

# Reservoir yield-reliability relationship and frequency of multi-year droughts for scaling and non-scaling reservoir inflows

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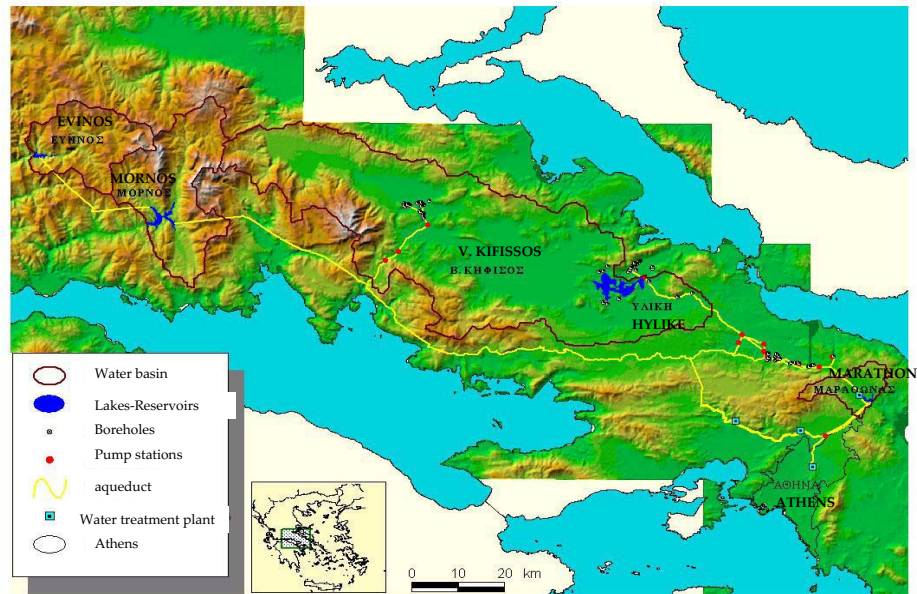
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## 1. Abstract

Being a group of undergraduate students attending the course of Stochastic Methods in Water Resources, we study, in cooperation with our tutors, the influence of the scaling behavior (also known as long-term persistence) of reservoir inflows to the reservoir yield-reliability relationship and to the frequency of multi-year droughts, in comparison to conventional, non-scaling, inputs. We perform an integrated monthly-scale simulation of the Hylike natural lake, which is one of the four reservoirs of the water resource system of Athens. Reservoir inflows, evaporation and precipitation on the lake surface, as well as leakage, which is significant due to the karstic subsurface of the lake, are all considered into the simulation. The reservoir inflows are generated by two alternative monthly stochastic models, a short term persistence model and a long term one, both cyclostationary. The resulting differences of the two approaches in the reservoir yield-reliability relationship and the frequency of multi-year drought periods (i.e. those in which demand is not fully satisfied) are discussed.

## 2. Problem statement and case study area

Hylike is a natural lake, one of the four reservoirs that constitute the water resource system of Athens. It has an average inflow of 410 hm<sup>3</sup>. This lake has a significant leakage due to its karstic subsurface. This case study aims to estimate the reliable release using stochastic simulation (Monte Carlo). While in reality the management of this reservoir is connected to that of the other reservoirs, here for simplicity we consider this reservoir as isolated.



Two different approaches, both at monthly scale, are examined and compared. The first one generates synthetic inflows using a short term persistence model PAR(1), whereas the second one reproduces the long term persistence effect (Hurst or Joseph phenomenon).

## 3. Assumptions on water balance simulation

- Inflows are simulated stochastically using a short term persistence model PAR(1) and a long term cyclostationary persistence model based on Symmetric Moving Average (Koutsoyiannis, 2000; Langousis & Koutsoyiannis, 2006).
- Evapotranspiration and precipitation are assumed constant per month with no overyear fluctuation, based on historical data (Aliartos, HNMS).
- Monthly leakage, which is substantial due to the karstic subsurface, is estimated taking into account an estimated water level-leakage and water level-storage reservoir curve (Koutsoyiannis & Nalbantis, 1989).
- The monthly variation of demand was estimated in a former investigation of the Athens supply system (Koutsoyiannis & Nalbantis, 1989).

Inflow statistical characteristics (historical data)

	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Year
<b>mean</b>	21,4	30,7	46,1	59,5	62,4	66,8	46,5	24,2	11,9	4,1	3,5	12,8	386,5
<b>st. dev</b>	10,9	17,7	31,9	30,8	34,2	31,1	27,1	16,3	11,6	7,3	5,2	8,7	159,7
<b>Cs*</b>	0,4	1,6	2,8	1,0	0,9	0,8	1,3	0,8	1,2	3,7	2,2	1,0	0,4
<b>Ck**</b>	0,0	3,7	9,7	0,7	0,4	1,1	3,9	0,7	1,9	18,3	6,6	4,1	-0,2

\*Coefficient of skewness

\*\*Coefficient of kurtosis

## 4. Short term persistence model

In order to generate stochastic hydrological time series based on historical data from the river of Boeotikos Kephisos (Karditsa station) the *Simple MULTivariate Stochastic Hydrologic* model (SMUSH; Koutsoyiannis, 2007) is used.

This model is based on Periodic AutoRegressive model PAR(1) which preserves monthly means, standard deviations and lag 1 autocorrelation coefficients.

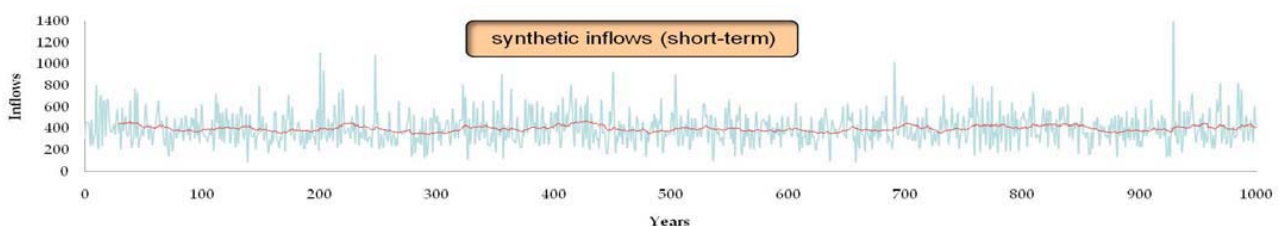
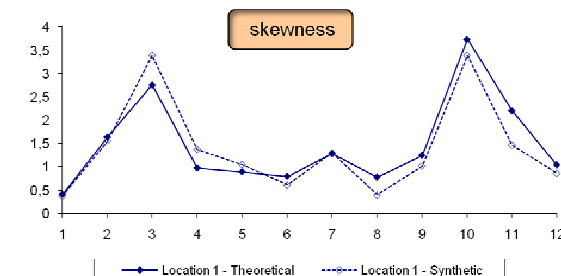
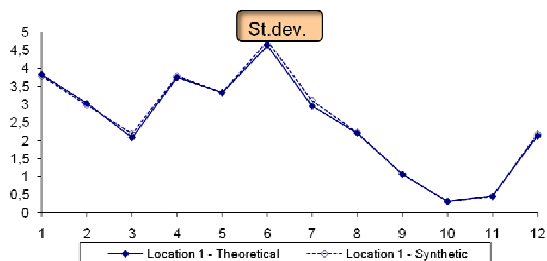
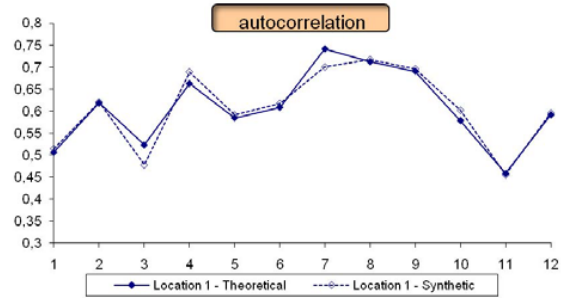
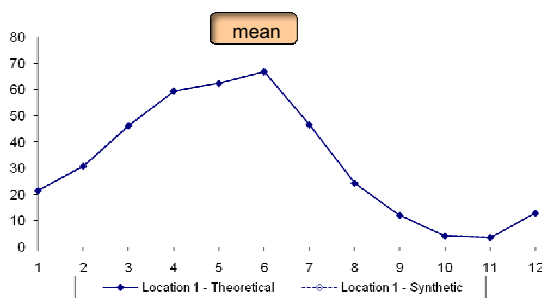
$$\mathbf{X}^s = \mathbf{a}^s \mathbf{X}^{s-1} + \mathbf{b}^s \mathbf{V}^s$$

The model is described in detail in Bras and Rodriguez-Iturbe (1985).

Comparison of inflow statistical characteristics

month	oct	nov	dec	jan	feb	mar	apr	may	jun	jul	aug	sep
Historical Parameters												
<b>mean</b>	21,4941	30,70943	46,13593	59,48294	62,42413	66,83735	46,50215	24,23299	11,93362	4,093282	3,505138	12,76133
<b>Std</b>	10,85642	17,56587	31,77702	30,7558	34,23147	31,06602	27,06437	16,31935	11,56949	7,260534	5,153484	8,742312
<b>Skew</b>	0,396073	1,647861	2,773698	0,973079	0,892059	0,794315	1,283071	0,775867	1,248103	3,739181	2,204278	1,03844
<b>Autocorrelation</b>	0,062996	3,753898	9,865417	0,749731	0,36506	1,139413	3,868989	0,696212	1,931455	18,27946	6,570551	4,067759
Simulated Parameters												
<b>mean</b>	22,078	31,740	48,004	61,259	64,337	67,161	46,776	24,192	12,629	5,465	4,225	13,506
<b>Std</b>	10,785	18,066	33,580	31,343	34,070	31,079	27,439	15,767	10,254	4,974	5,045	8,754
<b>Skew</b>	0,533	1,323	2,917	1,710	0,946	0,935	0,752	0,645	0,694	0,680	2,865	1,618
<b>Autocorrelation</b>	0,500	0,613	0,468	0,690	0,574	0,609	0,765	0,729	0,671	0,623	0,335	0,611

## 5. Preservation of statistics (SMUSH)

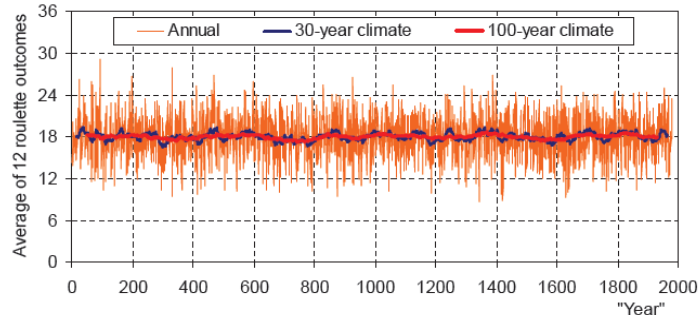


## 6. Hurst phenomenon and long term persistence

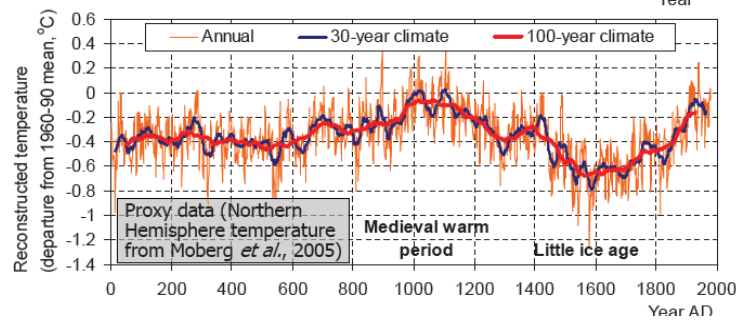
This model aims to reproduce the scaling behaviour also known as the Hurst or Joseph phenomenon. It was initially observed at the water level time-series of the river Nile (Roda Island) by Hurst but was also detected in numerous natural processes, including climatic time-series.



"Roulette climate"



Real-world or "Hurst-Kolmogorov" climate



Source : Koutsoyiannis, 2008

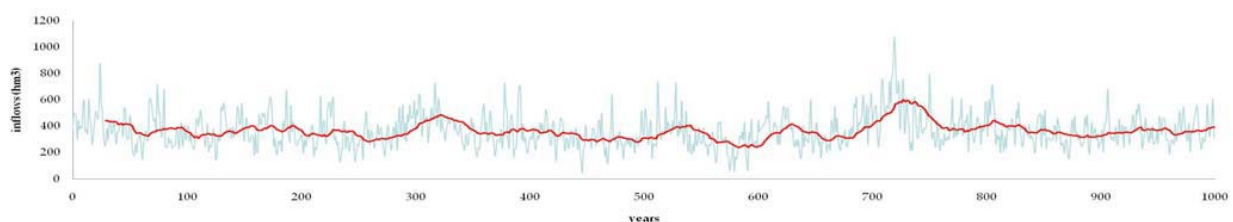
## 7. Long term persistence model

In order to generate stochastic hydrological time series with scaling behaviour a model based on SMA is used, consisting of the following steps, assuming that flow time series has been normalised by an appropriate transformation:

- Step 1: A set of random numbers is produced following the standard normal distribution
- Step 2: The sequences of Step 1 of consecutive months are made dependent to each other, with lag one autocorrelation.
- Step 3: The monthly sequences of step 2 are adapted to become consistent with the Hurst phenomenon using the SMA algorithm (Koutsoyiannis, 2000, 2002; Langousis & Koutsoyiannis, 2006).
- Step 4: The inverse non-linear transformation is applied, i.e.,

$$x(z) = c \left\{ \exp \left[ \left( \frac{z-a}{b} \right)^{\frac{1}{e}} \right] - 1 \right\}^{\frac{1}{d}}$$

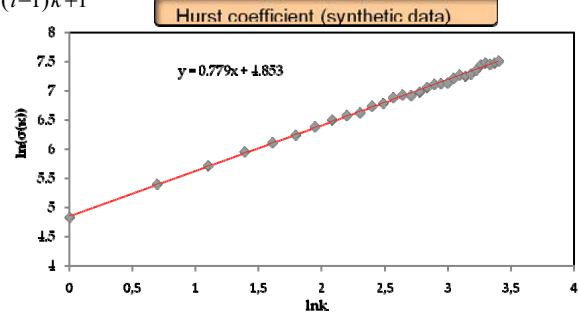
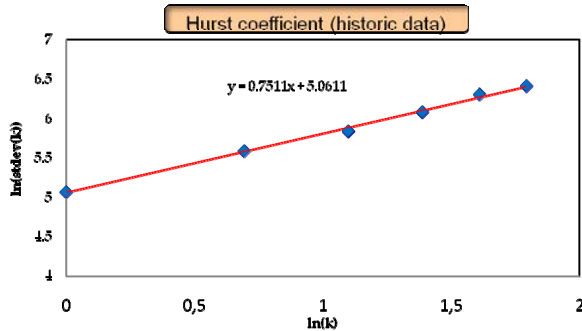
synthetic inflows (long-term)



# 8. Hurst coefficient and statistics

According to the mathematical expression of the Hurst- Kolmogorov phenomenon

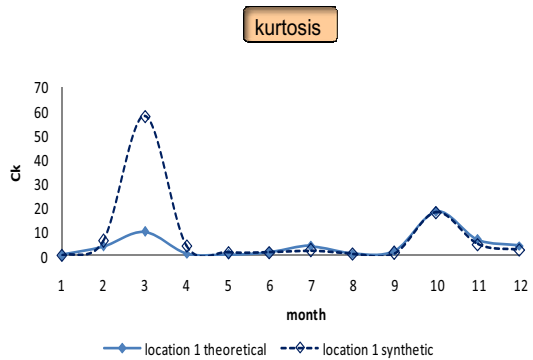
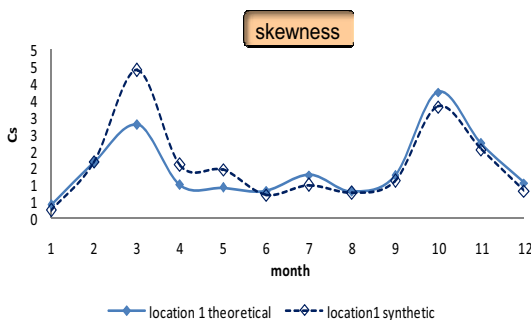
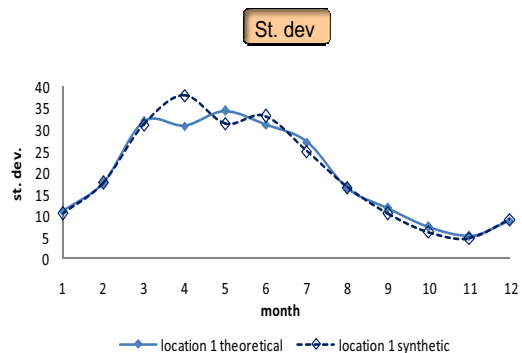
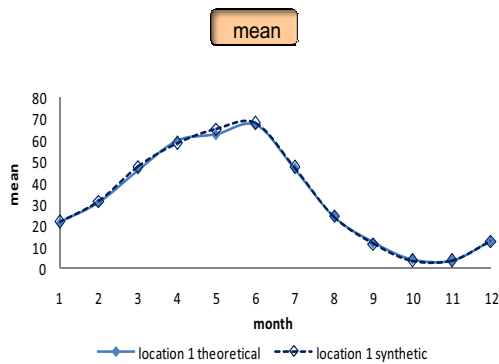
$$\gamma^{(k)} = Var(Z_i)^{(k)} = k^{2H} \gamma_o, \text{ where } Z^{(k)} = \sum_{l=(i-1)k+1}^{ik} X_l \text{ and } H \text{ is the Hurst coefficient}$$



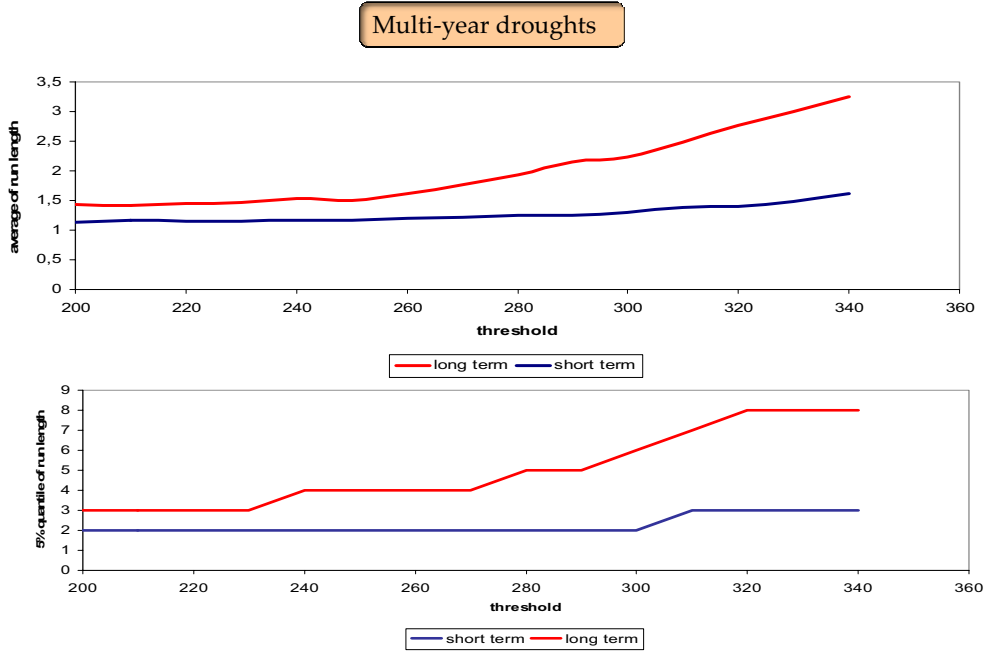
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<b>Synthetic data</b>												
mean	20,39	24,87	40,73	49,51	56,99	62,77	51,36	28,96	14,82	3,14	2,60	9,84
st.dev.	12,01	13,75	23,61	27,14	28,84	27,58	30,06	17,23	11,17	4,76	3,70	7,52
Cs	0,66	1,48	2,36	1,94	0,92	0,71	1,91	0,96	0,84	2,97	2,06	0,95
Ck	0,30	3,81	9,19	6,94	1,23	1,21	7,12	1,68	0,44	15,26	5,25	0,98

# 9. Preservation of statistics (SMA)

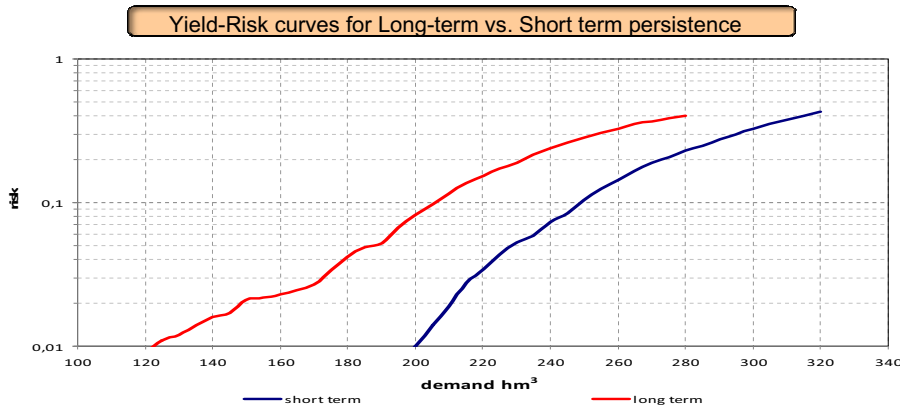
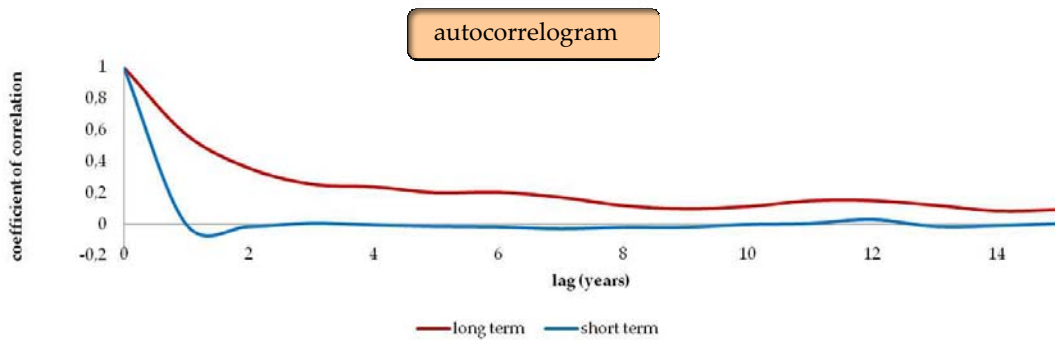


# 10. Statistics of Multi-year droughts



Evidently, the multi-year periods (runs) with low flows, below a specified threshold, are more frequent in the long term persistence model than in the short term persistence one.

# 11. Comparison Short-term vs. Long-term persistence





## 12. Conclusions

- Both models satisfactorily reproduce monthly means, standard deviations, skewness coefficients and lag 1 autocorrelation coefficients.
- In addition, the long term persistence model reproduces the Hurst phenomenon, yielding high autocorrelation values for large lags, and preserves the Hurst coefficient.
- The long term persistence model produces more frequent and longer (sometimes higher than 8 years) drought periods than the short term one (typically no more than 3 years).
- When the reliable reservoir yield is to be estimated, the differences of the two models are substantial, with the long term persistence model obviously being the more realistic and conservative.
- At a 99% reliability level, which is typical for water supply reservoirs, the short term persistence model overestimates the reliable release by about 80 hm<sup>3</sup> per year or about 60%. This makes the latter model totally inappropriate.

## References

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