The following equations replace (B1.4) and (B1.5) of Box 1 for the respective ranges and produce the errors given below, which unavoidably are larger than in the usual range:

Usual + small range (0.05 m $\leq D \leq$ 1 m, 0.1 m/s $\leq V \leq$ 3 m/s)

$$\beta = 0.32 + 0.0006\,\varepsilon_* + \frac{0.021}{1 + 12.1\,\varepsilon_*}, \quad \gamma = \frac{0.11}{1 + 0.32\,\varepsilon_*},$$

$$\alpha = 0.0033\,(1 + 1.92\,\varepsilon_*)^{0.32}\,(80\,000)^{\beta}, \quad N = 0.00648\,(1 + 1.92\,\varepsilon_*)^{0.16} \tag{A.1}$$

Maximum relative errors in estimation of J, D, V, Q: 9%, 2%, 5%, 5%, respectively.

Usual + large range (0.1 m $\leq D \leq$ 10 m, 0.3 m/s $\leq V \leq$ 10 m/s)

$$\beta = 0.25 + 0.0006\,\varepsilon_* + \frac{0.024}{1 + 7.2\,\varepsilon_*}, \quad \gamma = \frac{0.083}{1 + 0.42\,\varepsilon_*},$$

$$\alpha = 0.0045\,(1 + 2.47\,\varepsilon_*)^{0.28}\,(80\,000)^{\beta}, \quad N = 0.00757\,(1 + 2.47\,\varepsilon_*)^{0.14} \tag{A.2}$$

Maximum relative errors in estimation of J, D, V, Q: 8%, 2%, 5%, 5%, respectively.

Global range (0.05 m $\leq D \leq$ 10 m, 0.1 m/s $\leq V \leq$ 10 m/s)

$$\beta = 0.27 + 0.0008\,\varepsilon_* + \frac{0.043}{1 + 3.2\,\varepsilon_*}, \quad \gamma = \frac{0.1}{1 + 0.32\,\varepsilon_*},$$

$$\alpha = 0.0039\,(1 + 2.38\,\varepsilon_*)^{0.3}\,(80\,000)^{\beta}, \quad N = 0.00705\,(1 + 2.38\,\varepsilon_*)^{0.15} \tag{A.3}$$

Maximum relative errors in estimation of J, D, V, Q: 12%, 2%, 7%, 7%, respectively.

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Tables

Table 1 Optimal α, β, γ and N values for the most typical design roughness values used in Europe.

ε (mm)	0	0.1	0.3	1	3
Global application range (0.05 m $\leq D \leq$ 10 m; 0.1 m/s $\leq V \leq$ 10 m/s)*					
α	0.1273	0.1602	0.2200	0.3397	0.6458
β	0.31	0.28	0.28	0.29	0.32
γ	0.104	0.054	0.029	0.014	0.007
N (SI units: m, s)	0.0070	0.0093	0.0109	0.0128	0.0149
Usual application range (0.1 m $\leq D \leq$ 1 m; 0.2 m/s $\leq V \leq$ 2 m/s)**					
α	0.1376	0.1599	0.2115	0.3804	0.7886
β	0.33	0.30	0.29	0.31	0.35
γ	0.109	0.069	0.037	0.015	0.006
N (SI units: m, s)	0.0065	0.0083	0.0101	0.0121	0.0139

* Maximum relative errors in estimation of J, D, V, Q: 10%, 2%, 6%, 6%, respectively.

** Maximum relative errors in estimation of J, D, V, Q: 2.7%, 0.6%, 1.5%, 1.5%, respectively.

Table 2 Comparison of approximation errors and errors due to misspecification of roughness in the estimation of the energy gradient J (for the usual range of diameters and velocities).

Maximum approximation error of the power-law equation		5%
Maximum approximation error of the Manning equation		34%
Maximum approximation error of the Hazen-Willians equation		36%
Maximum error due to misspecification of ε in the region 0.1-1 mm (using the Colebrook-White equation),	… by a factor of 2	19%
	… by a factor of 5	36%
	… by a factor of 10	44%

Figures

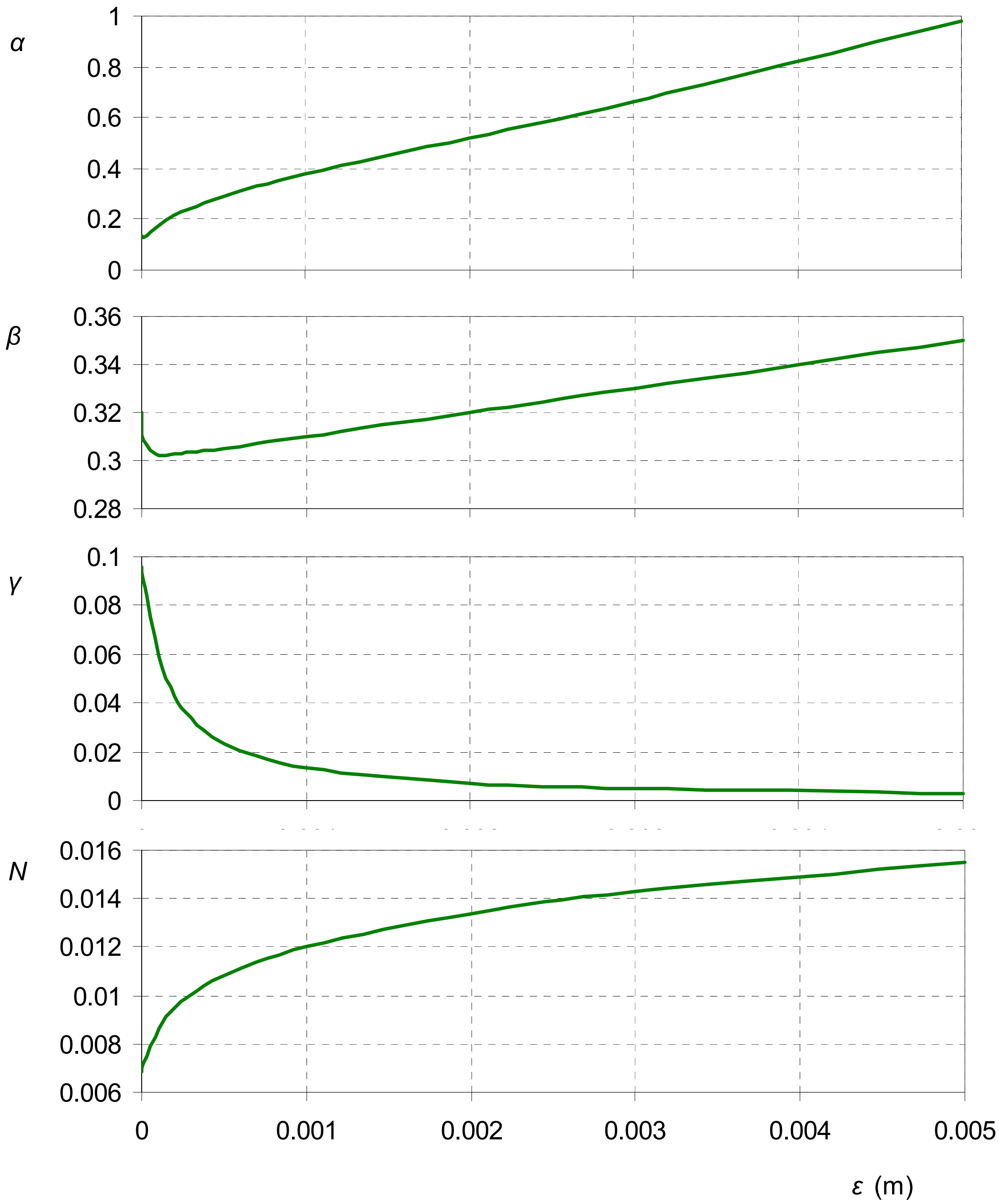

Figure 1 Variation with roughness ε of the dimensionless parameter α, β and γ, and the dimensional parameter N (in SI units m and s) in the generalized Manning equation (10) for the four ranges of diameters and velocities examined.

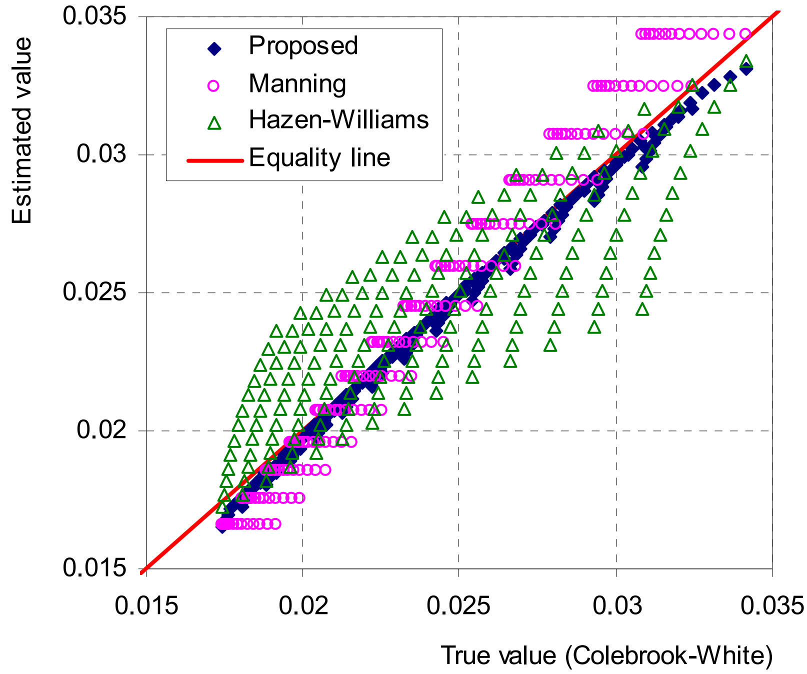

Figure 2 Comparison of approximated f values with the proposed generalized Manning equation (10), as well as the original Manning and Hazen-Williams equations, with the Colebrook-White values for the usual range of diameters and velocities and for $\varepsilon = 0.5$ mm.

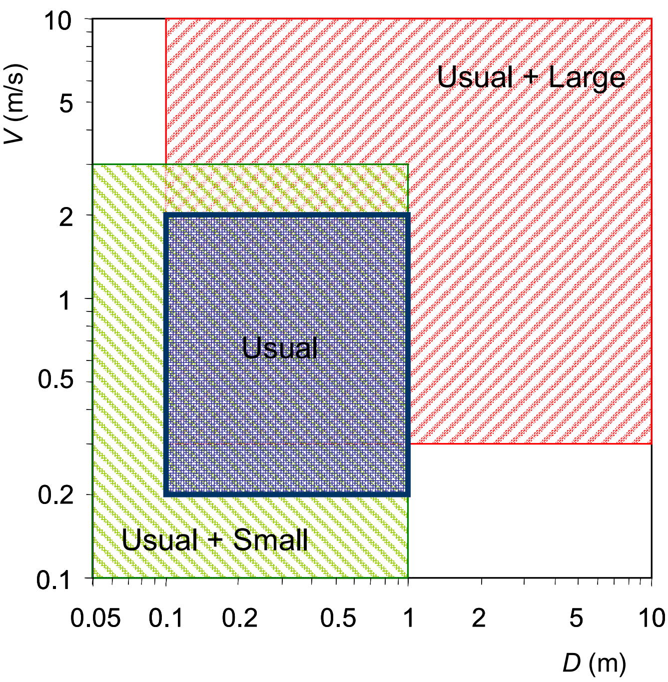

Figure A1 Definition sketch of the ranges of diameters and velocities.