1. Abstract
A three-dimensional (3D) stochastic simulation model is presented, which is a direct extension of the 1D simple scaling process (fractional Gaussian noise). The 3D process can generate time-varying 2D rainfall fields through a rather simple procedure, as well as other time-varying 2D spatial geophysical fields, consistent with the observed 2D long-term spatial persistence over time (3D slowly decaying autocorrelation over scale). Moreover, the differences between 1D (generating rainfall time series at a point), 2D (generating rainfall fields for specific time steps) and 3D (generating spatio-temporal rainfall fields) scaling processes are also being investigated through some applications based on observed rainfall fields.

2. Hurst phenomenon and the HK process
"High tendency of high/lower values to occur in natural events." Hurst (1951)  
- Slowly decaying autocorrelation over scale  
- Power-law behavior

3. Hurst coefficient (H) of the HK process
HK process depends on the characteristic parameter 0<1. Here, the estimation of the H coefficient is done via the minimization of the square error (SSE) of the empirical (S) and true (S-h) variance over scale k of the process. A method of Koutsoyiannis (2003) for the estimation of H was extended to the D-dimensional process.

4. Field Normalization
HK process generates random fields that follow the NO distribution. Here, the following transformation (Papalexiou et al., 2007) is used, where its coefficients  are estimated through minimizing the square error of the model and NO (3D) distribution function.

5. Autocorrelation function of the HK process
The autocorrelation function φ of the HK process is independent of the aggregated scale. For any displacement vector r (lag) is expressed by a power-law function. The latter can be integrated over a discrete space of points  and  so that the discrete spatio-temporal ac can be estimated. This integration has a analytical solution for D=1, Koutsoyiannis et al. (2000) proposed an approximated solution for D=2. Here, this solution is extended for D=3 (refers to KAS).

6. Simulation scheme for generating HK process
SMA stands for Symmetric Moving Average and it can be used to generate a stochastic process with any structure of autocorrelation or power spectrum (Koutsoyiannis, 2000). Here, the SMA scheme has been extended to three-spatio-temporal dimensions (direct extension from 1D and 2D schemes).

7. Spectral density and α4 coefficients of SMA
The spectral density F of the stochastic field can be determined via the Fourier transform of the discrete form of autocovariance γ. It can be shown that the Fourier transform of the field α4 is related to F (for p=q), thus the α4 field can then be estimated.

8. Case study on observed rainfall fields
The sample presented is based on an observed rainfall field in the India Ocean SE of India (coordinates: 30°N, 70°E). The data were acquired from TRMM NASA satellite system (available on-line).

9. Spatial - temporal resolution correspondence
The spatial resolution is estimated to be 25% of the initial unit (25 km), so that the two acts decay at the same rate (right figure). Thus the spatial grid is changed to 3x31 points (more coarse).

10. Normalization of rainfall field
As described in section 4, the transformation by Papalexiou et al. (2007) is used to normalize the observed rainfall field. The zero values of the natural field are replaced with the small value of log-5.

11. Stochastic simulation model (figures)

12. Stochastic simulation model (figures)

13. Conclusions
- A three-dimensional (3D) stochastic simulation model is proposed, which is a direct extension of the 1D simple scaling process (HK or FCN), that can generate time-varying 3D spatio-temporal fields consistent with 2D long-term spatial persistence over time (3D slowly decaying autocorrelation over scale).
- All of the framework used at the HK and SMA process is extended for any other dimension D of the field. The autocorrelation function over spatio-temporal displacement can be adequately approximated with the extended Koutsoyiannis et al. (2010) solution for D=3.
- The 3D model is verified through an application based on an observed rainfall field.