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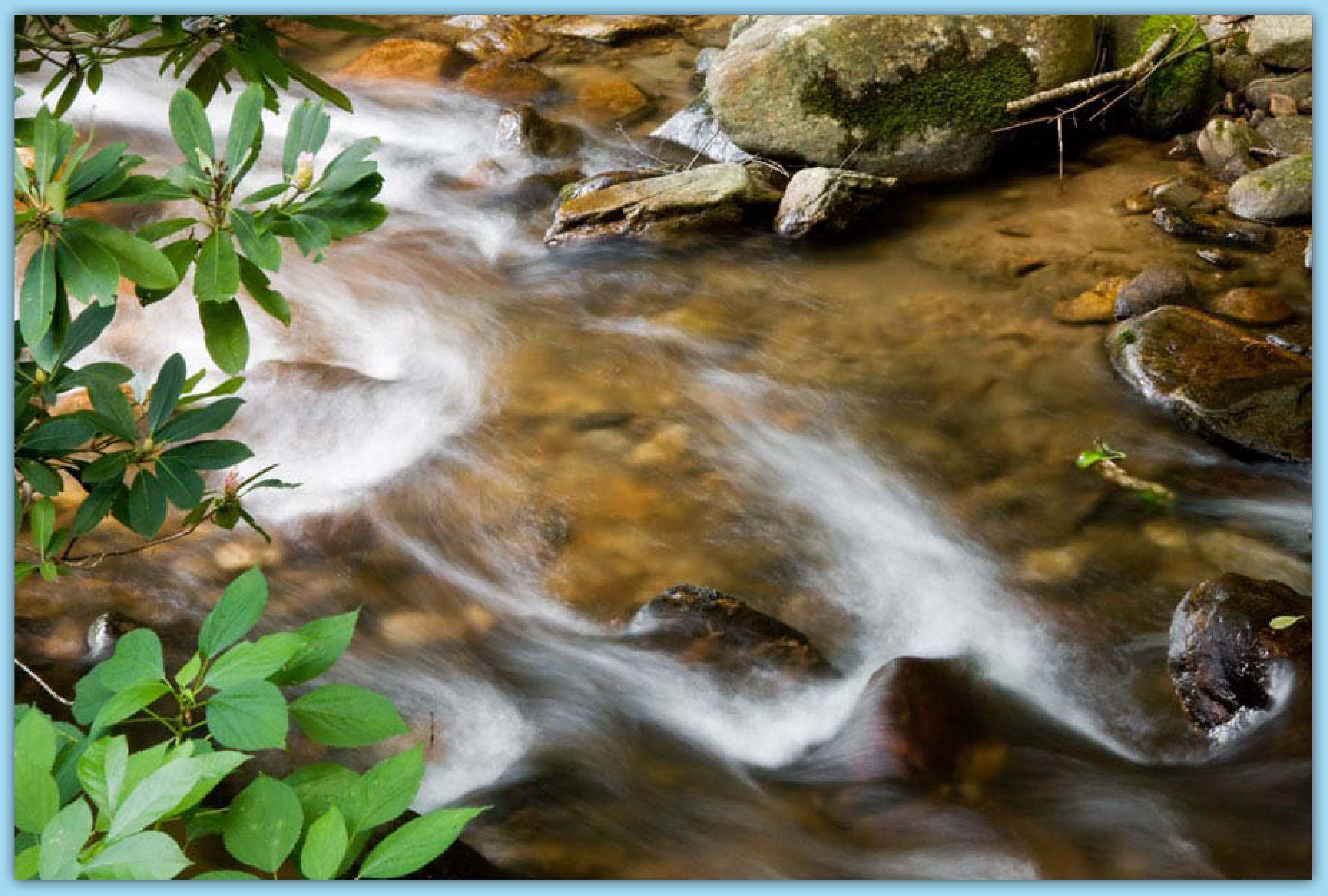

Hurst-Kolomogorov Processes and Uncertainty

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Abstract

The non-static, ever changing hydroclimatic processes are often described as nonstationary. However, revisiting the notions of stationarity and nonstationarity, which are defined within stochastics, it may be understood that claims of nonstationarity cannot stand unless the evolution in time of the statistical characteristics of the process is known in deterministic terms, in particular for the future. This however can hardly be the case, because deterministic predictions are difficult, especially of the future. Thus, change is not synonymous to nonstationarity, and even prominent change at a multitude of time scales, small and large, can be described satisfactorily by a stochastic approach admitting stationarity. This "novel" description does not depart from the 60- to 70-year old pioneering works of Hurst on natural processes and of Kolmogorov on turbulence. Contrasting stationary with nonstationary has important implications in engineering and management. The stationary description with Hurst-Kolmogorov stochastic dynamics demonstrates that nonstationary and classical stationary descriptions underestimate the uncertainty. This is illustrated using several examples of hydrometeorological time series, which also show the consistency of the Hurst-Kolmogorov approach with reality. A final example demonstrates how this framework was implemented in the planning and management of the water supply system of Athens, Greece, also in comparison with alternative nonstationary modelling approaches, including a trend-based and a climate-model-based approach.

«Αρχή σοφίας ονομάτων επίσκεψις» (Αντισθένης)
"The start of wisdom is the visit (study) of names" (Antisthenes; ~445-365 BC)

Introduction

Perhaps the most significant contribution of the intensifying climatic research is the accumulation of evidence that climate has never in the history of Earth been static. Rather, it has been ever changing at all time scales. This fact, however, has been hard, even for scientists, to accept, as displayed by the inflationary (and thus non scientific) term "climate change". The introduction of this term reflects a belief, or expectation, that climate would normally be static, and that its change is something extraordinary which to denote we need a special term ("climate change") and which to explain we need to invoke a special agent (e.g. anthropogenic influence). Examples indicating this problem abound, e.g., "*climate change is real*" (Tol, 2006) or "*there is no doubt that climate change is happening and that we should be taking action to address it now*" (Institute of Physics, 2010). More recently the scientific term "nonstationarity", contrasted to "stationarity", has also been recruited to express similar, or identical ideas to "climate change". Sometimes their use has been dramatized, perhaps to better communicate a non-scientific message, as in the recent popular title of a paper in Science: "*Stationarity is Dead*" (Milly *et al.*, 2008). We will try to show below, in section 2, that such use of these terms is in fact a diversion and misuse of the real scientific meaning of the terms.

Insisting on the proper use of the scientific terms "stationarity" and "nonstationarity" is not just a matter of semantics and of rigorous use of scientific terminology. Rather, it has important implications in engineering and management. As we demonstrate in section 2, nonstationary descriptions of natural processes use deterministic functions of time to predict their future evolution, thus explaining part of the variability and eventually reducing future uncertainty. This is consistent with reality only if the produced deterministic functions are indeed deterministic, i.e., exact and applicable in future times. As this is hardly the case as far as future applicability is concerned (according to a quotation attributed to Niels Bohr, “*prediction is difficult, especially of the future*”), the uncertainty reduction is a delusion and results in a misleading perception and underestimation of risk.

In contrast, proper stationary descriptions, which, in addition to annual (or sub-annual) variability, also describe the over-annual climatic fluctuations, provide more faithful representations of natural processes and help us characterize the future uncertainty in probabilistic terms. Such representations are based on the Hurst-Kolmogorov (HK) stochastic dynamics (section 3), which has essential differences from typical random processes. The HK representations are essential for water resources planning and management, which apparently demand long time horizons and can have no other rational scientific basis than probability (or its complement, reliability).

It is thus essential to illustrate the ideas discussed in this paper and the importance of rigorous use of scientific concepts through a real-world case study of water resources management. The case study we have chosen for this purpose is the complex water supply system of Athens. While Athens is a very small part of Greece (about 0.4% of the total area), it hosts about 40% of its population. The fact that Athens is a dry place (annual rainfall of about 400 mm) triggered the construction of water transfer works from the early stages of the long history of the city (Koutsoyiannis *et al.*, 2008b) . The modern water supply system transfers water from four rivers from up to about 200 km away from Athens.

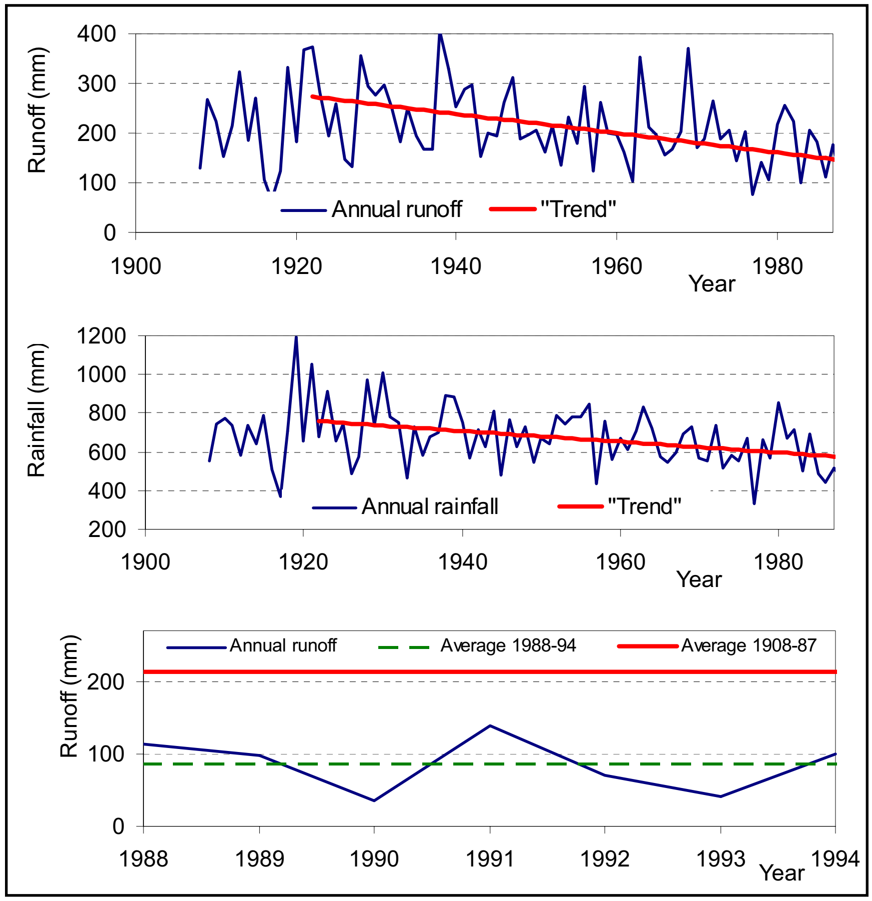

Figure 1. Time series of runoff (upper) and rainfall (middle) in the Boeoticos Kephisos River basin from the beginning of observations to 1987, with focus of the runoff during the severe, 7-year (1988-94) drought period (lower).

Figure 1 (upper panel) shows the evolution of the runoff of one of these rivers, the Boeoticos Kephisos River (in units of equivalent depth over its about 2,000 km^2 catchment) from the beginning of observations to 1987. A substantial falling trend is clearly seen in the time series. The middle panel of Figure 1 shows the time series of rainfall in a raingauge in the basin, where a trend is evident and explains (to a large extent) the trend in runoff. Most interesting is the runoff in the following seven years, 1988-1994, shown in the last panel of Figure 1, which is consistently below average, thus manifesting a long-lasting and severe drought that shocked Athens during that period. The average during these seven years is only 44% of the average of the previous years. A typical interpretation of such time series would be to claim nonstationarity, perhaps attributing it to anthropogenic global warming, etc. However, we will present a different interpretation of the observed behaviour and its implications on water resources planning and management (section 4). For Athens, these implications were particularly important even after the end of the persistent drought, because it was then preparing for the Olympic games—and apparently these would not be possible in water shortage conditions. Apparently, good planning and management demand a strong theoretical background and proper use of fundamental (but perhaps forgotten or abused) notions.

Visiting Names, Stationarity and Nonstationarity

Finding invariant properties within motion and change is essential to science. Newton’s laws are eminent examples. The first law asserts that, in the absence of an external force, the position x of a body may change in time t but the velocity $u := \mathrm{d}x/\mathrm{d}t$ is constant. The second law is a generalization of the first for the case that a constant force F is present, whence the velocity changes but the acceleration $a = \mathrm{d}u/\mathrm{d}t$ is constant and equal to F/m, where m is the mass of the body. In turn, Newton’s law of gravitation is a further generalization, in which the attractive force F (weight) exerted, due to gravitation, by a mass M on a body of mass m at a distance r is no longer constant. In this case, the quantity $G = F r^2/(m M)$ is constant, whereas in the application of the law for planetary motion another constant emerges, i.e., the angular momentum per unit mass, $(d\theta/dt)\, r^2$, where θ denotes angle.

However, whilst those laws give elegant solutions (e.g., analytical descriptions of trajectories) for simple systems comprising two bodies and their interaction, they can hardly derive the irregular trajectories of complex systems. Complex natural systems consisting of very many elements are impossible to describe in full detail and their future evolution is impossible to predict in detail and with precision. Here, the great scientific achievement is the materialization of macroscopic descriptions that need not model the details. This is essentially done using probability theory (laws of large numbers, central limit theorem, principle of maximum entropy). Here lies the essence and usefulness of the stationarity concept, which seeks invariant properties in complex systems.

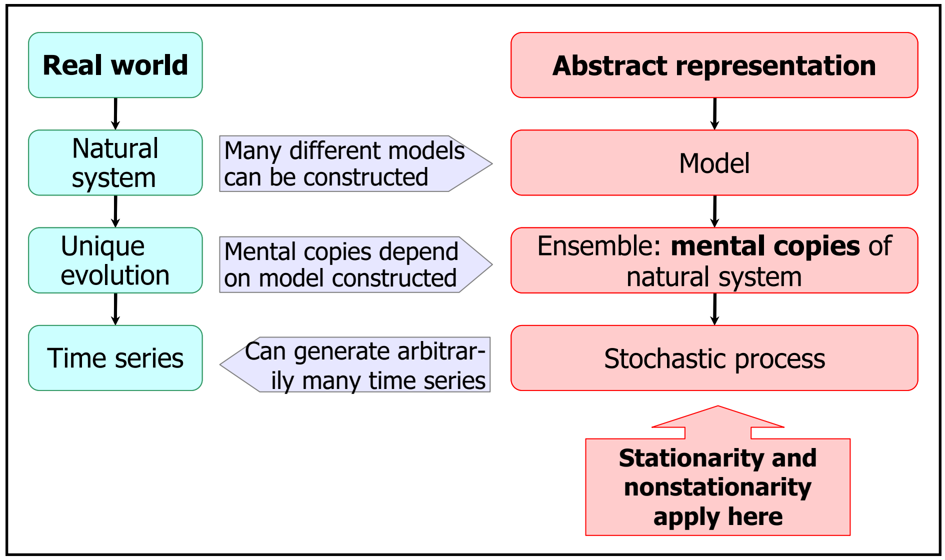

Figure 2 Schematic for the clarification of the notions of stationarity and nonstationarity.

According to the definitions quoted from Papoulis (1991), "*A stochastic process* $\underline{x}(t)$ *is called strict-sense stationary … if its statistical properties are invariant to a shift of the origin*" and "*… is called wide-sense stationary if its mean is constant* ($E[\underline{x}(t)] = \eta$) *and its autocorrelation depends only on [time difference]* τ*…*, ($E[\underline{x}(t+\tau)\,\underline{x}(t)] = R(\tau)$)". We can thus stress that the definition of stationarity applies to stochastic processes (rather than to time series; see also Koutsoyiannis, 2006b). Processes that are not stationary are called nonstationary and some of their statistical properties are *deterministic* functions of time. Figure 2 helps us to further clarify the definition. The left part of this graphic symbolizes the real world. Any natural system we study has a unique evolution (a unique trajectory in time), and if we observe this evolution, we obtain a time series. The right part of the graphic symbolizes the abstract world, the models. Of course, we can build many different models of the natural system, any one of which can give us an ensemble, i.e., mental copies of the real-world system. The idea of mental copies is due to Gibbs, known from statistical thermodynamics. An ensemble can also be viewed as multiple realization of a stochastic process, from which we can generate synthetic time series. Clearly, the notions of stationarity and non-stationarity apply here to the abstract objects—not to the real-world objects. In this respect, profound conclusions such as that hydroclimatic processes are nonstationary or that "stationarity is dead" may be pointless.

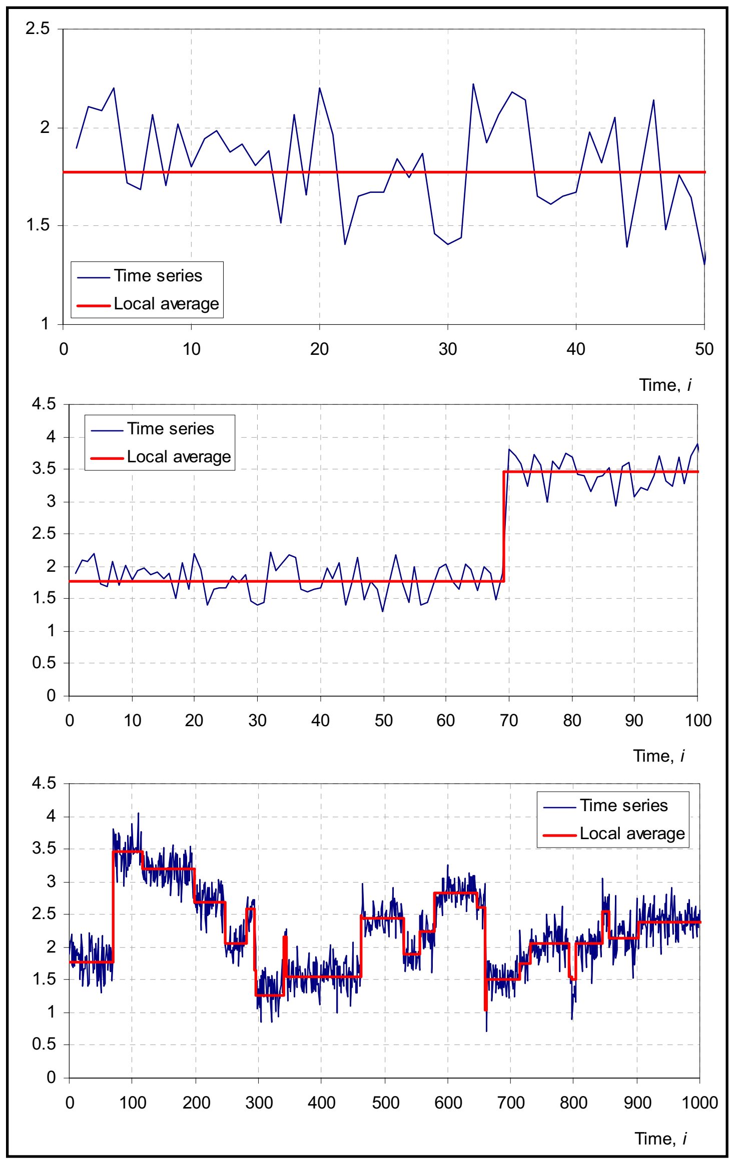

Figure 3. A synthetic time series for the clarification of the notions of stationarity and nonstationarity (see text); (upper) the first 50 terms; (middle) the first 100 terms; (lower) 1000 terms.

To further illustrate the notion of stationarity we use an example of a synthetic time series, shown in Figure 3, whose generating model will be unveiled below, along with some indication that it could be a plausible representation of a complex natural system. The upper panel of the figure depicts the first 50 terms of the time series. Looking at the details of this irregular trajectory, one could hardly identify any property that is constant. However, in a macroscopic—i.e., statistical—description one could assume that this time series comes from a stochastic process with a mean constant in time ($E[\underline{x}_i] = \mu$, where E denotes expected value, i denotes discrete time, x_i is the time series and $\underline{x}_i$ is the stochastic process). In a similar manner, one can assume that the process has a standard deviation σ constant in time (i.e., $E[(x_i - \mu)^2] = \sigma^2$) and so on. Both μ and σ are not material properties of the process (that for instance could be measured by a certain device), but abstract statistical properties.

The middle panel of Figure 3 depicts 100 terms of the time series. One could easily identify two periods, $i < 70$ with a local time average $m_1 = 1.8$ and $i \geq 70$ with a local time average $m_2 = 3.5$. One could then be tempted to use a nonstationary description, assuming a "change" or "shift" of the mean at time $i = 70$. But this is just a temptation (explained by the adherence to the classical views of natural phenomena as either "clockwork" or "dice throwing"; see Koutsoyiannis, 2009); it does not reflect any objective scientific truth and it is not the only option. Rather, a stationary description is still possible.

In fact, as is more evident from the lower panel of Figure 3, the stationary description corresponds to the actual model used to generate the time series. This model consists of the superposition of: (a) a stochastic process, with values m_j derived from the normal distribution $N(2, 0.5)$, each lasting a period τ_j exponentially distributed with $E[\underline{\tau}_j] = 50$ (the thick line with consecutive plateaus); and (b) white noise, with normal distribution $N(0, 0.2)$. Nothing in this model is nonstationary and, clearly, the process of our example is stationary.

In this example, distinguishing stationarity from nonstationarity is a matter of answering a simple question: Does the thick line of plateaus in Figure 3 represent a known (deterministic) function or an unknown (random) function? In the first case (deterministic function), we should adopt a nonstationary description, while in the second case (random function, which could be assumed to be a realization of a stationary stochastic process), we should use a stationary description. As stated above, contrasting stationary with nonstationary descriptions has important implications in engineering and management. To see this we have copied in Figure 4 the lower panel of Figure 3, now in comparison to a "mental copy", which was constructed assuming nonstationarity. We also did the same in Figure 5, but assuming stationarity. In Figure 4 (the nonstationary description), because nonstationarity implies that the sequence of consecutive plateaus is a deterministic

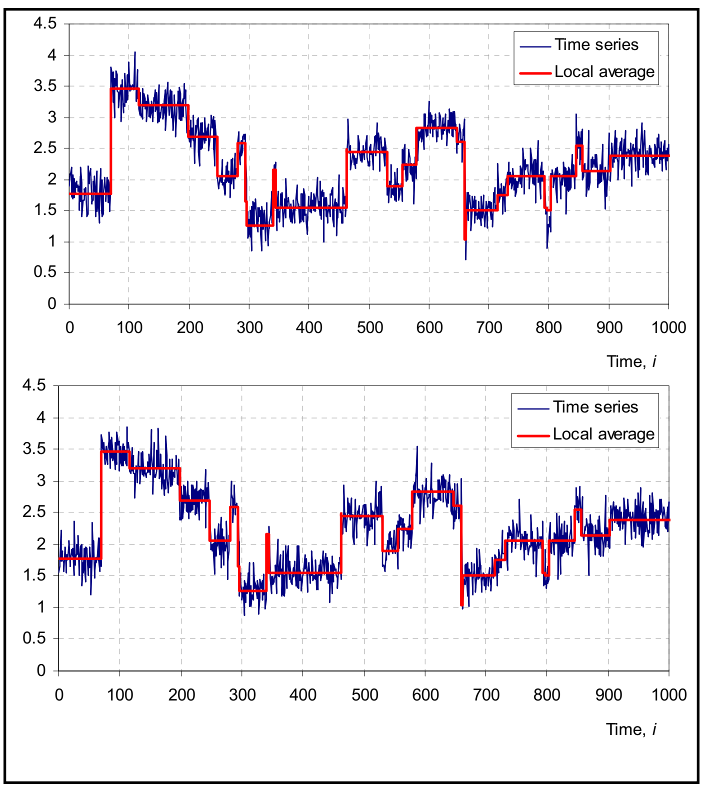

Figure 4. The time series of Figure 3 (upper) along with a mental copy of it (lower) assuming that the local average is a deterministic function and thus identical with that of the upper panel.

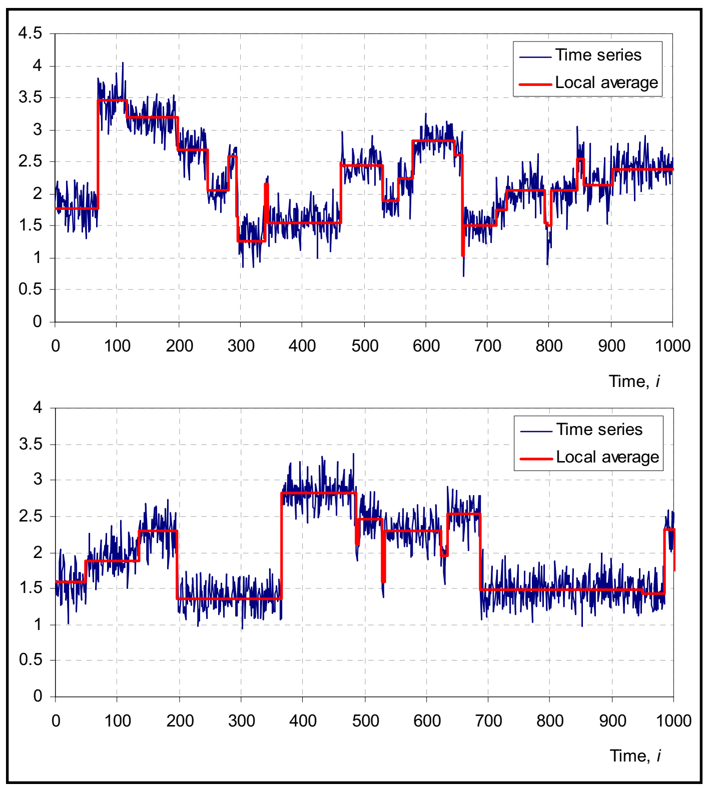

Figure 5. The time series of Figure 3 (upper) along with a mental copy of it (lower) assuming that the local average is a random function, i.e. a realization of the stochastic process described in text, different from that of the upper panel.

function of time, the thick lines of plateaus is exactly the same in the two copies. The uncertainty, expressed as the unexplained variance, i.e., the variance of differences between the thick line of plateaus and the rough line, is (by construction of the process) $0.2^2 = 0.04$. However, in Figure 5 (the stationary description) the two copies have different random realizations of the line of plateaus. As a result, the total variance (that of the "non-decomposed" time series) is unexplained, and this is calculated to be 0.38, i.e., almost 10 times greater than in the nonstationary description. Thus, a nonstationary description reduces uncertainty, because it explains part of the variability. This is consistent with reality only if the produced deterministic functions are indeed deterministic, i.e., exact and applicable in future times. As this is hardly the case, as far as future applicability is concerned, the uncertainty reduction is a delusion and results in a misleading perception and underestimation of risk.

In summary, the example illustrates that (a) stationary is not synonymous to static; (b) nonstationary is not synonymous to changing; (c) in a nonstationary process the change is described by a deterministic function; (d) nonstationarity reduces uncertainty (because it explains part of variability); and (e) unjustified/inappropriate claim of nonstationarity results in underestimation of variability, uncertainty and risk. In contrast, a claim of nonstationarity is justified and thus, indeed, reduces uncertainty, if the deterministic function of time is constructed by deduction (the Aristoteleian *apodeixis*), and not by induction (direct use of data). Thus, to claim nonstationarity, we must: (a) establish a causative relationship; (b) construct a quantitative model describing the change as a deterministic function of time; and (c) ensure applicability of the deterministic model in future time.

Because recently the inflationary use of the term "nonstationarity" in hydrology has been closely related to "climate change", it is useful to examine whether the terms justifying a nonstationary description of climate do hold true or not. The central question is: Do climate models (also known as general circulation models—GCMs) enable a nonstationary approach? More specific versions of these question are: Do GCMs provide credible deterministic predictions of the future climate evolution? Do GCMs provide good predictions for temperature and somewhat less good for precipitation (as often thought)? Do GCMs provide good predictions for global and continental scales and, after downscaling, for local scales? Do GCMs provide good predictions for the distant future (albeit less good for the nearer future, e.g., for the next 10-20 years—or for the next season or year)? To the author's opinion, the answers to all these questions should be categorically negative. Not only are GCMs unable to provide credible predictions for the future, but they also fail to reproduce the known past (see Koutsoyiannis *et al.*, 2008a; Anagnostopoulos *et al.*, 2009). An additional, very relevant question is: Is climate predictable in deterministic terms? Again the author's answer is negative (Koutsoyiannis, 2006a; 2009). Only stochastic climatic predictions could be scientifically meaningful. In principle, these could also include nonstationary descriptions wherever causative relationships of climate with its forcings are established. But until such a stochastic theory of climate, which includes nonstationary components, could be shaped, there is room for developing a stationary theory that characterizes future uncertainty as faithfully as possible; the main characteristics of such a theory are outlined in section 3 (see also Koutsoyiannis *et al.*, 2007).

While a nonstationary description of climate is difficult to establish or infeasible, in other cases, related to water resources, it may be much more meaningful. For example, in modelling of streamflow downstream of a dam we would use a nonstationary model with a shift in the statistical characteristics before and after the construction of the dam. Gradual changes in the flow regime, e.g., due to urbanization that evolves in time, could also justify a nonstationary description, provided that a solid information or knowledge (as opposed to ignorance) of the agents affecting a hydrological process is available. Even in such cases, as far as modelling of future conditions is concerned, a stationary model of the future is sought most frequently. A procedure that could be called "stationarization" is then necessary to adapt the past observations to the future conditions. For example, the flow data prior to the construction of the dam could be properly adapted, by deterministic modelling, so as to determine what the flow would be if the dam existed. Also, the flow data at a certain phase of urbanization could be adapted so as to represent the future conditions of urbanization. Such adaptations enable building a stationary model of the future.

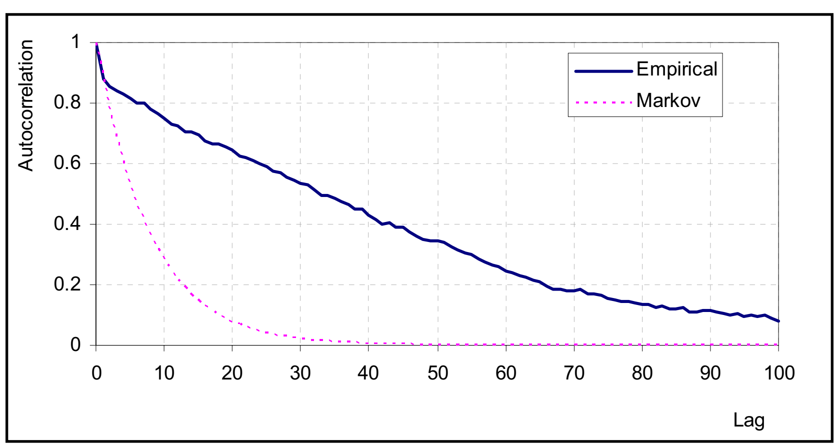

Figure 6. Empirical autocorrelogram of the time series of Figure 3 in comparison to the theoretical autocorrelogram of a Markovian process with lag one autocorrelation equal to the empirical.

Change Under Stationarity and the Hurst-Kolmogorov Dynamics

It was asserted earlier that nonstationarity is not synonymous to change. Even in the simplest stationary process, the white noise, there is change all the time. But in this case, which is characterized by independence in time, the change is only short-term. There is no change of long-term time averages. However, a process with dependence in time exhibits longer-term changes. Thus, change is tightly linked to dependence and long-term change to long-range dependence. Hence, stochastic concepts that have been devised to study dependence also help us to study change.

Here we remind of three such concepts, or stochastic tools, stressing that all are meaningful only for stationary processes (albeit this is sometimes missed). The autocorrelogram, which is a plot of the autocorrelation coefficient vs. lag time, provides a very useful characterization and visualization of dependence. Figure 6 depicts the empirical autocorrelogram estimated from the 1000 items of the time series of Figure 3. The fact that the autocorrelation is positive even for lags as high as 100 is an indication of long-range dependence. The classical Markovian dependence would give much lower autocorrelation coefficients, as also shown in Figure 6, whereas a white noise process would give zero autocorrelations, except in lag 0, which is always 1 irrespectively of the process. We recall that the process in our example involves no "memory" mechanism; it just involves change in two characteristic scales, 1 (the white noise components) and 50 (the average length of the plateaus). Thus, interpretation of long-range dependence as "long memory", despite being very common, is misleading; it is more insightful to interpret long-range dependence as long-term change (this has been first pointed out—or implied—by Klemes, 1974).

The power spectrum, which is the inverse finite Fourier transform of the autocorrelogram, is another stochastic tool for the characterization of change with respect to frequency. The power spectrum of our example is shown in Figure 7, where a rough line appears, which has an overall slope of about −1. This negative slope, which indicates the importance of variation at lower frequencies relative to the higher ones, provides a clue of long-range dependence. However, the high roughness of the power spectrum does not allow accurate estimations. A better depiction is provided in Figure 8 by the climacogram (from the Greek climax, i.e., scale), which provides a multi-scale stochastic characterization of the process. Based on the process $\underline{x}_i$ at scale 1, we define a process $\underline{x}_i^{(k)}$ at any scale $k \geq 1$ as:

$$\underline{x}_i^{(k)} := \frac{1}{k} \sum_{l=(i-1)k+1}^{ik} \underline{x}_l \qquad (1)$$

A key multi-scale characteristic is the standard deviation $\sigma^{(k)}$ of $\underline{x}_i^{(k)}$. The climacogram is a plot (typically double logarithmic) of $\sigma^{(k)}$ as a function of the scale $k \geq 1$. While the power spectrum and the autocorrelogram are related to each other through a Fourier transform, the climacogram is related to the autocorrelogram by a simpler transformation, i.e.,

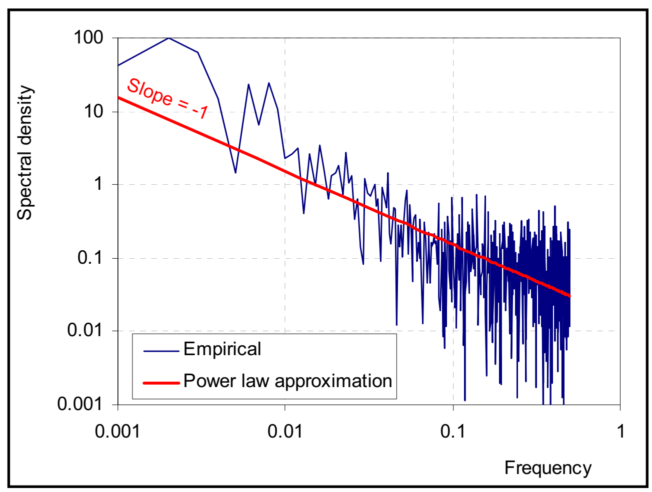

Figure 7. Empirical power spectrum of the time series of Figure 3.

$$\sigma^{(k)} = \frac{\sigma}{\sqrt{k}}\sqrt{\alpha_k}\,, \quad \alpha_k = 1 + 2\sum_{j=1}^{k-1}\left(1-\frac{j}{k}\right)\rho_j \quad \text{<-->} \quad \rho_j = \frac{j+1}{2}\alpha_{j+1} - j\alpha_j + \frac{j-1}{2}\alpha_{j-1} \quad (2)$$

To estimate the climacogram, the standard deviation $\sigma^{(k)}$ could be calculated either from the autocorrelogram by means of (2) or directly from time series $\underline{x}_i^{(k)}$ aggregated by (1). It is readily verified (actually this is the most classical statistical law) that in a white noise process, $\sigma(k) = \sigma/\sqrt{k}$, which implies a slope of −1/2 in the climacogram. Positively autocorrelated processes yield higher $\sigma^{(k)}$ and perhaps milder slopes of the climacogram. Figure 8 illustrates the constant slope of −1/2 of a white-noise process, which is also asymptotically the slope of a Markovian process, while the process of our example suggests a slope of –0.25 for scales k near 100.

Recalling that our example involves two time scales of change (1 and 50), we can imagine a process with additional time scales of change. The simplest case of such a process (which assumes theoretically infinite time scales of fluctuation, although practically, three such scales suffice; Koutsoyiannis, 2002), is the one whose climacogram has a constant slope $H - 1$, i.e.

$$\sigma^{(k)} = k^{H-1}\sigma \qquad (3)$$

This simple process, which is essentially defined by (3), has been termed the Hurst-Kolmogorov (HK) process (after Hurst, 1951, who first analyzed statistically the long-term behaviour of geophysical time series, and Kolmogorov, 1940, who, in studying turbulence, had proposed the mathematical form of the process, also known as simple scaling stochastic model or fractional Gaussian noise). The constant H is called the Hurst coefficient and in positively-dependent processes ranges between 0.5 and 1. The elementary statistical properties of the HK process are shown in Table 1, where it can be seen that all properties appear to be power laws of scale, lag and frequency.

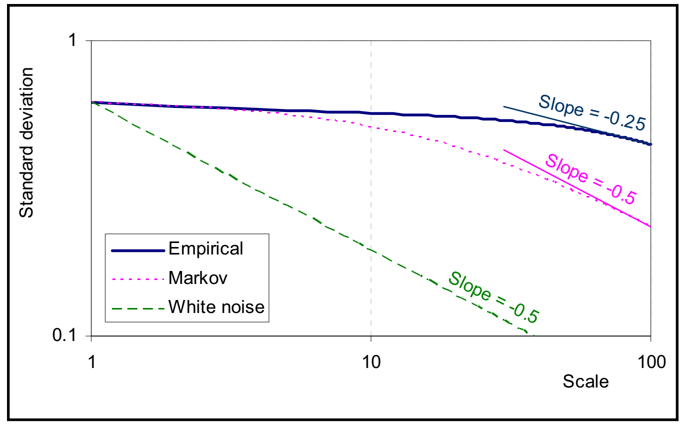

Figure 8. Empirical climacogram of the time series of Figure 3 in comparison to the theoretical climacograms of a white-noise and a Markovian process.

Figure 9. Development of turbulence in a hydraulic jump in a controlled experiment in laboratory, whose window, with the help of reader's imagination, reveals the outer uncontrolled turbulence (courtesy of Panos Papanicolaou).

Fluctuations at multiple temporal or spatial scales, which may suggest HK stochastic dynamics, are common in Nature. One characteristic example for visualization is the hydraulic jump shown in Figure 9. In this case we have molecular motion or change, as well as micro-turbulence, because the Reynolds number is high; downstream of the hydraulic jump (in the right part of the photo), we have also macro-turbulence, i.e., turbulence at larger scales. The energy associated with each scale increases with scale length (e.g., without the macro-turbulence of the hydraulic jump, the energy loss due to molecular motion and micro-turbulence would be much lower).

We owe the most characteristic example of a large spatial-scale phenomenon that exhibits HK temporal dynamics to the Nilometer time series, the longest available instrumental record. Figure 10 shows the record of the Nile minimum water level from the 7th to the 15th century AD (813 years). Comparing this Nilometer time series with synthetically generated white noise, also shown in Figure 10 (lower panel), we clearly see a big difference on the 30-year scale. The fluctuations in the real-world process are much more intense and frequent than the stable curve of the 30-year average in the white noise process. The climacogram of the Nilometer series, shown in Figure 11, suggests that the HK model is a very good

Statistical property	At scale k = 1 (e.g. annual)	At any scale k
Standard deviation	$\sigma \equiv \sigma^{(1)}$	$\sigma^{(k)} = k^{H-1}\sigma$
Autocorrelation function (for lag j)	$\rho_j \equiv \rho_j^{(1)} = \rho_j^{(k)} \approx H(2H-1)\lvert j\rvert^{2H-2}$	
Power spectrum (for frequency ω)	$s(\omega) \equiv s^{(1)}(\omega) \approx 4(1-H)\sigma^2(2\omega)^{1-2H}$	$s^{(k)}(\omega) \approx 4(1-H)\sigma^2 k^{2H-2}(2\omega)^{1-2H}$

Table 1. Elementary statistical properties of the HK process

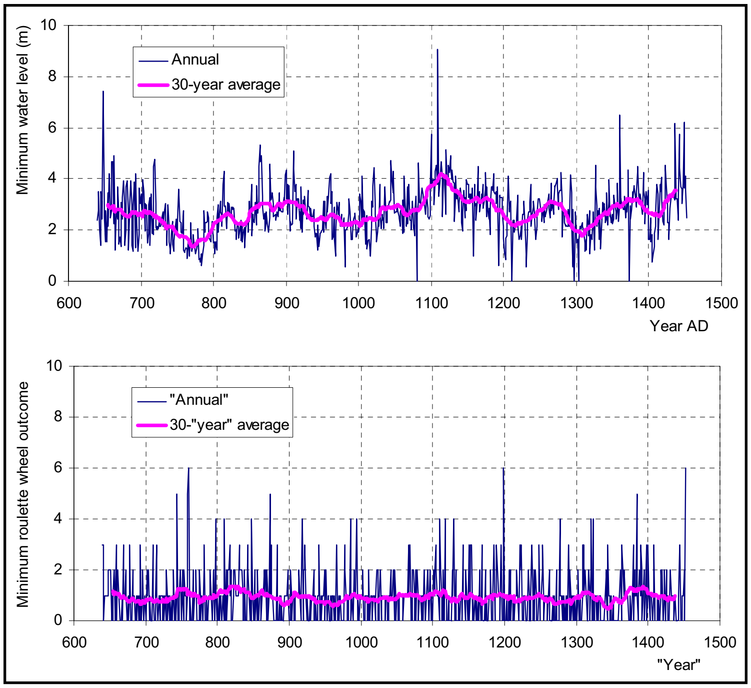

Figure 10. The annual minimum water level of the Nile River from the Nilometer (upper) and, for comparison, a synthetic series, each value of which is the minimum of 36 outcomes of a roulette wheel (lower); both time series have equal standard deviation (about 1.0).

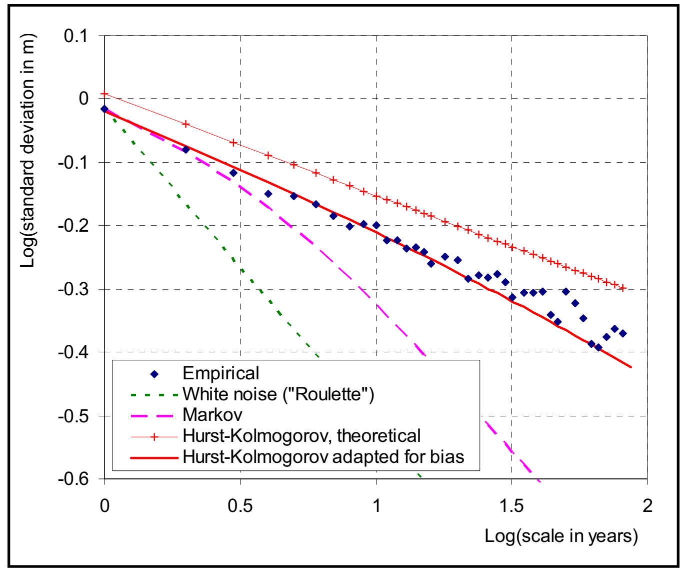

Figure 11. Climacogram of the Nilometer time series of Figure 10.

True values →	Mean, μ	Standard deviation, σ	Autocorrelation ρ_l for lag l
Standard estimator	$\underline{\bar{x}} := \frac{1}{n}\sum_{i=1}^{n} \underline{x}_i$	$\underline{s} := \sqrt{\frac{1}{n-1}}\sqrt{\sum_{i=1}^{n}(\underline{x}_i - \underline{\bar{x}})^2}$	$\underline{r}_l := \frac{1}{(n-1)\underline{s}^2} \cdot \sum_{i=1}^{n-l}(\underline{x}_i - \underline{\bar{x}})(\underline{x}_{i+l} - \underline{\bar{x}})$
Relative bias of estimation, CS	0	≈ 0	≈ 0
Relative bias of estimation, HKS	0	$\approx \sqrt{1 - \frac{1}{n'}} - 1 \approx -\frac{1}{2n'}$ (−22%) (−22%)	$\approx -\frac{1/\rho_l - 1}{n' - 1}$ (−79%)
Standard deviation of estimator, CS	$\frac{\sigma}{\sqrt{n}}$ (10%)	$\approx \frac{\sigma}{\sqrt{2(n-1)}}$ (7.1%)	
Standard deviation of estimator, HKS	$\frac{\sigma}{\sqrt{n'}}$ (63%)	$\approx \frac{\sigma\sqrt{(0.1\,n + 0.8)^{\lambda(H)}(1 - n^{2H-2})}}{\sqrt{2(n-1)}}$ where $\lambda(H) := 0.088\,(4H^2 - 1)^2$ (9.3%)	

Table 2. Impacts to statistical estimation: Hurst-Kolmogorov statistics (HKS) vs. classical statistics (CS) (sources: Koutsoyiannis, 2003; Koutsoyiannis and Montanari, 2007).

Notes (a) $n' := n^{2-2H}$ is the "equivalent" or "effective" sample size: a sample with size n' in CS results in the same uncertainty of the mean as a sample with size n in HKS; (b) the numbers in parentheses are numerical examples for n = 100, σ = 1, H = 0.90 (so that n′ = 2.5) and l = 10.

representation of reality. The Hurst coefficient is $H = 0.84$ and the same value is verified from the simultaneous record of maximum water levels and from the modern record (131 years) of the Nile flows at Aswan.

The same behaviour can be verified is several geophysical time series; examples are given in most related publications referenced herein. Two additional examples are depicted in Figure 12, which refers to the monthly lower tropospheric temperature, and in Figure 13, which refers to the monthly Atlantic Multidecadal Oscillation (AMO) index. Both examples suggest consistency with HK behaviour with a very high Hurst coefficient, $H = 0.99$.

One of the most prominent implications of the HK behaviour concerns the typical statistical estimation. The HK dynamics implies dramatically higher intervals in the estimation of location statistical parameters (e.g., mean) and highly negative bias in the estimation of dispersion parameters (e.g., standard deviation). The HK framework allows calculating the statistical measures of bias and uncertainty of statistical parameters, as summarized in Table 2, and even of future predictions (Koutsoyiannis *et al.*, 2007). It is thus striking that in most of the literature the HK behaviour is totally neglected and even studies recognizing the presence of HK dynamics usually miss to account for these implications in statistical estimation and testing.

Naturally, the implications magnify as the "intensity" of the HK behaviour increases, i.e., as H approaches 1. Table 2 provides, in addition to the theoretical formulae, a numerical example for $n = 100$ and $H = 0.90$, whereas Figure 12 and Figure 13 depict the huge bias in the standard deviation when $H = 0.99$. This bias increases with increased time scale because the sample size for higher time scales becomes smaller. Obviously, the comparison of the sample standard deviation, estimated by the classical statistical estimator, with the theoretical one of the HK model must be done after subtraction of the bias from the latter.

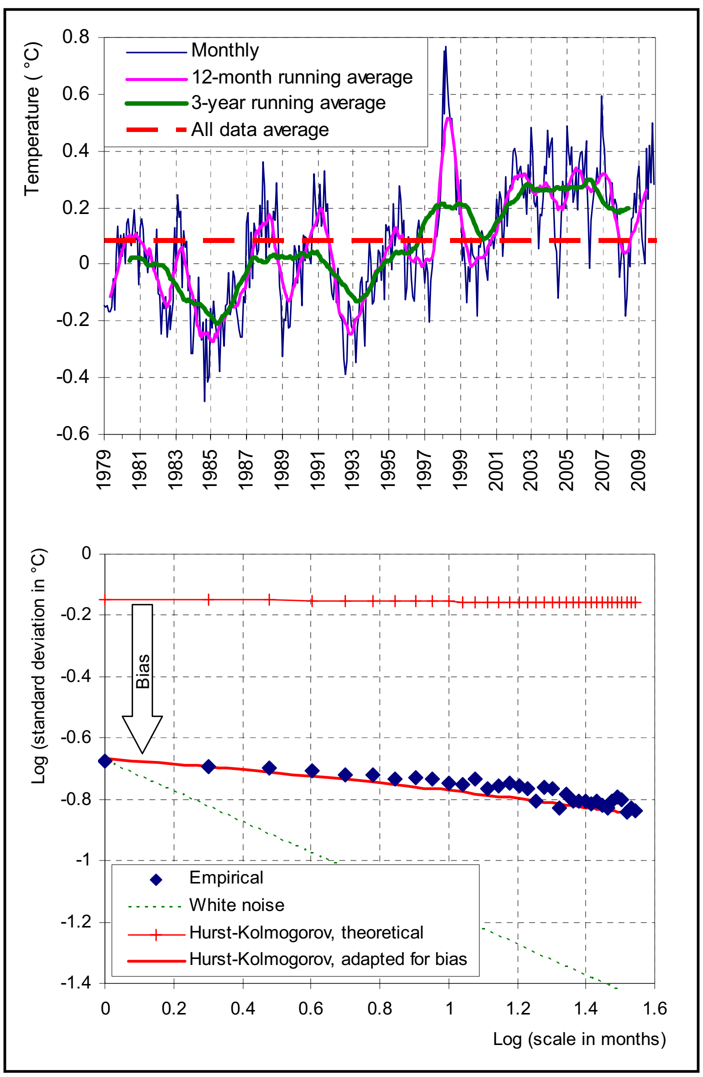

Figure 12. Monthly time series (upper) and climacogram (lower) of the global lower tropospheric temperature (data for 1979-2009, from http://vortex.nsstc.uah.edu/public/msu/t2lt/tltglhmam_5.2).

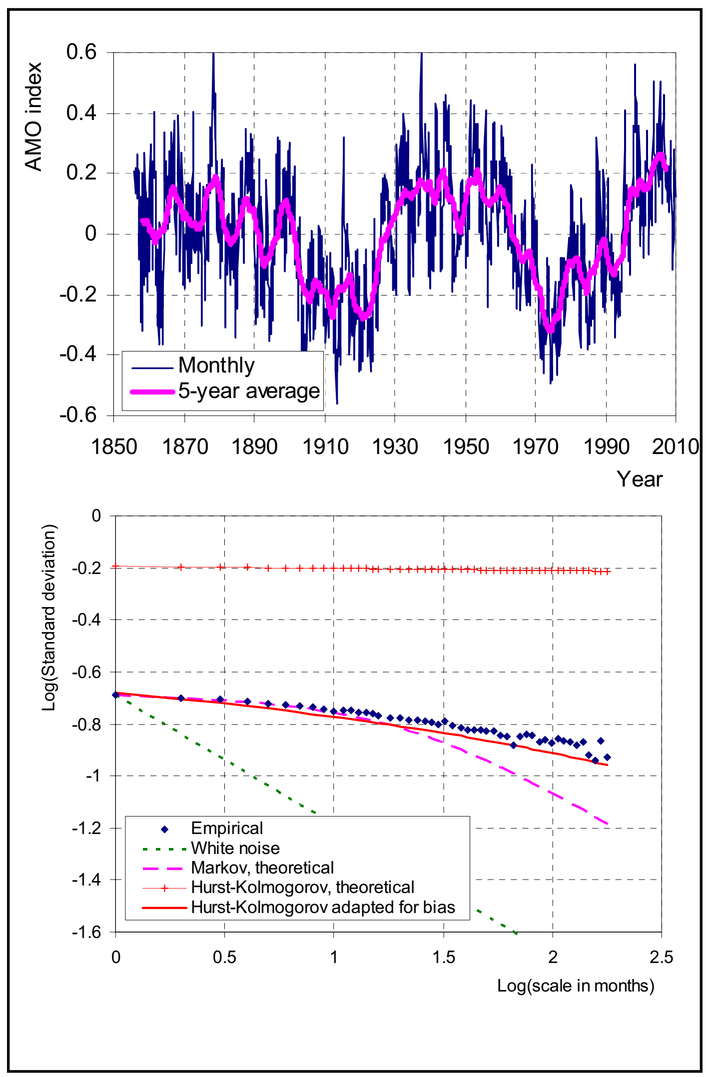

Figure 13. Monthly time series (upper) and climacogram (lower) of the Atlantic Multidecadal Oscillation (AMO) index (data for 1856-2009, from NOAA, http://www.esrl.noaa.gov/psd/data/timeseries/AMO/).

Implications in engineering design and water resources management

Coming back to the Athens water supply system, it is interesting to estimate the return period of the multi-year drought mentioned in the Introduction. Assuming that the annual runoff in the Boeoticos Kephisos basin can be approximated by a Gaussian distribution and that the multi-year standard deviation at scale (number of years) *k* is given by the classical statistical law, $\sigma^{(k)} = \sigma/\sqrt{k}$, we can easily assign a theoretical return period to the lowest (as well as to the highest) recorded value for each time scale. Figure 14 shows the assigned return periods of the lowest and highest values for time scales $k = 1$ to 10. Empirically, since the record length is about 100 years, we expect that the return period of lowest and highest values would be of the order of 100 years for all time scales. This turns out to be true for $k = 1$ to 2, but the return periods reach 10,000 years at scale $k = 5$. Furthermore, the return period of the lowest value at scale $k = 10$ (10-year drought) reaches 100,000 years!

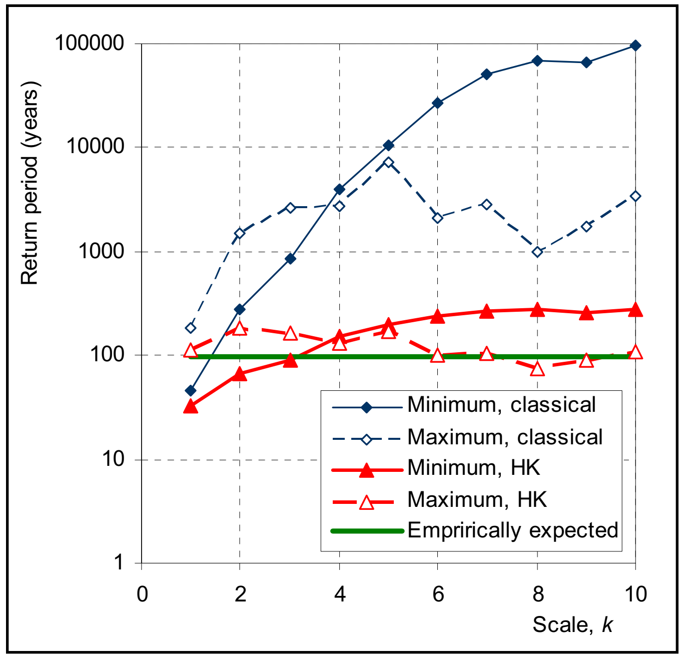

Figure 14. Return periods of the lowest and highest observed annual runoff, over time scale k = 1 to 10 years, of the Boeoticos Kephisos basin assuming normal distribution (adapted from Koutsoyiannis et al., 2007).

Is this sufficient evidence that Athens experienced a very infrequent drought event, which happens on the average once every 100,000 years, in our lifetime? In the initial phase of our involvement in this case study we were inclined to believe that we witnessed an event that extraordinary, but gradually, we understood that the answer should be negative. History is the key to the past, to the present, and to the future; and the longest available historical record is that of the Nilometer (Figure 10). This record offers a precious empirical basis of long-term changes. It suffices to compare the time series of the Beoticos Kephisos runoff (shown in its entirety in Figure 15) with that of the Nilometer series. We observe that a similar pattern had appeared in the Nile flow between 680 and 780 AD: a 100-year falling trend (which, notably, reverses after 780 AD), with a clustering of very low water level around the end of this period, between 760 and 780 AD. Such clustering of similar events was observed in several geophysical time series by Hurst (1951), who stated: "*Although in random events groups of high or low values do occur, their tendency to occur in natural events is greater. This is the main difference between natural and random events.*"

Thus, the Athens story simply tells us that we should replace the classical statistical framework with a HK framework. As shown in Figure 15 (lower panel) the Boeticos Kephisos runoff time series is consistent with the HK model, with a Hurst coefficient $H = 0.79$. Redoing the calculations of return period, we find that the return period for scale *k* reduces from the extraordinary value of 100,000 years to a humble value of 270 years. Also, the HK framework renders the observed downward trend a natural and usual behaviour (Koutsoyiannis, 2003). The Boeticos Kephisos runoff is another "naturally trendy" process (Cohn and Lins, 2005).

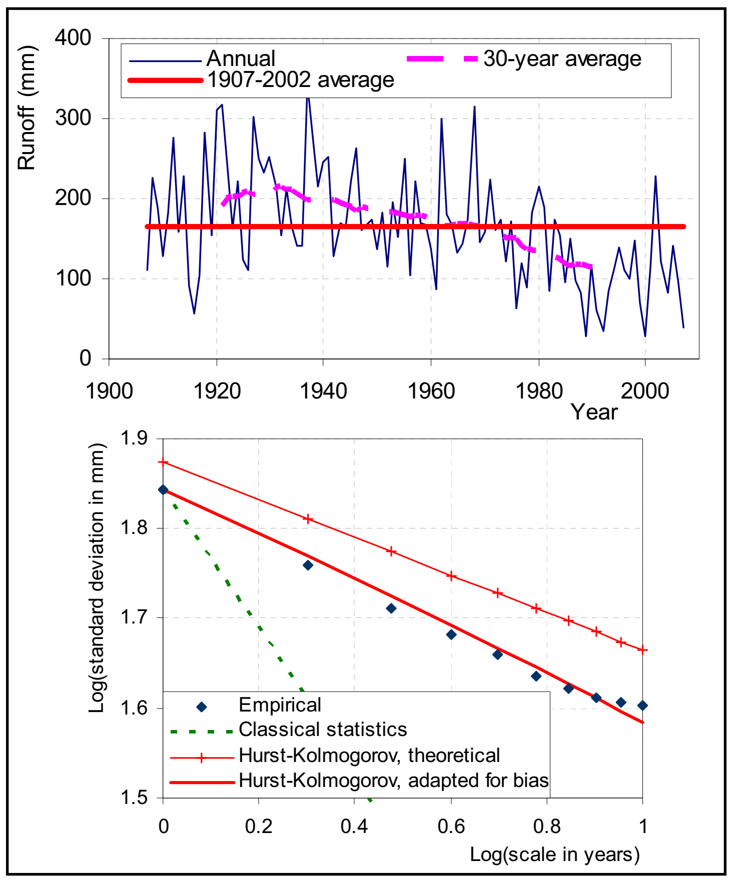

Figure 15. The entire annual time series (upper) and the climacogram (lower) of the Boeoticos Kephisos runoff.

Thus, the HK framework implies a perspective of natural phenomena that is very different from that of classical statistics, particularly in aggregate scales. This is further demonstrated in Figure 16, which depicts normal probability plots of the distribution quantiles of the Boeoticos Kephisos runoff at the annual and the climatic, 30-year, time scale. At the annual time scale (k = 1) the classical and the HK statistics yield the same point estimates of distribution quantiles (i.e. the same amount of uncertainty due to variability), but the estimation (or parameter) uncertainty, here defined by the 95% confidence limits constructed by a Monte Carlo method, is much greater according to the HK statistics. The confidence band is narrow in classical statistics (shaded area in Figure 16) and becomes much wider in the HK case. More interesting is the lower panel of Figure 16, which refers to the typical climatic time scale (k = 30). The low variability and uncertainty in the classical model is depicted as a narrow, almost horizontal, band in the lower panel of Figure 16. Here, the HK model, in addition to the higher parameter uncertainty, results in uncertainty due to variability much wider than in the classical model. As a result, while the total uncertainty (by convention defined as the difference of the upper confidence limit at probability of exceedence 97.5% minus the lower confidence limit at probability of exceedence 2.5%) is about 50% of the mean in the classical model, in the HK case it becomes about 200% of the mean, or four times larger. Interestingly, it happens that the total uncertainty of the classical model *at the annual scale* is 200% of the mean. In other words, the total uncertainty (due to natural variability and parameter estimation) at the annual level according to the classical model equals the total uncertainty at the 30-year scale according to HK model. This allows paraphrasing a common saying (which sometimes has been used to clarify the definition of climate, e.g., NOAA Climate Prediction Center, 2010) that "*climate is what we expect, weather is what we get*" in the following way: "*weather is what we get immediately, climate is what we get if you keep expecting for a long time*".

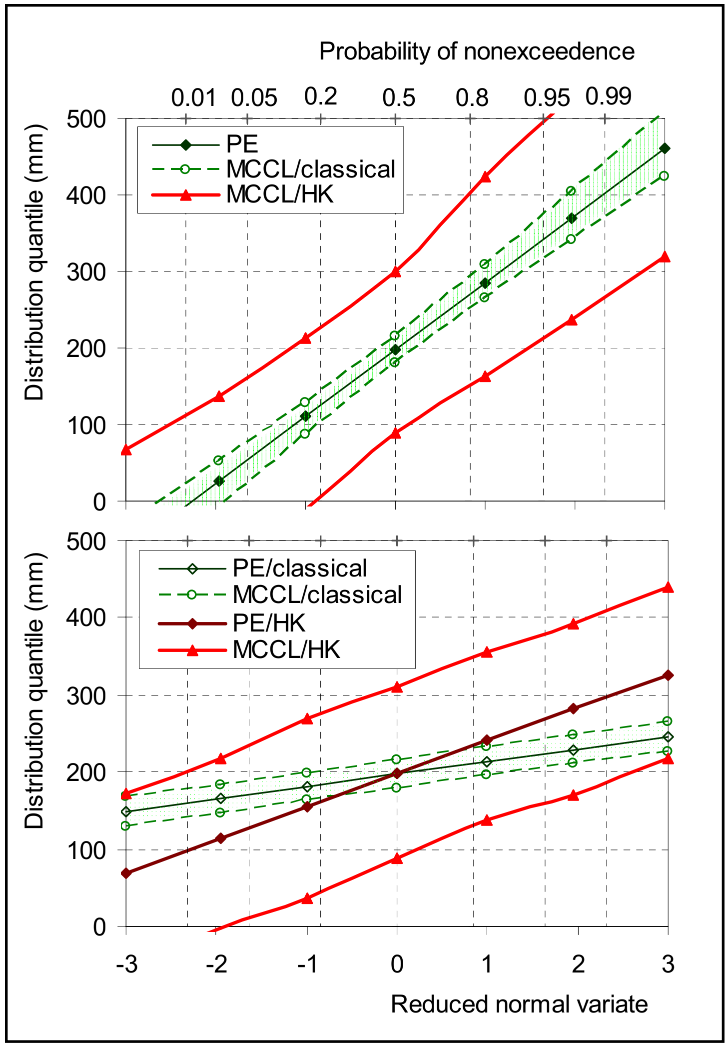

Figure 16. Point estimates (PE) and 95% Monte Carlo confidence limits (MCCL) of the distribution quantiles of the Boeoticos Kephisos runoff at the annual (upper) and climatic (30-year; lower) time scales, both for classical and HK statistics (adapted from Koutsoyiannis et al., 2007).

For reasons that should be obvious from the above discussion, the current planning and management of the Athens water supply system are based on the HK framework. Appropriate multivariate stochastic simulation methods have been developed (Koutsoyiannis, 2000, 2001) that are implemented within a general methodological framework termed parameterization-simulation-optimization (Nalbantis and Koutsoyiannis, 1997; Koutsoyiannis and Economou, 2003; Koutsoyiannis *et al.*, 2002, 2003; Efstratiadis *et al.*, 2004). The whole framework assumes stationarity, but simulations always use the current initial conditions (in particular, the current reservoir storages) and the recorded past conditions: apparently, in a Markovian framework, only the latest observations affect the future probabilities, but in the non-Markovian HK framework the entire record of past observations should be taken into account to condition the simulations of future (Koutsoyiannis, 2000).

Nonetheless, it is interesting to discuss two alternative methods that are more commonly used than the methodology developed for Athens. The first alternative approach, which is nonstationary, consists of the projection of the observed "trend" into the future. As shown in Figure 17, according to this approach the flow would disappear by 2050. Also this approach would lead to reduced uncertainty (because it assumes that the observed "trend" explains part of variability); the initial standard deviation of 70 mm would decrease to 55 mm. Both these implications are glaringly absurd.

The second alternative, again admitting nonstationarity, is to use outputs of climate models and to feed them in hydrological models to predict the future runoff. This approach is illustrated in Figure 18, also in comparison to the HK stationary approach and the classical statistical approach. Outputs from three different GCMs (ECHAM4/OPYC3, CGCM2, HadCM3), each one for two different scenarios, were used, thus shaping 6 combinations shown in the legend of Figure 18 (each line of which corresponds to each of the three models in the order shown above; see more details in Koutsoyiannis *et al.*, 2007). To smooth out the annual variability, the depictions of Figure 18 refer to the climatic (30-year) scale. In fact, outputs of the climate models exhibited huge departures from reality (highly negative efficiencies at the annual time scale and above); thus, adjustments, also known as "statistical downscaling", were performed to make them match the most recent observed climatic value (30-year average). Figure 18 shows plots of the GCM-based time series after the adjustments. For the past, despite adjustments, the proximity of models with reality is not satisfactory (they do not capture the falling trend, except one part reflecting the more intense water resources exploitation in recent years). Even worse, the future runoff obtained by adapted GCM outputs is too stable. All different model trajectories are crowded close to the most recent climatic value. Should one attempt to estimate future uncertainty by enveloping the different model trajectories, this uncertainty would be lower even from that produced by the classical statistical model. Hence, the GCM-based approach is too risky, as it predicts a future that is too stable, whereas the more consistent HK framework entails a high future uncertainty (due to natural variability and unknown parameters), which is also shown in Figure 18. The planning and management of the Athens water supply system is based on the latter uncertainty.

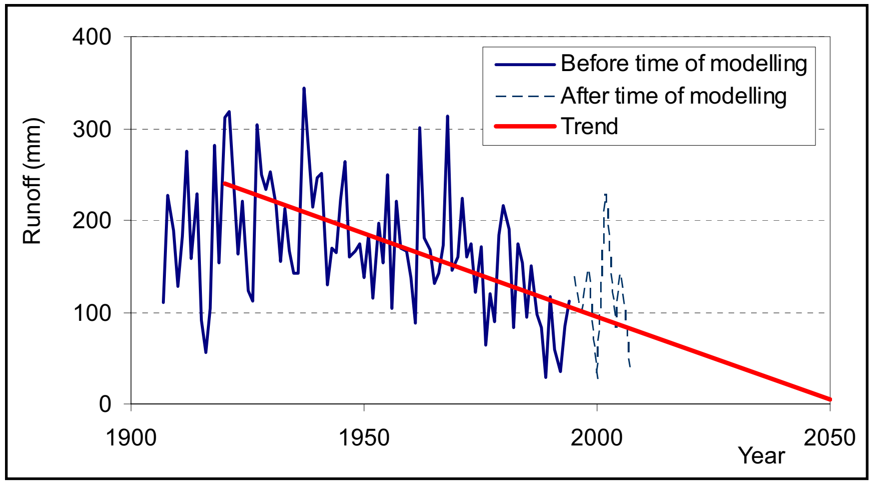

Figure 17. Illustration of the alternative method of trend projection into the future for modelling of the Boeoticos Kephisos runoff.

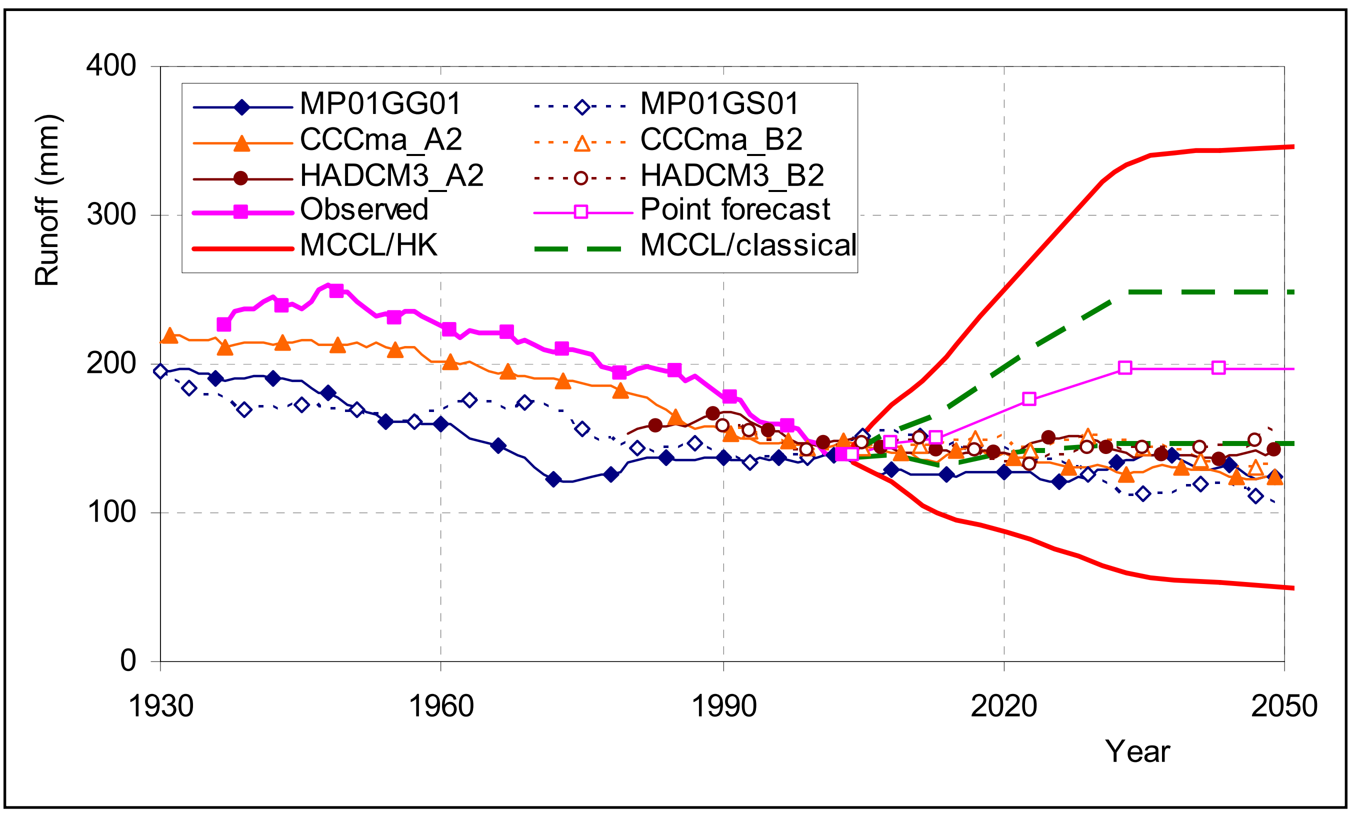

Figure 18. Illustration the alternative GCM-based method for modelling of the Boeoticos Kephisos runoff, vs. the uncertainty limits (Monte Carlo Confidence Limits—MCCL) estimated for classical and HK statistics; runoff is given at climatic scale, i.e. runoff y at year x is the average runoff of a 30-year period ending at year x (adapted from Koutsoyiannis et al., 2007).

Additional Remarks

While this exposition has focused on climatic averages and low extremes (droughts), it may be useful to note that change, which underlies the HK dynamics, also affects high extremes such as intense storms and floods. This concerns both the marginal distribution tail as well as the timing of high intensity events. For example, Koutsoyiannis (2004) has shown that an exponential distribution tail of rainfall may shift to a power tail if the scale parameter of the former distribution changes in time; and it is well known that a power tail yields much higher rainfall amounts in comparison to an exponential tail for high return periods. Also, Blöschl and Montanari (2010) demonstrated that five of the six largest floods of the Danube at Vienna (100 000 km^2 catchment area) of the 19th century were grouped in its last two decades. This is consistent with Hurst's observation about grouping of similar events and should properly be taken into account in flood management—rather than trying to speculate about human-induced climate effects. (Interestingly, Blöschl and Montanari, by plotting the 19th century peak flows in a separate graph so that the grouping appear as if it indeed were in "the last two decades", try to tease the recent "trend" to regard the most recent hydroclimatic phenomena as extraordinary and human induced).

Overall, the "new" HK approach exposed herein is as old as Kolmogorov's (1940) and Hurst's (1951) expositions. It is stationary (not nonstationary) and demonstrates how stationarity can coexist with change at all time scales. It is linear (not nonlinear) thus emphasizing the fact that stochastic dynamics need not be nonlinear to produce realistic trajectories (while, in contrast, trajectories from linear deterministic dynamics are not representative of the evolution of complex natural systems). The HK approach is simple, parsimonious, and inexpensive (not complicated, inflationary and expensive) and is honest (not deceitful) because it does not hide uncertainty and it does not pretend to predict the distant future deterministically.

Conclusions

- Change is nature's style.
- Change occurs at all time scales.
- Change is not nonstationarity.
- Hurst-Kolmogorov dynamics is the key to perceive multi-scale change and model the implied uncertainty and risk.
- In general, the Hurst-Kolmogorov approach can incorporate deterministic descriptions of future changes, if available.
- In the absence of credible predictions of the future, Hurst-Kolmogorov dynamics admits stationarity.

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Biography

Demetris Koutsoyiannis received his diploma in Civil Engineering from the National Technical University of Athens (NTUA) in 1978 and his doctorate from NTUA in 1988. Currently, he is professor of the NTUA in Hydrology and Analysis of Hydrosystems; also professor of Hydraulics in the Hellenic Army's Postgraduate School of Technical Education of Officers Engineers; Editor of Hydrological Sciences Journal; member of the editorial board of Hydrology and Earth System Sciences, and formerly of Journal of Hydrology and Water Resources Research; and Chair of the Sub-Division on Precipitation & Climate of the Division on Hydrological Sciences of the European Geosciences Union (EGU). He received the Henry Darcy Medal in 2009 by EGU for his outstanding contributions to the study of hydrometeorological variability and to water resources management. He teaches undergraduate and postgraduate courses in hydrometeorology, hydrology, hydraulics, hydraulic works, water resource systems, water resource management, and stochastic modelling. He is an experienced researcher in the areas of hydrological modelling, hydrological stochastics, climate stochastics, analysis of hydrosystems, water resources engineering and management, hydroinformatics, and ancient hydraulic technologies. He has participated in over 40 research projects and 60 engineering studies as a consultant. His record includes more than 500 scientific and technological contributions (research articles, books and educational notes, conference and workshop talks, research reports, engineering studies and miscellaneous publications), among which 74 publications in peer reviewed journals. (More information: http://www.itia.ntua.gr/dk/)

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